TITLE: **PPKBZ9: Symbolic–Numeric Model in Artificial Satellite Theory**

AUTHOR: Juan F. San-Juan

AFFILIATION: Dept. Matemáticas y Computación. Universidad de La Rioja

MAIL ADDRESS:
Universidad de la Rioja. Edificio Vives
C/ Luis de Ulloa s/n.
260004 Logroño (La Rioja), SPAIN

E-MAIL: `juanfelix.sanjuan@unirioja.es`

Abstract

Analytical orbit propagators based on Brouwer-Lyddane theory [4, 10] are frequenly used for orbit determination and orbit prediction. For example, SGP4, SDP4 or PPT3 are some of these orbit propagators. These programs requiere mean orbital elements for the initial position of the satellite.

In this work we present a closed second order analytical theory of an artificial satellite around an Earth-like planet and the corresponding orbit propagator derived from this theory. Our Orbit propagator uses osculating orbital elements for the initial position of the satellite and also produces osculating orbital elements as output.

In polar-nodal variables $(r, \theta, \nu, R, \Theta, N)$, the Hamiltonian for a satellite perturbed by the first eight zonal harmonics coefficients of the gravity potential of an Earth-like planet is given by

$$\mathcal{H} = \frac{1}{2}\left(R^2 + \frac{\Theta^2}{r^2}\right) - \frac{\mu}{r} + \frac{\mu}{r}\sum_{n\geq 2}^{9} J_n \left(\frac{\alpha}{r}\right)^n P_n(\sin i\, \sin\theta),$$

where P_n is the Legendre polynomial of degree n, μ is the gravitational constant, α is the equatorial radius of the planet, and J_n are the zonal harmonic coefficients. Remark that the cualitative features of this Hamiltonian have been described in [5].

We first reduce the problem to one degree of freedom. By combining two Lie transformations, the elimination of the Parallax [6] and the elimination of the Perigee [3], we remove the long period terms, due to the argument of the perigee, to the transformed Hamiltonian $\mathcal{H}''$, while the short period terms, due to mean anomaly, still remain in $\mathcal{H}''$. These two Lie transformations are developed in close form of the eccentricity and lead us to an integrable problem in the variables (r'', R''). Traditionally to complete the theory and obtain the mean elements like Brouwer, a further reduction is done in Delaunay variables (l, g, h, L, G, H) by the Delaunay normalization [7] that averages the problem over the mean anomaly.

In this work instead of applying the Delaunay normalization, we replace the time and the variable r'' by the perturbed true anomaly and the inverse of r'' in the Hamiltonian $\mathcal{H}''$, respectively. Then the equations of motion become one-dimensional perturbed harmonic oscillator and the Krylov-Bogoliubov-Mitropolsky [8] method is used to integrate it. Except for the critical inclination, this theory is valid for small eccentricities and inclinations.

The analytical expressions and the orbit propagator program, coded in lenguage C, were performed by the symbolic-numeric environment *MathATESAT*. Previous version of this software has been used to created analytical theories and orbit propagators program in the cases of the Earth orbiter [11], Mars orbiter [14], JIMO mission [9] and Moon orbiter [2, 15].

Finally, prediction accuracy given by our orbit propagator program, PPKBZ9, was investigated by using more than 30 data of different types of orbits around the Earth and Mars: Low Earth Orbit (LEO), Near Circular Orbit (NCO), Highly Eccentric Orbit (HEO) and Low Mars Orbit (LMO). The results of the PPKBZ9 orbit propagator have been compared with a 8th order Runge-Kutta method. We show in Figures 1 the along-track, across-track and radial error for 1-day propagation for the four types of orbit consider in this work. Moreover, Table 1 contains a summary of the some characteristics and the maximun along-track error for each type of Earth orbits. All comparisons were performed for a 30-day propagation period.

Table 1: Summary of the some characteristics and the maximun along-track error for Earth orbits.

Type	eccentricity	Perigee height (km)	Along-track error (m)
LEO	$e < 0.2$	below 450	below 230
NCO	$e < 0.015$	from 700 to 1500	below 140
HEO	$e > 0.2$	below 420	below100

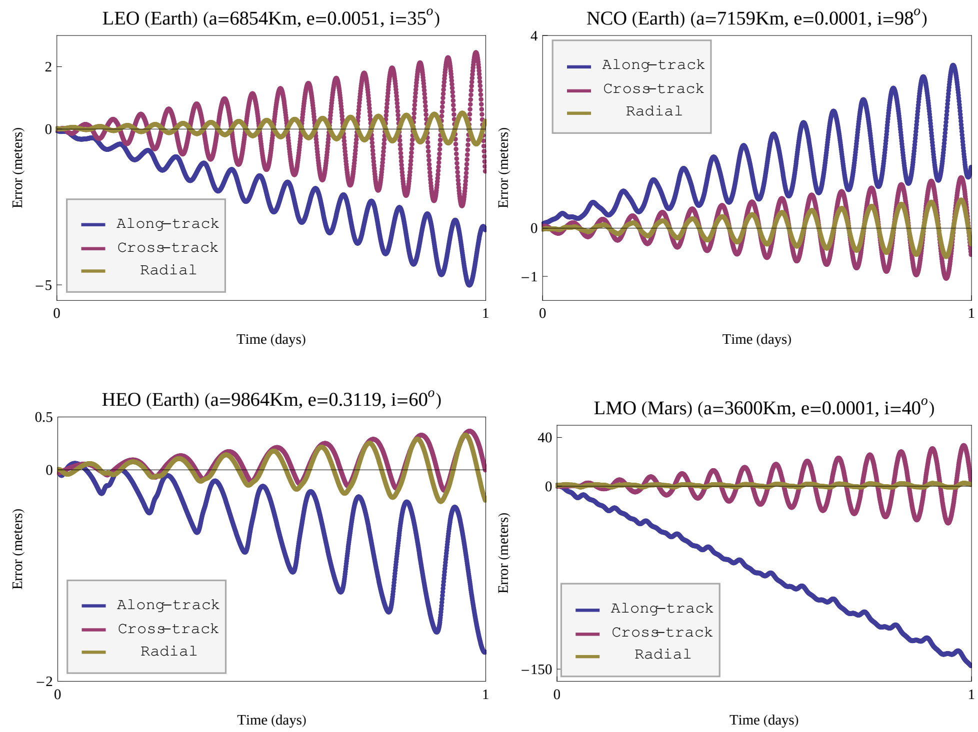

Figure 1: Along-track, cross-track and radial error in predicted satellite position.

References

[1] Abad, A., San Juan, J.F. and Gavín, A: 2001, Short term evolution of artificial satellites. *Celestial Mechanics & Dynamical Astronomy* **79**: 277–296.

[2] Abad, A., Elipe, A., San Juan, J.F., San Martín, M.:2007, An analytical Model for a Lunar Orbiter, *Acta Academiae Aboensis*, Ser. B, **67** (2): 134–143.

[3] Alfriend, K. T. and Coffey, S. L.: 1984, Elimination of the Perigee in Satellite Problem. *Celestial Mechanics*, **32**: 163–172.

[4] Brouwer, D.: 1959, Solution of the Problem of Artificial Satellite Theory Without Drag. *Astron. J.*, **64**: 379–397.

[5] Coffey, S. L., Deprit, A. and Deprit, E.: 1994, Frozen orbits for satellites close to an Earth-like planet. *Celestial Mechanics & Dynamical Astronomy*, **59**: 37–72.

[6] Deprit, A.: 1981, The Elimination of the Parallax in Satellite Theory. *Celestial Mechanics*, **24**: 111-153.

[7] Deprit, A.:1982, Delaunay Normalizations. *Celestial Mechanics*, **26**: 9–21.

[8] Krylov, N. and Bogoliubov, N. N.: 1947, Introduction to Nonlinear Mechanics. Princeton University Press, Princeton N.Y.

[9] Lara, M. and San Juan, J. F.: 2005, On the dynamical behavior of an orbiter around Europa, *Journal of Guiance, Control and Dynamics*, **28** (2): 291–297.

[10] Lyddane, R. N.: 1963, Small eccentricities or inclinations in Brouwer theory of the artificial satellite. *Astron. J.*, **68**: 555–558.

[11] Palacián, J., San Juan, J. F. and Yanguas, P.: 1997, Analytical Theory for the Spot Satellite, *Advances in the Astronautical Sciences*, **95**(Part I), 375–382.

[12] San Juan, J. F.: 1994, ATESAT: Automatization of theories and ephemeris in the artificial satellite problem, Tech. rep. CT/TI/MS/MN/94-250, CNES, France.

[13] San Juan, J. F.: 1996, Manipulación algebraica de series de Poisson. Aplicación a la teoría del satélite artificial, Ph.D. thesis, University of Zaragoza.

[14] San Juan, J. F. and Serrano, S.: 2000, Application of the Z6PPKB ATESAT–model to compute the orbit of an artificial satellite around Mars, Tech. rep. DTS/MPI/MS/MN/2000-057. CNES, Francia.

[15] San Juan, J. F., Abad, A., Elipe, A. and Tresaco, E.: 2008, “Analytical theory for Moon Orbiter.” AAS/AIAA Space Flight Mechanics Meeting. Galveston, Texas, January 27–30, 2008. Paper AAS 08-184.

[16] San Juan, J. F.: 2008, *MathATESAT*: A symbolic-numeric environment in Astrodynamic, *in preparation*.