Symmetries in $\mathcal{2HDM}$, $\mathcal{CP}$ violation and heavy Higgs effects

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Based on papers with

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Outline

- **Lagrangian of $\mathcal{2HDM}$**
Form invariance and rephasing invariance
- **Z_2 symmetry and its violation**
- **Explicit $\mathcal{CP}$ violation description**
- **Heavy Higgs bosons with decoupling and without it**
- **Natural parameters range**

The simplest extension of the $\mathcal{SM}$ —
a Two Higgs Doublet Model ($2\mathcal{HDM}$):

$\mathcal{L} = \mathcal{L}^{SM}_{gf} + \mathcal{L}_H + \mathcal{L}_Y$;

$\mathcal{L}^{SM}_{gf}$ – $\mathcal{SM}$ interaction, gauge bosons + fermions

$\mathcal{L}_H \equiv T - V$ – Higgs lagrangian,

T – Higgs kinetic term, V – Higgs potential,

$\mathcal{L}_Y$ – Yukawa interaction of fermions to scalars.

$$T = (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + (D_\mu\phi_2)^\dagger(D^\mu\phi_2)$$
$$+\varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2) + \varkappa^*(D_\mu\phi_2)^\dagger(D^\mu\phi_1)\,,$$
$$V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)$$
$$+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + h.c.\right]$$
$$+\left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + h.c.\right\} + \mathcal{M}(\phi_i)$$
$$\mathcal{M}(\phi_i) = -\frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2)\right.$$
$$\left.+\left[m_{12}^2(\phi_1^\dagger\phi_2) + h.c.\right]\right\}\,.$$

λ_{5-7}, $\varkappa$ and m_{12} are generally complex.

1. Overall phase freedom

$\mathcal{L}_H$ is invariant under the global phase transformation $\phi_i \to \phi_i e^{-i\rho_0}$.

2. Reparameterization invariance

in the space of Lagrangians with coordinates

λ_i, m_{ij}^2, $\varkappa$:

The physical reality corresponding to a particular choice of Lagrangian does not change with the change of Lagrangian

under the global transformation

$$\mathcal{F}\ :$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = e^{-i\rho_0} \begin{pmatrix} \cos\theta\, e^{i\rho'} & \sin\theta\, e^{i\tau} \\ -\sin\theta\, e^{-i\tau} & \cos\theta\, e^{-i\rho'} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

accompanied by compensating transformation of λ_i, m_{ij}, $\varkappa$ and renormalization of fields η_i.

It is governed by 3 angles θ, ρ', τ similar to Euler angles.

Particular case

at $\theta = 0$:

3. Rephasing invariance

under the global rephasing transformation

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad (i = 1, 2),$$

$$\rho_0 = (\rho_1 + \rho_2)/2, \quad \rho = \rho_2 - \rho_1 (\equiv 2\rho'),$$

accompanied by transformation

$$\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{ii}^2 \to m_{ii}^2\,, \quad m_{12}^2 \to m_{12}^2 e^{i\rho}$$

$$\lambda_5 \to \lambda_5\, e^{2i\rho}\,, \quad \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,, \quad \varkappa \to \varkappa\, e^{i\rho}\,.$$

ρ – rephasing gauge parameter,

(ρ_0 – overall phase parameter.)

Some choice of ρ – rephasing representation.

This invariance is extended to the description of a whole system of scalars and fermions by adding of similar transformations for the phases of fermion fields and Yukawa couplings.

The z_2 symmetry and its violations

The $2\mathcal{HDM}$ generally give a $\not{C}$Pat $\mathcal{EWSB}$.
In the most general form of $\mathcal{L}_Y$ large $\mathcal{FCNC}$ effects become possible.
Experiment: $\not{C}$P and $\mathcal{FCNC}$ effects are weak.

$$\Downarrow$$

The natural construction of $2\mathcal{HDM}$ should start with the lagrangian having additional symmetry which forbids a $\not{C}$P and $\mathcal{FCNC}$ effects.

$$\Downarrow$$

That is $\boldsymbol{Z_2}$ symmetry under independent transformations for both fields

$$\begin{array}{c} \boldsymbol{\phi_1 \to -\phi_1,\ \phi_2 \to \phi_2,} \\ \boldsymbol{\phi_1 \to \phi_1,\ \phi_2 \to -\phi_2,} \end{array} ,$$

which forbids (ϕ_1, ϕ_2) mixing.
This symmetry can be weakly broken to open door for weak $\not{C}$P and $\mathcal{FCNC}$ effects.

Z_2 conserving case: $m_{12} = \lambda_6 = \lambda_7 = \varkappa = 0$. Soft violation of Z_2: dim. 2 operator with m_{12} (retained unmixed ϕ_i fields at small distances). Hard violation of Z_2: + dim. 4 operators with λ_6, λ_7, $\varkappa$ – looks unnatural since (ϕ_1, ϕ_2) mixing retains at small distances.

Hard violation of Z_2

1) The (ϕ_1, ϕ_2) mixing retains at small distances – very unnatural .

2) The mixed kinetic terms (with $\varkappa$, $\varkappa^*$) can be removed by the nonunitary transformation:

$$(\phi_1', \phi_2') \to \left(\frac{\sqrt{\varkappa^*}\phi_1 + \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1+|\varkappa|)}} \pm \frac{\sqrt{\varkappa^*}\phi_1 - \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1-|\varkappa|)}}\right). \quad \blacklozenge$$

Starting from the case $\varkappa = 0$, $\lambda_{6,7} \neq 0$, the renormalization of quadratically divergent, non-diagonal two-point functions leads to $\varkappa \neq 0 \Rightarrow$ λ_6, λ_7, $\varkappa$ are running $\Rightarrow$ all of these terms should be included in Lagrangian on the same footing $\Rightarrow$ the treatment of the hard violation of Z_2 symmetry without $\varkappa$ terms (as in most of papers considering this "most general $2\mathcal{HDM}$ potential") is inconsistent.

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The diagonalization $\blacklozenge$ destroy relatively simple relations for the masses of the Higgs bosons, usually written.

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We present relations for a case of hard violation of Z_2 symmetry at $\varkappa = 0$ keeping in mind that the loop corrections can change results significantly.

Hidden Z_2 symmetry with its soft violation:

Reparameterization transformation

$\mathcal{F}(\mathcal{L}_{\text{softly broken } Z_2 \text{ symmetry}})$

$\Downarrow$

the potential with λ_6, $\lambda_7 \neq 0$, mixed kinetic term $\varkappa = 0$ —

Lagrangian of hidden Z_2 symmetry with possible soft violation ($hidZ_2s$)

This case mimic the case of hard violation of Z_2 symmetry but with constraints. $\Rightarrow$ Total number of parameters of general potential is 14 (except $\varkappa$). Total number of independent parameters in $hidZ_2s$ is 13: 10 in the initial potential + θ, ρ', τ.

$\Downarrow$

The case with hard Z_2 symmetry contains 1 additional parameter in potential as compare $hidZ_2s$ + 2 parameters $Re\varkappa$, $Im\varkappa$ $\Rightarrow$ $\varkappa$ cannot be eliminated.

Transformation to the observable Higgs fields h_i, etc. gives terms like $\lambda_{6,7}$ in the obtained potential. The correlations between quartic couplings in the case of soft Z_2 symmetry (or in its hidden form) prevent running mixing between fields ϕ_i at small distances.

The minimum of the potential

defines the v.e.v.'s $\langle\phi_i\rangle$ via

$$\frac{\partial V}{\partial \phi_i}\left(\phi_1 = \langle\phi_1\rangle,\ \ \phi_2 = \langle\phi_2\rangle\right) = 0$$

with $\quad \langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix};$

$$v_1 = v\cos\beta\,,\ \ v_2 = v\sin\beta\,,\ \ \beta \in (0,\, \pi/2).$$

The $\mathcal{SM}$ constraint $v = \left(G_F\sqrt{2}\right)^{-1/2} = 246$ GeV.

At the rephasing transformation $\boxed{\xi \to \xi - \rho}$

$\Downarrow$

Rephasing invariant quantities

$$\overline{\lambda}_{1-4} = \lambda_{1-4}, \quad \overline{\lambda}_5 \equiv \lambda_5 e^{2i\xi}, \quad \overline{\lambda}_{6,7} \equiv \lambda_6 e^{i\xi},$$
$$\overline{\varkappa} \equiv \varkappa e^{i\xi}, \quad \overline{m}_{12}^2 \equiv m_{12}^2 e^{i\xi}.$$

$$\overline{\lambda}_{345} = \lambda_3 + \lambda_4 + Re(\overline{\lambda}_5), \; \overline{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 + \frac{v_2}{v_1}\overline{\lambda}_7,$$
$$\tilde{\lambda}_{67} = \frac{1}{2}\left(\frac{v_1}{v_2}\overline{\lambda}_6 - \frac{v_2}{v_1}\overline{\lambda}_7\right),$$

The minimum condition

allow to express m_{ii}^2 via $\overline{\lambda}_i$, v_j and parameter $\nu = Re(\overline{m}_{12}^2)/(2v_1v_2)$,

gives no limitation for quantity ν,

imaginary part $\delta = Im(\overline{m}_{12}^2)/(2v_1v_2)$ is expressed via $Im(\overline{\lambda}_{5-7})$:

$$\overline{m}_{12}^2 = 2v_1v_2(\nu + i\delta),$$
$$\delta = Im\Big(\underbrace{0}_{Z_2\ sym} + \underbrace{\overline{\lambda}_5/2}_{soft} + \underbrace{\overline{\lambda}_{67}/2}_{hard}\Big).$$

We mainly use zero rephasing gauge – rephasing representation with $\xi = 0$.

Starting from arbitrary set λ_i, m_{ij}, one should derive v_i and v.e.v. phase ξ, and then come to rephasing invariant (overlined) parameters:

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\begin{aligned}
V = &\frac{\overline{\lambda}_1}{2}\left[(\phi_1^\dagger\phi_1) - \frac{v_1^2}{2}\right]^2 + \frac{\overline{\lambda}_2}{2}\left[(\phi_2^\dagger\phi_2) - \frac{v_2^2}{2}\right]^2 \\
&+\overline{\lambda}_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \overline{\lambda}_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+\frac{1}{2}\left[\overline{\lambda}_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] \\
&+\left\{\left[\overline{\lambda}_6(\phi_1^\dagger\phi_1) + \overline{\lambda}_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} \\
&-\frac{1}{2}\left(\overline{\lambda}_{345} + Re\overline{\lambda}_{67}\right)[v_2^2(\phi_1^\dagger\phi_1) + v_1^2(\phi_2^\dagger\phi_2)] \\
&-v_1v_2\, Re[\overline{\lambda}_6(\phi_1^\dagger\phi_1) + \overline{\lambda}_7(\phi_2^\dagger\phi_2)] \\
&+\nu(v_2\phi_1 - v_1\phi_2)^\dagger(v_2\phi_1 - v_1\phi_2) \\
&+2\delta\cdot v_1v_2 Im(\phi_1^\dagger\phi_2).
\end{aligned}
$$

Mass term here is written via v_1, v_2 and $\overline{\lambda}$'s plus a single free dimensionless parameter ν. The mentioned relation

$$Im(\overline{m}_{12}^2) = Im(\overline{\lambda}_5 + \overline{\lambda}_{67})v_1v_2$$

$\Rightarrow$ constraint for potential

in this representation

The standard decomposition of the fields ϕ_i in terms of physical fields (in zero rephasing gauge):

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix} \quad (i = 1, 2).$$

Goldstone boson fields

$$G^0 = \cos\beta\,\chi_1 + \sin\beta\,\chi_2,$$

$$G^\pm = \cos\beta\,\varphi_1^\pm + \sin\beta\,\varphi_2^\pm.$$

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Charged Higgs boson fields

$$H^\pm = \sin\beta\,\varphi_1^\pm + \cos\beta\,\varphi_2^\pm \text{ with}$$

$$M_{H^\pm}^2 = v^2\left[\nu - \frac{1}{2}Re(\lambda_4 + \overline{\lambda}_5 + \overline{\lambda}_{67})\right].$$

Neutral Higgs sector. By definition η_i are standard $C-$ and $P-$ even (scalar) fields while $A = -\sin\beta\,\chi_1 + \cos\beta\,\chi_2$ is C–odd (in the interactions with fermions it behaves as $P-$ odd particle - a pseudoscalar). The mass-squared matrix $\mathcal{M}$ in the η_1, η_2, A basis is

$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix}, \qquad \text{with}$$

$$M_{11} = \left[c_\beta^2\,\lambda_1 + s_\beta^2\,\nu + s_\beta^2 Re(\bar{\lambda}_{67}/2 + 2\tilde{\lambda}_{67})\right] v^2,$$
$$M_{22} = \left[s_\beta^2\,\lambda_2 + c_\beta^2\,\nu + c_\beta^2 Re(\bar{\lambda}_{67}/2 - 2\tilde{\lambda}_{67})\right] v^2,$$
$$M_{12} = -\left(\nu - \bar{\lambda}_{345} - \frac{3}{2} Re\bar{\lambda}_{67}\right) c_\beta s_\beta v^2,$$
$$M_{13} = -\left(\delta + Im\tilde{\lambda}_{67}\right) s_\beta v^2,$$
$$M_{23} = -\left(\delta - Im\tilde{\lambda}_{67}\right) c_\beta v^2,$$
$$M_{33} = \left[\nu - Re(\bar{\lambda}_5 - \frac{1}{2}\bar{\lambda}_{67})\right] v^2 \equiv M_A^2,$$
$$c_\beta = \cos\beta, \qquad s_\beta = \sin\beta.$$

M_A is CP–odd Higgs boson mass in the CP conserving case.

The masses squared of the physical neutral states h_i – eigenvalues of the matrix $\mathcal{M}$, the Higgs eigenstates h_i have no definite $\mathcal{CP}$ parity since they are mixtures of fields η_i and A with opposite $\mathcal{CP}$ parities (provided by M_{13} and M_{23}):

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix} \quad \text{with } R\mathcal{M}R^T = diag(M_1^2,\, M_2^2,\, M_3^2)\,.$$

The diagonalizing matrix

$$R = R_3 R_2 R_1 \equiv \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

$$R_1 = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_2 = \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}.$$

(R_i are rotation matrices, α_i are Euler angles, $c_i = \cos\alpha_i$, $s_i = \sin\alpha_i$).

Starting point:

Diagonalization of scalar (12) sector

$$\begin{pmatrix} h \\ -H \\ A \end{pmatrix} = R_1 \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix} \quad \text{with}$$

$$R_1 \mathcal{M} R_1^T = \mathcal{M}_1 \equiv \begin{pmatrix} M_h^2 & 0 & M'_{13} \\ 0 & M_H^2 & M'_{23} \\ M'_{13} & M'_{23} & M_A^2 \end{pmatrix},$$

$$M'_{13} = c_1 M_{13} + s_1 M_{23}, \quad M'_{23} = -s_1 M_{13} + c_1 M_{23}.$$

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If $\mathcal{CP}$ conserves (at $M_{13} = M_{23} = 0$), $h_1 = h$, $h_2 = -H$, $h_3 = A$. So, notations customary for $\mathcal{CP}$ conserving case:

$$\boxed{\alpha = \alpha_1 - \pi/2}, \quad \alpha \in (-\pi/2, \pi/2)\,.$$

$$H = \cos\alpha\,\eta_1 + \sin\alpha\,\eta_2\,, \quad h = -\sin\alpha\,\eta_1 + \cos\alpha\,\eta_2\,,$$

$$M_{h,H}^2 = (M_{11} + M_{22} \mp \mathcal{N})/2,$$

$$\mathcal{N} = \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}\,,$$

$$\sin 2\alpha = \frac{2M_{12}}{M_H^2 - M_h^2} \Rightarrow \frac{\sin 2\alpha}{\sin 2\beta} = \frac{v^2(\bar{\lambda}_{345} - \nu)}{M_H^2 - M_h^2}\,,$$

$$M'_{13} = -v^2[\delta\cos(\beta+\alpha) - Im\tilde{\lambda}_{67}\cos(\beta-\alpha)]\,,$$

$$M'_{23} = v^2[\delta\sin(\beta+\alpha) - Im\tilde{\lambda}_{67}\sin(\beta-\alpha)]\,.$$

Complete diagonalization

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R_3 R_2 \begin{pmatrix} h \\ -H \\ A \end{pmatrix} \quad \text{with}$$

$$R\,\mathcal{M}\,R^T = R_3 R_2 \mathcal{M}_1 R_2^T R_3^T = \begin{pmatrix} M_1^2 & & \\ & M_2^2 & \\ & & M_3^2 \end{pmatrix}$$

The angles α_2 and α_3 describe mixing of $\mathcal{CP}-$even states h, H with $\mathcal{CP}-$odd state A.

Mass sum rule

$$M_1^2 + M_2^2 + M_3^2 = M_h^2 + M_H^2 + M_A^2 = M_{11} + M_{22} + M_{33}$$

Special cases

• If $\delta = 0$ and $Im\tilde{\lambda}_{67} = 0$, CP symmetry does not violated, h, H and A are physical Higgs bosons and $\alpha_2 = \alpha_3 = 0$.

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• If $|M'_{13}/(M_A^2 - M_h^2)| \ll 1 \;\Rightarrow$
$\alpha_2 \approx 0 \Rightarrow h_1 \approx h$ (*practically* $\mathcal{CP}$ *–even*),
h_2, h_3 *generally have no definite* $\mathcal{CP}$ *parity*

$$\tan 2\alpha_3 \approx \frac{2M'_{23}}{M_A^2 - M_H^2}.$$

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• If $|M'_{23}/(M_A^2 - M_H^2)| \ll 1 \;\Rightarrow$
$\alpha_3 \approx 0 \Rightarrow h_2 \approx -H$ (*practically* $\mathcal{CP}$ *–even*),
h_1, h_3 *generally have no definite* $\mathcal{CP}$ *parity*

$$\tan 2\alpha_2 \approx \frac{2M'_{13}}{M_A^2 - M_h^2}.$$

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• Case of weak $\mathcal{CP}$ violation joins 2 above cases.

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• Intensive coupling regime $M_h \approx M_H \approx M_A$.
$\Rightarrow$ CP violating mixing of fields is naturally strong, spacing between M_i is increased due to this mixing.

Relative couplings of Higgs boson h_i:

$$\chi_a^i \stackrel{def}{=} g_a^i/g_a^{SM}, \qquad a = q,\, \ell,\, V(=Z,\, W)$$

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Couplings to gauge bosons

$$\chi_V^{(i)} = \cos\beta\, R_{i1} + \sin\beta\, R_{i2}, \quad i = 1-3, \quad V = W,\, Z$$

with sum rule followed from the unitarity of transformation matrix R (Gunion et al.)

$$\sum_{i=1}^{3} (\chi_V^{(i)})^2 = 1\,.$$

In particular, for the case with weak violation of $\mathcal{CP}$ symmetry approximately

$$\chi_V^{(1)} = \sin(\beta-\alpha), \quad \chi_V^{(2)} = -\cos(\beta-\alpha),$$
$$\chi_V^{(3)} = -s_2 \sin(\beta-\alpha) + s_3 \cos(\beta-\alpha).$$

Yukawa interaction

General Yukawa Lagrangian

$$-\mathcal{L}_{\mathrm{Y}} = \bar{Q}_L[(\Gamma_1\phi_1 + \Gamma_2\phi_2)d_R$$
$$+(\Delta_1\tilde{\phi}_1 + \Delta_2\tilde{\phi}_2)u_R] + \text{h.c.}$$
$$+ \textit{ lepton terms}$$

Γ and Δ — 3–dimensional in the family space matrices with generally complex coefficients.

If they are non diagonal in family index, the $\mathcal{FCNC}$ appears.

> To have only soft violation of Z_2 symmetry (to keep separate fields ϕ_i at small distances), each right-handed fermion should couple to only one field, either ϕ_1 or ϕ_2.

Otherwise, e.g. in Model III, hard violation of Z_2 symmetry appears via one–loop corrections.

The case $\Gamma_2 = \Delta_2 = 0$ – Model I,

the case $\Gamma_2 = \Delta_1 = 0$ – Model II.

Model II

$$-\mathcal{L}_Y^{II} = \sum_{k=1,2,3} g_{dk}\bar{Q}_{Lk}\phi_1 d_{Rk} + \sum_{k=1,2,3} g_{uk}\bar{Q}_{Lk}\tilde{\phi}_2 u_{Rk}$$
$$+ \sum_{k=1,2,3} g_{\ell k}\bar{\ell}_{Lk}\phi_1 \ell_{Rk} + \text{h.c.}$$

For the physical Higgs fields it result in (for two-component spinors)

$$\chi_u^{(i)} = \frac{1}{\sin\beta}[R_{i2} - i\cos\beta\, R_{i3}],$$
$$\chi_d^{(i)} = \frac{1}{\cos\beta}[R_{i1} - i\sin\beta\, R_{i3}].$$

For the interaction of the charged Higgs bosons with fermions, independent on details of the Higgs potential, one has for 4-component spinors

$$\mathcal{L}_{H^-tb} = \frac{M_t}{v\sqrt{2}}\cot\beta\, \bar{b}(1+\gamma^5)H^- t$$
$$+\frac{M_b}{v\sqrt{2}}\tan\beta\, \bar{b}(1-\gamma^5)H^- t + h.c.$$

Pattern relation and sum rules

based on the unitarity of the mixing matrix R.

• The pattern relation among the basic relative couplings of each neutral Higgs particle h_i (GKO):

$$(\chi_u^{(i)} + \chi_d^{(i)})\chi_V^{(i)} = 1 + \chi_u^{(i)}\chi_d^{(i)}, \qquad (pr)$$

Besides,

$$\tan^2\beta = \frac{(\chi_V^{(i)} - \chi_d^{(i)})^\dagger}{\chi_u^{(i)} - \chi_V^{(i)}} = \frac{1 - |\chi_d^{(i)}|^2}{|\chi_u^{(i)}|^2 - 1}.$$

• A horizontal sum rule for each neutral Higgs boson h_i (Gunion et al)

$$|\chi_u^{(i)}|^2 \sin^2\beta + |\chi_d^{(i)}|^2 \cos^2\beta = 1. \qquad (hsr)$$

• A vertical sum rule for each basic relative coupling χ_j to all three neutral Higgs bosons h_i (Gunion et al):

$$\sum_{i=1}^{3} (\chi_j^{(i)})^2 = 1 \qquad (j = V, d, u). \qquad (vsr)$$

For couplings to gauge bosons this sum rule takes place independently on a particular form of the Yukawa interaction.

The consequences for some cases

with possible CP violation everywhere

(i) $\chi_V^{(2)} \approx \pm 1 \Rightarrow \chi_V^{(1)} \approx \chi_V^{(3)} \approx 0$ independently on the form of Yukawa sector $\Leftarrow$ vsr.

(ii) $\chi_V^{(2)} \approx \pm 1 \Rightarrow (1 \mp \chi_d^{(2)})(1 \mp \chi_d^{(2)}) \approx 0 \Leftarrow$ pr.

(iii) $\chi_V^{(2)} \approx \pm 1 \Rightarrow \chi_u^{(1)} \chi_d^{(1)},\ \chi_u^{(3)} \chi_d^{(3)} \approx -1 \Leftarrow$ pr, vsr.

(iv) The couplings to fermions are generally complex $\chi_{u,d}^{(2)} \approx \pm 1 \Rightarrow \chi_{u,d}^{(1)} \approx \pm(\mp) i \chi_{u,d}^{(3)} \Leftarrow$ vsr.

(v) $\chi_u^{(i)} \approx \pm 1 \Rightarrow \chi_d^{(i)} \approx \pm(\mp) 1 \Leftarrow$ hsr.

(vi) $|\chi_{u,d}^{(i)}| \gg 1 \Rightarrow \chi_{d,u}^{(i)} \approx 0 \Leftarrow$ hsr.

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In the $\mathcal{CP}$ conserving case

$$\chi_{H^\pm}^{(\phi)} \equiv -\frac{v g_{hH^+H^-}}{2M_{H^\pm}^2}$$

$$= \left(1 - \frac{M_\phi^2}{2M_{H^\pm}^2}\right) \chi_V^{(\phi)} + \frac{M_\phi^2 - \nu v^2}{2M_{H^\pm}^2} (\chi_u^{(\phi)} + \chi_d^{(\phi)}).$$

Constraints for parameters of Higgs potential

were written only in the case of soft violation of Z_2 symmetry without $\mathcal{CP}$ violation. We extend these results to the case with $\mathcal{CP}$ violation.

- Positivity (vacuum stability) constraints.

The potential must be positive at large quasi–classical values of fields $|\phi_i|$ for an arbitrary direction in the (ϕ_1, ϕ_2) plane:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0,$$
$$\lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.$$

- Minimum constraints — conditions ensuring that the condition for vacuum is a local minimum for all directions in (ϕ_1, ϕ_2) space, except the Goldstone modes (the physical fields provide the basis in the coset).

• Unitarity constraints. The quartic terms of Higgs potential lead, in the tree approximation, to a s–wave Higgs-Higgs and $W_L W_L$ and $W_L H$, etc. scattering amplitudes for different elastic channels. These amplitudes should not overcome unitary limit for partial wave. The earlier constraints for the case without $\mathcal{CP}$ violation (Akeroyd et al.) – with real λ_5 extends to the case with $\mathcal{CP}$ violation by the change $\lambda_5 \to |\lambda_5|$(IFG, Ivanov).

These constraints give bounds for the Higgs-boson masses which strongly depend on the quadratic mass parameter ν.

Large ν $\Rightarrow$ all M_H, M_A, $M_{H^\pm}$ are large (decoupling limit).

Small ν $\Rightarrow$ moderately large upper bound of $600 \div 700$ GeV for M_H, M_A, $M_{H^\pm}$.

The correspondence between the tree-level unitarity limit and realization of the Higgs field as more or less narrow particle, as in minimal $\mathcal{SM}$, takes place in the $2\mathcal{HDM}$ only in the case when all unitarity constraints are violated simultaneously. In the case when only some of these constraints are violated the physical picture become more complex.

Heavy Higgs bosons in $2\mathcal{HDM}$

Many analyses of $2\mathcal{HDM}$ assume that the lightest Higgs boson h_1 is similar to the Higgs boson of the $\mathcal{SM}$, all other Higgs bosons are very heavy (with mass $\sim M$).
Usual additional hidden requirement (?!?):
The theory must have explicit decoupling property: the mention features remain valid at $M \to \infty$ (decoupling property).
In fact, the mentioned physical picture can be realized in the $2\mathcal{HDM}$ both with and without decoupling property.

Two scenarios of generation of heavy Higgs masses.

Decoupling of heavy Higgs bosons

$\nu \gg |\lambda_i|$.

$\Rightarrow M'_{13} \sim \lambda_i v^2 \;\; \Rightarrow |M'_{13}| \ll M_A^2 - M_h^2 \approx \nu v^2 \Rightarrow$ $h_1 \approx h$, etc. as it was discussed earlier, $\beta - \alpha \approx \pi/2$,

$$M_h^2 = v^2 \left(\underbrace{c_\beta^4 \lambda_1 + s_\beta^4 \lambda_2 + 2 s_\beta^2 c_\beta^2 \overline{\lambda}_{345}}_{soft} - \underbrace{2 s_\beta^2 c_\beta^2 Re \overline{\lambda}_{67}}_{hard} \right)$$

$$M_H^2 = v^2 \left\{ \underbrace{\nu + s_\beta^2 c_\beta^2 (\lambda_1 + \lambda_2 - 2\overline{\lambda}_{345})}_{soft} + \right.$$

$$\left. \underbrace{Re \left[2 s_\beta c_\beta (\overline{\lambda}_6 + \overline{\lambda}_7) + \left(-\frac{3}{2} + 4 s_\beta^2 c_\beta^2 \right) \overline{\lambda}_{67} \right]}_{hard} \right\},$$

$$\alpha \equiv \alpha_1 - \frac{\pi}{2} = \beta - \frac{\pi}{2} + \delta_\alpha \,,$$

$$\delta_\alpha = -\frac{\sin 2\beta [\overline{\lambda}_{345} \cos 2\beta + c_\beta^2 \lambda_1 - s_\beta^2 \lambda_2 + \mathcal{O}(Re \overline{\lambda}_{6,7})]}{\nu}$$

Decoupling. Lightest Higgs boson h_1.

$\beta-\alpha\approx\pi/2 \Rightarrow$ all couplings of h_1 are close to those in $\mathcal{SM}$ and also selfcouplings, $h_1h_1h_1$ and $h_1h_1h_1h_1$, are very close to the corresponding $\mathcal{SM}$ couplings. Besides, h_1 practically decouple from $H^\pm$, since the quantity $\chi_{H^\pm}^{(1)}\sim\mathcal{O}(|\lambda_i|/\nu)$.

Higgs bosons h_2, h_3 are almost degenerate in masses, since

$$M_A\approx M_H(\approx M_2\approx M_3)=v\sqrt{\nu}\,(1+\mathcal{O}\,(|\lambda|/\nu))\,.$$

Besides, $M_{H^\pm}\approx M_2\approx M_3$.

The $\mathcal{CP}$ violating mixing angle α_3 can be large, $\tan 2\alpha_3\approx\dfrac{2M'_{23}}{M_A^2-M_H^2}$, and

$$\chi_u^{(2)}=i\chi_u^{(3)}=-\cot\beta\, e^{i\alpha_3},$$
$$\chi_d^{(2)}=i\chi_d^{(3)}=\tan\beta\, e^{-i\alpha_3}.$$

while couplings of h_2, h_3 to gauge bosons and $H^\pm$ are small,

$$\chi_V^{(2)}=\cos\alpha_3\delta_\alpha\,,\ \ \chi_V^{(3)}=\sin\alpha_3\delta_\alpha\,,$$
$$\chi_{H^\pm}^{(2,3)}\sim\mathcal{O}(|\lambda_i|/\nu)\,.$$

Heavy Higgs bosons without decoupling.

The option, when except one neutral h_1 all other Higgs bosons are heavy enough, can also be realized in $2\mathcal{HDM}$ without decoupling.

Sets of parameters of potential, satisfying unitarity constraints, for light h (mass 120 GeV) and heavy H, $H^{\pm}$, non-decoupling case.

	$\tan\beta$	λ_1	λ_2	λ_3	λ_4	λ_5	ν
(1)	50	1	6	5.5	-6	-6	0.24
(2)	0.02	6	1	5.5	-6	-6	0.24
(3)	1	6.25	6.25	6.25	-6	-6	0
(4)	10	4	8	4.4	-9	-0.5 $+0.3i$	0.24

	M_h	M_H	M_A	$M_{H^\pm}$	s_2	s_3
(1)	120	600	600	600	-	-
(2)	120	600	600	600	-	-
(3)	120	600	600	600	-	-
(4)	120	700	206	556	0.09	0.02

Lines (1-3) – the case without $\mathcal{CP}$ violation, (4) – with $\mathcal{CP}$ violation.

Natural set of parameters

To have $\not{C}P$ in the Higgs sector $\Leftarrow Im\,(\overline{m}_{12}^2) \neq 0$ (simultaneously $Im\,(\overline{\lambda}_5) \neq 0$). This $\not{C}P$ is presumably weak if

$$Im\,(\overline{m}_{12}^2) \ll |M_A^2 - M_h^2|,\ |M_A^2 - M_H^2|\,.$$

This simple form of condition is valid only for zero rephasing gauge. In other rephasing gauges this condition includs both $Im\,(m_{12}^2)$ and $Re\,(m_{12}^2)$. Naturally, this condition must be formulated independently on the rephasing gauge $\Rightarrow$ for the natural set of parameters of $2\mathcal{HDM}$ we require that $|m_{12}^2| \ll |M_A^2 - M_h^2|,\ |M_A^2 - M_H^2|$, i.e.

$|\nu|,\ |\lambda_5| \ll |\lambda_{1-4}|$ *(natural set of parameters).*

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In the decoupling case $Re\,(\overline{m}_{12}^2) \gg Im\,(\overline{m}_{12}^2) \Rightarrow$ unnatural case.

Weak $\not{C}P$ in Higgs sector looks unnatural if $|m_{12}|$ is large, i.e. a weak $\mathcal{CP}$ violation naturally correspond to weakly broken Z_2 symmetry with $|\nu| < |\lambda_i|$.

Different scenarios in $\mathcal{2HDM}$

The $\mathcal{SM}$ is verified now with high precision apart from mechanism of $\mathcal{EWSB}$.

Two opportunity for next generation of colliders:

■ We meet clear signals of New Physics (new particles, strong deviations from $\mathcal{SM}$ in some processes) at LHC or e^+e^- LC.

■ The physical picture coincide with that expected in $\mathcal{SM}$ — **$\mathcal{SM}$ –like scenario,** determined for the fixed time:

- No new particles and interactions will be discovered at the Tevatron, LHC and e^+e^- LC except the Higgs boson.
- The couplings squared of Higgs boson to W, Z and quarks coincide with those predicted in $\mathcal{SM}$ within experimental precision.

Different realizations of the $\mathcal{SM}$ – like scenario.

The $\mathcal{SM}$ – like scenario can be realized both in the $\mathcal{SM}$ and in other models. In the $2\mathcal{HDM}$ it can be realized by many ways:

type	notation	observed Higgs	χ_V	$\tan\beta =$	
$\chi_V \approx \chi_u \approx \chi_d$	A_{h+}	$h_1 \approx h$	$\approx +1$	$\sqrt{\left\lvert\frac{\epsilon_d}{\epsilon_u}\right\rvert}$	$\lessgtr 1$
	A_{H+}	$h_2 \approx -H$	$\approx +1$		
	A_{h-}	$h_1 \approx h$	≈ -1		$\ll 1$
	A_{H-}	$h_2 \approx -H$	≈ -1		$\gg 1$
$\chi_V \approx \chi_u \approx -\chi_d$	B_{h+d}	$h_1 \approx h$	$\approx +1$	$\sqrt{\frac{2}{\epsilon_V}} \gtrsim 10$	
	$B_{H\pm d}$	$h_2 \approx -H$	$\approx \pm 1$		
$\chi_V \approx \chi_d \approx -\chi_u$	$B_{h\pm u}$	$h_1 \approx h$	$\approx \pm 1$	$\sqrt{\frac{\epsilon_V}{2}} \lesssim 0.1$	
	B_{H+u}	$h_2 \approx -H$	$\approx +1$		
$\chi_i = g_i^{obs}/g_i^{SM} = \pm(1-\epsilon_i)$					

The numbers here correspond to the anticipated accuracy of measurements at TESLA.

The decoupling case – particular case of A_{h+}.

• The observed Higgs boson can be either $h_1 \approx h$ or $h_2 \approx -H$. The other Higgs bosons are practically decoupled to gauge bosons and cannot be seen at e^+e^- LC in the standard processes. If its mass below 350 GeV and $\tan\beta \ll 1$, it can be seen in $\gamma\gamma \to \gamma\gamma$ process at Photon Collider (see M. Muehlleitner, M. Spira, P. Zerwas for H and A) (for h – if use low energy part of photon spectrum) and in $e^+e^- \to t\bar{t}H$ at $2E > 2M_t + M_H$

• If $h_{2,3}$ and $H^\pm$ cannot be seen at LHC and the first stage of LC since they are heavy, one can distinguish models via measurement of two–photon width of observed $\mathcal{SM}$ – like Higgs boson. (I.Ginzburg, M. Krawczyk, P. Olsen).

The two photon width is calculated via the measured at e^+e^- LC Higgs couplings to the matter. For natural set of parameters of $2\mathcal{HDM}$ we find: For solutions A and B_d the deviation from $\mathcal{SM}$, given by contributions of heavy charged Higgs bosons, is about $\sim$ 10% (compare with anticipated 2% accuracy). For solutions B_u change of relative sign of contributions of t–loop and W–loop increase the observed cross section more than twice in comparison with $\mathcal{SM}$. Therefore, measurement of two-photon width at Photon Collider can resolve these models reliably.

For $M_{H^\pm} = 800$ GeV the ratio of the two-photon Higgs width to its $\mathcal{SM}$ value is shown in Figure.