

The study of Markov processes on 3D Schur graph

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The three-dimensional Schur graph is an infinite graded graph whose vertices are three-dimensional strict Young diagrams (strict planar partitions). Young graph and Schur graph are related to various problems of asymptotic combinatorics. Some of these problems show the connection between the combinatorics of these graphs and special Markov processes on them. From this point of view, the most important Markov processes are those which generate a central measure [1].

A central measure is a measure where the probabilities of different paths between given pair of diagrams are the same. For two-dimensional case there exists a central process called Plancherel process. Papers [2, 3, 4] were devoted to investigation of sequences produced by Plancherel process on two-dimensional Young and Schur graphs. Unfortunately, there are no known central processes on three-dimensional Young and Schur graphs. Markov processes on three-dimensional Young graph which generate asymptotically central measure were investigated in [5, 6]. These are so-called pseudo-Plancherel processes.

Here we construct an analogous process on three-dimensional Schur graph. In order to show the asymptotic centrality, we study the ratios of probabilities of different paths between a pair of diagrams. We define the normalized dimension for three-dimensional strict Young diagrams. We investigate both random and greedy paths for pseudo-Plancherel processes on Schur graph. A greedy path is a deterministic sequence of diagrams built in the following way: on each step the box with the maximum possible probability is added to the diagram. Also we investigate the growth and oscillations of normalized dimensions along greedy trajectories of processes. We study the limit shape of a strict three-dimensional diagram produced by pseudo-Plancherel process.

References

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