Direct CP Violation in B decays with $\rho^0 - \omega$ Mixing

Phenomenological Approach

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1 Physical Motivations for $B \to V_1 V_2$

$\star$ $V_{1,2}$ Vector Meson 1^{--} decaying into Pseudoscalar Mesons 0^{-+}

$\star$ Weak Decay of the B meson $\Longrightarrow$ $V_{1,2}$ are polarized

$\Rightarrow$ Specific Angular Distributions and Correlations among final particles

$\Downarrow$

Cross-checks of the Standard Model predictions

$\Downarrow$

Test of Models Beyond the SM (BSM)

(Dunietz, Lipkin et al Phys.Rev. D34 (1991), 2134)

• Special case of $B \to V^0 V^0$ or $B \to V^+ V^-$

$$s_B = 0 \Rightarrow \ell = S = 0, 1, 2 \Longrightarrow CP = (-1)^\ell$$

$\Longrightarrow$ Mixing of Different CP Eigenstates $\equiv$ CP non-conservation

$\Longrightarrow$ Looking for Direct CP Violation, DCPV, in a Model Independent way.

$\Longrightarrow$ Constraint on any model BSM

2 Kinematics and Helicity Formalism

$\star$ Helicity defined by $\lambda \ = \ \vec{s}\cdot\vec{p}/p$ (s = spin of the particle, $\vec{p}$ its momentum)

$\star$ $B \to X\ Y$ (s_1 and s_2 are Spin of ANY particles X and Y)

$\star$ $\vec{s_B} = \vec{\ell} + \vec{s_1} + \vec{s_2} \ = \ \vec{0}$

$\star$ Projection on the momentum-axis ($\Delta_{\vec{p}}$) in the B rest-frame

$$\Downarrow$$

$$\lambda_1 \ = \ \lambda_2$$

So, Number of helicity states :

Decay Channel	Helicity values	Nb of amplitudes
P_1P_2	0	1
VP	0	1
V_1V_2	-1, 0, +1	3
γV	-1, +1	2
γP	FORBIDDEN	0

N.B : In any Radiative Decay, Photon Helicity : $\lambda_\gamma = \pm 1$

- Analytical formulation of the Decay Amplitude, $B \rightarrow V_1 V_2 \rightarrow Hadrons$

$$\mathcal{H}_W = Weak\ Hamiltonian$$

$$\text{Partial Amplitude } H_\lambda = < V_1(\lambda) V_2(\lambda) | \mathcal{H}_W | B >$$

$$V_i(\lambda) \rightarrow a_i + b_i \quad \text{where} \quad a_i, b_i = 0^{-+}$$

$\star$ For a given Helicity State, the amplitude factorizes according to :

$$M_\lambda\big(B \rightarrow \sum_{i=1}^{2}(a_i + b_i)\big) = H_\lambda(B \rightarrow V_1 + V_2) \times \prod_{i=1}^{2} A_i(V_i(\lambda) \rightarrow a_i + b_i)$$

(Kramer-Palmer, Phys.Rev.D45 (1992), 193)
with :

$\star$ $A_i(V_i \rightarrow a_i + b_i) \equiv$ Decay Amplitude of V_i due to Strong or E.M. interaction

Using Jacob-Wick-Jackson convention and Wigner-Eckart theorem

$$A_1(V_1 \to a_1 + b_1) = c_1 D^1_{\lambda m_1}(\phi_1, \theta_1, 0)$$
$$A_2(V_2 \to a_2 + b_2) = c_2 D^1_{\lambda m_2}(\phi_2, \theta_2, 0)$$
$$m_1 \;=\; m_2 \;=\; 0$$

using

- c_1 , c_2 $=$ Dynamical Decay Parameters related to resonances V_1 and V_2
- $D^1_{\lambda m_i}(\phi_i, \theta_i, 0)$ $=$ Wigner rotation matrix element for a spin-1 particle
- $\lambda(a_i)$, $\lambda(b_i)$ $=$ Helicities of the final particles a_i and b_i in the V_i rest frame
- θ_1 $=$ Polar Angle of meson a_1 in the V_1 helicity frame
- Decay Plane of V_1 $\equiv$ (X-Z) Plane $\Rightarrow$ Azimuthal Angle $\phi_1 \;=\; 0$
- θ_2 , ϕ_2 $\equiv$ Polar and Azimuthal angles of meson a_2 in the V_2 helicity frame
- $m_i \;=\; \lambda(a_i) - \lambda(b_i)$

$\star$ Convention for the $D^1_{\lambda m_i}(\alpha, \beta, \gamma)$ (Rose, Dover 1957)

$$D^1_{\lambda m_i}(\alpha, \beta, \gamma) = \exp[-i(\lambda\alpha + m_i\gamma)]\; d^1_{\lambda m_i}(\beta) \;.$$

$\star$ General form of the Decay Amplitude $\mathcal{M}\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big)$:

Linear Superposition of the previous amplitudes $M_\lambda\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big)$

$$\Longrightarrow \mathcal{M}\Big(B \to \sum_{i=1}^{2}(a_i + b_i)\Big) = \sum_{\lambda} M_\lambda\Big(B \to \sum_{i=1}^{2}(a_i + b_i)\Big) \ .$$

$\star$ Decay Width $\Gamma(B \to V_1 V_2)$

$$d^3\Gamma(B \to V_1 V_2) \propto \Big|\sum_{\lambda} M_\lambda\Big(B \to \sum_{i=1}^{2}(a_i + b_i)\Big)\Big|^2 = \sum_{\lambda,\lambda'} h_{\lambda\lambda'} F_{\lambda\lambda'}(\theta_1) G_{\lambda\lambda'}(\theta_2, \phi) \ ,$$

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3 Kinematic Parameters : $\theta_1,\ \theta_2,\ \phi$

3 Density-Matrices : $h_{\lambda\lambda'},\ F_{\lambda\lambda'}(\theta_1),\ G_{\lambda\lambda'}(\theta_2, \phi)$

Correlations among final particles

$\star$ $h_{\lambda\lambda'} = H_\lambda H^*_{\lambda'} \Longrightarrow$ Density-Matrix related to the B decay.
$\Rightarrow$ It contains all the Decay Dynamics

$\star$ $F_{\lambda\lambda'}(\theta_1) \Longrightarrow$ Density-Matrix related to the decay $V_1 \to a_1 + b_1$

$\star$ $G_{\lambda\lambda'}(\theta_2, \phi) \Longrightarrow$ Density-Matrix related to the decay $V_2 \to a_2 + b_2$.

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Very General Analytic Formulation

$\star$ Independent of the Nature of the Intermediate Resonances

$\star$ Independent of their Decay Modes (except for the Spin of the final particles)

Physical Informations which can be deduced

⋆ Angular Distribution of each final particle $a_i(b_i)$ in the V_i rest frame

⋆ Correlations among the final B products

• "Practical" Applications

$$B^0 \to K^{*0}\rho^0(\omega)\ ,\quad K^{*0} \to K^+\pi^-\ ,\ \rho^0 \to \pi^+\pi^-$$

$$B^\pm \to K^{*(\pm)}\rho^0(\omega)\ ,\quad K^{*(\pm)} \to K^\pm\pi^0\ ,\ \rho^0 \to \pi^+\pi^-$$

$$B^\pm \to \rho^\pm\rho^0(\omega)\ ,\quad \rho^\pm \to \pi^\pm\pi^0\ ,\ \rho^0 \to \pi^+\pi^-$$

$$B^\pm \to K^{*0}\rho^\pm\ ,\quad K^{*0} \to K^+\pi^-\ ,\ \rho^\pm \to \pi^\pm\pi^0$$

$$\frac{d^3\Gamma(B\to V_1V_2)}{d(\cos\theta_1)d(\cos\theta_2)d\phi} \propto (h_{++}+h_{--})\sin^2\theta_1\sin^2\theta_2/4 + h_{00}\cos^2\theta_1\cos^2\theta_2$$
$$+\Big\{Re(h_{+0})\cos\phi - Im(h_{+0})\sin\phi + Re(h_{0-})\cos\phi - Im(h_{0-})\sin\phi\Big\}\sin 2\theta_1\sin 2\theta_2/4$$
$$+\Big\{Re(h_{+-})\cos 2\phi - Im(h_{+-})\sin 2\phi\Big\}\sin^2\theta_1\sin^2\theta_2/2$$

Physical Consequences

- Expression Completely Symmetric in θ_1 and θ_2

$\Longrightarrow$ Angular Distribution of a_1 in the V_1 frame is identical to that of a_2 in the V_2 frame.

- Probabilty Distribution Functions (PDF) of the different angles :

$$\begin{aligned} f(\cos\theta_{1,2}) &= (3h_{00}-1)\cos^2\theta_{1,2} + (1-h_{00}) \\ g(\phi) &= 1 + 2\ Re(h_{+-})\cos 2\phi - 2\ Im(h_{+-})\sin 2\phi\ . \end{aligned}$$

3 Decay Dynamics and Final State Interactions

$*$ How to compute the decay amplitudes ?

$*$ Interplay between Weak Interaction (initial decay) and the Strong one.

• Weak Decay : Completely described by the Standard Model and the CKM mechanism at the Quark level

BUT.... Composite Hadrons observed in the final states and not Quarks....

$\Longrightarrow$ Final State Interactions, FSI, are Important and Unavoidable

• 1^{st} stage : Light Quarks exchange Gluons amongst themselves.
• 2^{nd} stage : Hadronization of the Quarks.

$\Longrightarrow$ Two regimes : Perturbative and Non-Perturbative regimes

How to deal with FSI in a realistic manner ??

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Effective Lagrangean approach

PR $\rightarrow$ Calculations at Next-To-Leading (NLO) performed for an energy scale $\geq m_B$
NPR$\rightarrow$ Effects inserted for an energy scale $\leq m_B$

FACTORIZATION

(Naive Factorization)

$*$ Matrix Element of a four-fermion Operator replaced by the product of the M.E. of two Currents

$**$ Involving Form Factors and Decay Constants

$***$ Modification of the Effective Number of Colors, N_c

$$\frac{1}{(N_c^{eff})_i} = \frac{1}{3} + \xi_i \quad , \quad \xi_\mathrm{i} \text{ describes Non} - \text{Factorizable effects}$$

Example : $B^{\pm 0} \Longrightarrow K^* \rho^0(\omega)$

$\star$ Two Matrix Elements :

$\langle \rho^0(\omega)|(\bar{u}u)|0\rangle\langle K^*|(\bar{q}_i q_j)|B^{\pm,0}\rangle$ and $\langle K^*|(\bar{q}_i q_j)|0\rangle\langle \rho^0(\omega)|(\bar{u}b)|B^{\pm,0}\rangle$

$\star$ q_i , q_j could be either u, d or s

$\star$ Transition Matrix Element for $B \to X^*$

$$\langle X^*|J_\mu|B\rangle = \frac{2}{m_B + m_{X^*}}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p_{X^*}^\sigma V(k^2) + i\Bigg\{\epsilon_\mu^*(m_B + m_{X^*})A_1(k^2) - \frac{\epsilon^* \cdot k}{m_B + m_{X^*}}(P_B + P_{X^*})_\mu A_2(k^2) - \frac{\epsilon^* \cdot k}{k^2}2m_{X^*} \cdot k_\mu A_3(k^2)\Bigg\} + i\frac{\epsilon^* \cdot k}{k^2}2m_{X^*} \cdot k_\mu A_0(k^2)$$

where :

- $J_\mu = \bar{q}\gamma^\mu(1 - \gamma_5)b$ with $q = u, d, s$
- $k = p_B - p_{X^*}$
- ϵ_μ is the polarization vector of X^*
- A_0, A_1, A_2, A_3, V $\equiv$ Form Factors describing the transition $0^- \to 1^-$.

Effective Hamiltonian

Wilson Operator Product Expansion

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(\mu) O_i(\mu)$$

where

- G_F = Fermi Constant
- V_{CKM} = CKM Matrix Elements
- $C_i(\mu)$ are the Wilson Coefficients
- $O_i(\mu)$ are the Operators from the OPE
- $\mu \equiv$ Renormalization Scale

$\star$ OPE represents physics processes for an Energy Scale lower than $\mu \equiv$ NPR

$\star$ $C_i(\mu)$ represent Perturbative parts computed at Energy Scale greater than μ.

General Form of a Wilson Operator

$$O_i = (\bar{q}_i \Gamma_{n1} q_j)(\bar{q}_k \Gamma_{n2} q_l)$$

$\star$ Γ_{n1} , Γ_{n2} $\equiv$ Combination of Dirac Gamma Matrices
$\star$ q_i $\equiv$ Quark Flavor.
$\star$ Three Categories of Operators :

Tree Operators (O_1, O_2)

QCD Penguin Operators (O_3 to O_6)

EW Penguin Operators (O_7 to O_{10})

Wilson Coefficients

$\star$ W.C. are calculated perturbatively at NLO by Renormalization Group techniques (Buras, Fleischer hep-ph/9806471)

$\star$ Energy Scale $\mu = m_B$.

4 Explicit Calculations and $\rho^0 - \omega$ Mixing

FSI are illustrated by Phase-Shift Angles between different amplitudes contributing to the same process

- Quark Contribution

$\star$ Absorptive Part of the Scattering Amplitude due to the Penguin Diagram

$\Longrightarrow$ BSS Mechanism (M.Bander, Phs.Rev.Lett.43 (19179), 242)

$\Longrightarrow$ Generation of a Phase Shift according to the Tree Diagram

$$\mathcal{A} = \mathcal{A}^T \; + \; \mathcal{A}^P \; = \; \langle K^* \pi^- \pi^+ | H^T | B \rangle [1 + r e^{i\delta} e^{i\phi}]$$

$\star$ $\phi =$ Weak Phase arising from the CKM Matrix Elements $= \arg[(V_{tb} V_{ts}^*)/(V_{ub} V_{us}^*)]$

$\star$ $\delta =$ Strong Phase from Penguin diagram "a la BSS"

$$r \equiv \left| \frac{\langle \rho^0(\omega) K^* | H^P | B \rangle}{\langle \rho^0(\omega) K^* | H^T | B \rangle} \right| .$$

- The $\rho^0 - \omega$ Mixing Contribution

$$\rho^0 \to \pi^+\pi^- \approx 100\%$$

$$\omega \to \pi^+\pi^- \approx 2\%$$

$$\Downarrow$$

$\rho^0 - \omega$ Mixing is natural in the study of $B \to \pi\pi\ V$

$$\Downarrow$$

Generation of an Additionnal Phase-Shift δ_R
when $\pi^+\pi^-$ mass is in the vicinity of the Resonance ω

Final Expression of the Amplitude

$$\begin{aligned}
\langle K^*\pi^-\pi^+|H^T|B\rangle &= \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega + \frac{g_\rho}{s_\rho}t_\rho \ , \\
\langle K^*\pi^-\pi^+|H^P|B\rangle &= \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho \ .
\end{aligned}$$

with

- t_V $(V = \rho \text{ or } \omega)$ is the Tree Amplitude for producing a Vector Meson V

- p_V , Penguin Amplitude for producing V
- g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$
- $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho^0 - \omega$ Mixing Amplitude
- s_V is the inverse propagator of the vector meson V

• $s_V = s - m_V^2 + i m_V \Gamma_V$ with $\sqrt{s}$ $\equiv$ Invariant Mass of the $\pi^+\pi^-$ pair.

• $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_w^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$

where :

$Re\ \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300\ \mathrm{MeV}^2, \quad Im\ \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300\ \mathrm{MeV}^2$, and $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04.$

$\Longrightarrow \Longrightarrow \Longrightarrow$

$$re^{i\delta}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega}t_\omega + s_\omega t_\rho}$$

• Final Phase-Shift δ depends both on the Quark Loop and the Resonance Mixing.

Combining Wilson OPE, $\rho^0 - \omega$ Mixing and Helicity Formalism

$$
\begin{aligned}
H_\lambda\big(B \to \rho^0(\omega) V_2\big) = & \Big(V_{ub}V_{us}^* c_{t_1}^\rho - V_{tb}V_{ts}^* c_{p_1}^\rho\Big)\Big\{\beta_1^\rho \varepsilon_{\alpha\beta\gamma\delta}\epsilon_{V_2}^{*\alpha}(\lambda)\epsilon_\rho^{*\beta}(\lambda)P_B^\gamma P_{V_2}^\delta \\
& + i\Big(\beta_2^\rho \epsilon_{V_2}^*(\lambda)\epsilon_\rho^*(\lambda) - \beta_3^\rho(\epsilon_{V_2}^*(\lambda)P_B)(\epsilon_\rho^*(\lambda)P_B)\Big)\Big\} + \Big(V_{ub}V_{us}^* c_{t_2}^\rho - V_{tb}V_{ts}^* c_{p_2}^\rho\Big) \\
& \Big\{\beta_4^\rho \varepsilon_{\alpha\beta\gamma\delta}\epsilon_\rho^{*\alpha}(\lambda)\epsilon_{V_2}^{*\beta}(\lambda)P_B^\gamma P_\rho^\delta + i\Big(\beta_5^\rho \epsilon_\rho^*(\lambda)\epsilon_{V_2}^*(\lambda) - \beta_6^\rho(\epsilon_\rho^*(\lambda)P_B)(\epsilon_{V_2}^*(\lambda)P_B)\Big)\Big\} \\
& + \frac{\tilde{\Pi}_{\rho\omega}}{(s_\rho - m_\omega^2) + i m_\omega \Gamma_\omega}\Bigg[\Big(V_{ub}V_{us}^* c_{t_1}^\omega - V_{tb}V_{ts}^* c_{p_1}^\omega\Big)\Big\{\beta_1^\omega \varepsilon_{\alpha\beta\gamma\delta}\epsilon_{V_2}^{*\alpha}(\lambda)\epsilon_\omega^{*\beta}(\lambda)P_B^\gamma P_{V_2}^\delta \\
& + i\Big(\beta_2^\omega \epsilon_{V_2}^*(\lambda)\epsilon_\omega^*(\lambda) - \beta_3^\omega(\epsilon_{V_2}^*(\lambda)P_B)(\epsilon_\omega^*(\lambda)P_B)\Big)\Big\} + \Big(V_{ub}V_{us}^* c_{t_2}^\omega - V_{tb}V_{ts}^* c_{p_2}^\omega\Big) \\
& \Big\{\beta_4^\omega \varepsilon_{\alpha\beta\gamma\delta}\epsilon_\omega^{*\alpha}(\lambda)\epsilon_{V_2}^{*\beta}(\lambda)P_B^\gamma P_\omega^\delta + i\Big(\beta_5^\omega \epsilon_\omega^*(\lambda)\epsilon_{V_2}^*(\lambda) - \beta_6^\omega(\epsilon_\omega^*(\lambda)P_B)(\epsilon_{V_2}^*(\lambda)P_B)\Big)\Big\}\Bigg]
\end{aligned}
$$

with

- $\epsilon_{V_2,\rho,\omega}(\lambda)$ being the K^*, ρ^0 and ω polarization 4-vectors expressed in the B rest frame
- $\varepsilon_{\alpha\beta\gamma\delta}$ Antisymmetric Tensor in Minkowski space.
- β_i , Form Factors describing transitions between Vector Mesons.

$\star$ Simplification of the Helicity Amplitude in the B rest-frame : $P_B = (m_B, \vec{0})$

$$H_\lambda\big(B \to \rho^0(\omega)V_2\big) = iB^\rho_\lambda(V_{ub}V^*_{us}c^\rho_{t_1} - V_{tb}V^*_{ts}c^\rho_{p_1}) + iC^\rho_\lambda(V_{ub}V^*_{us}c^\rho_{t_2} - V_{tb}V^*_{ts}c^\rho_{p_2}) +$$
$$\frac{\tilde{\Pi}_{\rho\omega}}{(s_\rho - m^2_\omega) + im_\omega\Gamma_\omega}\Big[iB^\omega_\lambda(V_{ub}V^*_{us}c^\omega_{t_1} - V_{tb}V^*_{ts}c^\omega_{p_1}) + iC^\omega_\lambda(V_{ub}V^*_{us}c^\omega_{t_2} - V_{tb}V^*_{ts}c^\omega_{p_2})\Big]$$

$$\Downarrow$$

$$h_{\lambda\lambda'} = H_\lambda\big(B \to \rho^0(\omega)V_2\big)H^*_{\lambda'}\big(B \to \rho^0(\omega)V_2\big)\ .$$

$\star$ Hermiticity of the Density-Matrix $(h_{\lambda\lambda'}) \Longrightarrow$ Only Six Elements have to be calculated.

Form Factor Models

- Two Models of FF have been used :

⋆ Pioneering Model of BSW (Bauer et al, Z.Phys.C34(1987),103) ≡ Relativistic Oscillator describing potential among Quarks.

⋆ Modified BSW Model by Guo and Huang (GH, Phys.Rev.D43(1991),2931) which uses the light-cone approach.

$$V(k^2) = \frac{h_V}{\left(1 - \frac{k^2}{m_V^2}\right)} \ , \qquad A_j(k^2) = \frac{h_{A_j}}{\left(1 - \frac{k^2}{m_{A_j}^2}\right)} \ ,$$

where

- m_{A_j} , m_V are the Pole Masses associated with the transition current
- h_V , h_{A_j} are the values of the Form Factors at $k^2 = 0$.

Simulations and Main Results

Input Values

⋆ Wolfenstein Parameters

$$\lambda_c = 0.2237\ ,\quad A = 0.8113\ ,\quad 0.190 < \rho < 0.268\ ,\quad 0.284 < \eta < 0.366\ .$$

In the numerical simulations, the Average Values of ρ and η are used.

⋆ Decay Constants of Vector Mesons :

$$f_{K^*} = 214\ \mathrm{MeV}\ ,\ f_\rho = 221\ \mathrm{MeV}\ ,\ f_\omega = 195\ \mathrm{MeV}$$

Free Parameters

⋆ Ratio ${q^2}/{{m_b}^2}$ related to the Penguin Diagram, where q^2 is the Gluon squared mass

$$\Rightarrow\quad 0.3 \leq {q^2}/{{m_b}^2} \leq 0.5$$

(R.Fleischer, Int.J.Mod.Phys. A12(1997), 2459)

⋆ Form Factor Models : BSW or GH

⋆ Color Effective Number, N_c^{eff} $\Rightarrow$ It depends on the Neutral Transition $b \to d$ $(N_c^{eff} = 2.01)$ or $b \to s$ $(N_c^{eff} = 2.84)$

Determination of Density Matrix h_{ij}

⋆ h_{ij} elements depend essentially on the masses of the Resonances.

⋆ Mass of each Vector Resonance generated according to a Relativistic Breit-Wigner distribution :

$$\frac{d\sigma}{dM^2} = C_N \frac{\Gamma_R M_R}{\left(M^2 - M_R^2\right)^2 + \left(\Gamma_R M_R\right)^2}$$

- C_N = Normalization Constant
- M_R and Γ_R respectively the Mass and the Width of the Vector Meson

$\star$ Simplified Simulation of the $\rho^0 - \omega$ Mixing

The ρ^0-propagator is replaced by the following one :

$$\frac{1}{s_{\rho\omega}} = \frac{1}{s_\rho} + \frac{T_\omega}{T_\rho}\frac{\Pi_{\rho\omega}}{s_\rho s_\omega} \ ,$$

(P.Langacker, Phys.Rev. D20(1979), 2983)

$\star$ $\Pi_{\rho\omega}$ = Mixing amplitude
$\star$ Natural Choice $T_\omega/T_\rho = 1$

Main Results

1) Spectrum of h_{ij} too wide because of the Resonance Widths, especially the ρ^0 width, $\Gamma_\rho = 150\ \mathrm{MeV/c^2}$

2) The Longitudinal Polarization $h_{00} = |H_0|^2$ is largely Dominant

$B^0 \to \rho^0(\omega) K^{*0} \implies$ Mean Value of h_{00} is $\approx 87\%$

$B^+ \to \rho^0(\omega) \rho^+ \implies$ Mean Value of h_{00} is $\approx 90\%$

3) $h_{--} = |H_{-1}|^2$ very tiny $\leq 0.5\%$

4) Non-Diagonal matrix elements h_{ij} :

- Smallness of both Real and Imaginary parts
- $Im/Re \approx 0.001 \Rightarrow 0.1$
- Special case of $B^+ \to \rho^0(\omega)\rho^+$, $Im(h_{i,j}) \approx 0.0$

First Conclusion

"Universal behavior" of the Density-Matrix Elements, whatever the decay $B \to \pi^+\pi^- V$ is !! ($V = K^{*0}, K^{*\pm}$, $\rho^\pm$)

Consequences on Angular Distributions

$\star$ Spin $s_B = 0 \Longrightarrow$ Isotropic Angular Distribution of V_i in B rest-frame.

$\star$ Helicity Frame of each Vector Meson V_i :

Angular Distributions given by the DM elements h_{ij}

$\Longrightarrow \; g(\phi) = 1 + 2\ Re(h_{+-})\cos 2\phi - 2\ Im(h_{+-})\sin 2\phi$

Smallness value of $\langle h_{+-} \rangle \Rightarrow$ Azimuthal angle distribution $\approx$ FLAT

$\Longrightarrow \; f(\cos\theta_{1,2}) = (3h_{00} - 1)\cos^2\theta_{1,2} + (1 - h_{00})$

Polar Angle distribution $\approx \cos^2\theta$ because of the dominant longitudinal part h_{00}.

4.1 Branching Ratios and Asymmetries

⋆ Vector Mesons with large widthes like $\Gamma_\rho = \ 150\mathrm{MeV}/c^2$

$\Longrightarrow$ Branching Ratio of each channel computed by Monte-Carlo methods according to :

$$d\Gamma(B \to V_1 V_2) \ = \ \frac{1}{8\pi^2 M} \ |\mathcal{M}(B \to V_1 V_2)|^2 \ \frac{d^3\vec{p}_1}{2E_1} \ \frac{d^3\vec{p}_2}{2E_2} \ \delta^4(P - p_1 - p_2) \ .$$

⋆ For a fixed value of ${q^2}/{{m_b}^2}$, BRs depend strongly on the Form Factor model
$\Longrightarrow$ Variation up to a factor 2

⋆ Relative Difference between two conjugate branching ratios,
$Br(B \to f)$ and $Br(\bar{B} \to \bar{f})$,
is $\approx$ independent of the form-factor models.

⋆ Global Asymmetry :

$$\mathcal{A}_{CP} = \frac{Br(B \to f) - Br(\bar{B} \to \bar{f})}{Br(B \to f) + Br(\bar{B} \to \bar{f})} \quad \leq 2\%$$

4.2 Asymmetry according to $\pi\pi$ Invariant Mass

$\star$ New and unexpected effect "discovered" wth the Differential Asymmetry according to the $\pi\pi$ invariant mass.

$$a_{CP}(m) = \frac{\Gamma_m(B \to f) - \bar{\Gamma}_m(\bar{B} \to \bar{f})}{\Gamma_m(B \to f) + \bar{\Gamma}_m(\bar{B} \to \bar{f})}$$

$\Longrightarrow$ $a_{CP}(m)$ is amplified in the vicinity of the ω Resonance mass :

$\pm 20\ \mathrm{MeV}/c^2$ around $M_\omega = 782\mathrm{MeV}/c^2$

$\star$ $a_{CP}(m)$ is $\approx 15\%$ in the case of $B^0 \to K^{*0}\rho^0(\omega)$ and

$\star$ $a_{CP}(m) = 24\%$ in the channel $B^\pm \to \rho^\pm\rho^0(\omega)$

$\Downarrow$

Asymmetry almost independent of the Form Factor model

Only explanation of this effect : $\rho^0 - \omega$ MIXING

$\star$ Results predicted analytically in the channel $B \to VP \Rightarrow \pi^+\pi^-\pi$
(O.Leitner, X.Guo, A.W.Thomas, Phys.Rev. D66(2002), 096008)

4.3 Ratio P/T

$$\frac{P}{T} = r \exp i\phi \exp i\delta \quad , \quad r = r' \left| \frac{V_{tb} V_{tq}^*}{V_{ub} V_{uq}^*} \right|$$

- r' $\equiv$ "Naked" ratio P/T

$*$ r' is almost constant over the $\pi\pi$ interval mass , but it varies very sharply in the ω interval, 760 MeV $\rightarrow$ 820 MeV.

$*$ Variation almost independent of q^2/m_b^2

$*$ Variation depending on N_c^{eff} .

Channel	Usual Values	ω Interval
$B^0 \rightarrow K^{*0}\rho^0(\omega)$	$0.08 \leq r' \leq 0.30$	0.60
$B^+ \rightarrow K^{*+}\rho^0(\omega)$	$0.04 \leq r' \leq 0.05$	0.06
$B^+ \rightarrow \rho^+\rho^0(\omega)$	$r' \approx 0.016$	0.05

4.4 Final Phase-Shift δ

• $\delta =$ Strong phase difference between the Penguin and the Tree diagrams

It depends on :

$*$ Intrinsic Phase-Shift induced by the Top Quark in the Penguin diagram.

$*$ Phase-Shift induced by the complex Wilson Coefficients.

$* \rho^0 - \omega$ mixing in the $\pi\pi$ final state interactions.

• δ depends strongly on N_c^{eff} and on the ratio ${q^2}/{m_b}^2$

$* \delta$ is almost constant in all the $\pi\pi$ invariant mass interval, except in the ω Resonance window $\equiv 770 \Rightarrow 790$ MeV.

$\star \Delta\delta = 80^\circ \rightarrow 100^\circ$ in the channel $K^{*0}\rho^0(\omega)$
$\star \Delta\delta = 5^\circ \rightarrow 25^\circ$ in the channel $K^{*+}\rho^0(\omega)$
and $\Delta\delta$ has an Unexpected Behaviour in the $\rho^+\rho^0(\omega)$ case !?

Why the behaviour of δ is so IMPORTANT ?

because ...

Physical Consequences

$\star$ New Method to detect and to measure Direct CP Violation both in B^0 and $B^{\pm}$ decays.

$\star$ According to the analytical expression :

$$a_{CP}^{dir} = \frac{A^2 - \bar{A}^2}{A^2 + \bar{A}^2} = \frac{-2\ r\ \sin\delta\ \sin\phi}{1 + r^2 + 2\ r\ \cos\delta\ \cos\phi}\ .$$

a_{CP}^{dir}, ratio r and δ depend on $\pi\pi$ Invariant Mass .

$\Longrightarrow$ Possibility of measuring $\sin\Phi$, where Φ is a Weak Angle resulting from the CKM Matrix elements :

Channel $B \to \rho^0(\omega)K^*$ $\Longrightarrow \Phi = Arg[V_{tb}V_{ts}^*/V_{ub}V_{us}^*] = \gamma$

Channel $B \to \rho^0(\omega)\rho^{\pm}$ $\Longrightarrow \Phi = Arg[V_{tb}V_{td}^*/V_{ub}V_{ud}^*] = \beta + \gamma = \Pi - \alpha$

Branching Ratios

channel	$\frac{q^2}{m_b^2}$	BSW	GH
$B^0 \to K^{*0}\rho^0(\omega)$	0.3	2.0	1.0
	0.5	1.6	0.80
$B^+ \to K^{*+}\rho^0(\omega)$	0.3	6.1	3.6
	0.5	4.0	2.4
$B^+ \to \rho^+\rho^0(\omega)$	0.3	22.3	12.5
	0.5	22.8	12.2
$B^0 \to K^{*+}\rho^-$	0.3	5.9	3.2
	0.5	3.3	1.8

Table 1: B^0, B^+ branching ratios (in units of 10^{-6}) using either the BSW or GH form factor models, for $q^2/m_b^2 = 0.3(0.5)$, with $N_{cmax}^{b\to s} = 2.84(2.82)$, $N_{cmax}^{b\to d} = 2.01(1.95)$, $\rho = 0.229$ and $\eta = 0.325$.

Differential Asymmetries

channel	$\frac{q^2}{m_b^2}$	BSW	GH
$B^0 \to K^{*0}\rho^0(\omega)$	0.3	-13	-16
	0.5	-11	-14
$B^+ \to K^{*+}\rho^0(\omega)$	0.3	-2	-1
	0.5	-25	-24
$B^+ \to \rho^+\rho^0(\omega)$	0.3	20.0	23.0
	0.5	17.0	19.0

Table 2: Differential Asymmetries $(\%)$ in the vicinity of the ω resonance using either the BSW or GH form factor models, for $q^2/m_b^2 = 0.3(0.5)$, with $N_{cmax}^{b\to s} = 2.84(2.82)$, $N_{cmax}^{b\to d} = 2.01(1.95)$, $\rho = 0.229$ and $\eta = 0.325$.

Comparison with recent Experimental Results

First Results of CDF (Phys.Rev.Let.85, 4668)

$B \to J/\psi K^*$ $\Longrightarrow$ Longitudinal Decay Amplitude dominates :

$$|H_0|^2 = 0.59 \pm 0.06 \pm 0.01$$

Belle Collaboration

$$\mathrm{Br}(B^+ \to \rho^+ \rho^0) = \left(31.7 \pm 7.1(stat.)^{+3.8}_{-6.7}(sys.)\right) \times 10^{-6}$$

Longitudinal Polarization

$$\Gamma_L/\Gamma \;=\; (94.8 \pm 10.6(stat.) \pm 2.1(syst.))\%$$

$\star$ Computed Value of Longitudinal Polarization : $h_{00} \approx 90\%$

Babar Collaboration (hep-ex/0307026)

Channel	Br($\times 10^{-6}$)	$f_L = \|H_0\|^2$	A_{CP}
$\rho^0 K^{*+}$	$10.6^{+3.0}_{-2.6} \pm 2.4$	$0.96^{+0.04}_{-0.15} \pm 0.04$	$0.20^{+0.32}_{-0.29} \pm 0.04$
Our results	$2.3 \rightarrow 5.8$	87%	$-6.4\% \rightarrow -22\%$
$\rho^0 \rho^+$	$22.5^{+5.7}_{-5.4} \pm 5.8$	$0.97^{+0.03}_{-0.07} \pm 0.04$	$-0.19 \pm 0.23 \pm 0.03$
Our results	$11.0 \rightarrow 20.0$	90%	$-8.5\% \rightarrow -10\%$

5 Perspectives and Conclusion

• Helicity Formalism $\equiv$ Very Powerful Tool

$\Longrightarrow$ Rigorous and Detailed Calculations carried out completely

$\Longrightarrow$ Formalism available to any $B \to VV$ channel

$\Longrightarrow$ Implemented in the LHCb generator code

• Naive Factorization very useful in Weak Hadronic B decays.
BUT... It involves some uncertainties.

$\Longrightarrow$ replaced soon by QCDFactorization.

• Longitudinal Polarization largely dominating $\to$ confirmed by recent experimental results (BaBar and Belle).

• Importance of the Form Factor Model, especially for the estimation of the different Branching Ratios.

• Importance of the $\rho^0 - \omega$ Mixing in the enhancement of Direct CPV with regard to $\pi^+\pi^-$ Invariant Mass.

$\Downarrow\Downarrow\Downarrow\Downarrow$

$\star$ New Way to look for CP Violation

$\star$ New Methods (in the future) for measuring Angles γ and α

$\star$ Performing Generators require good collaboration between Theorists and Experimentalists

$\Downarrow\Downarrow\Downarrow\Downarrow$

Good Success for Phenomenology

$\star$ Waiting with (im)patience for LHC and for High Statistics Data

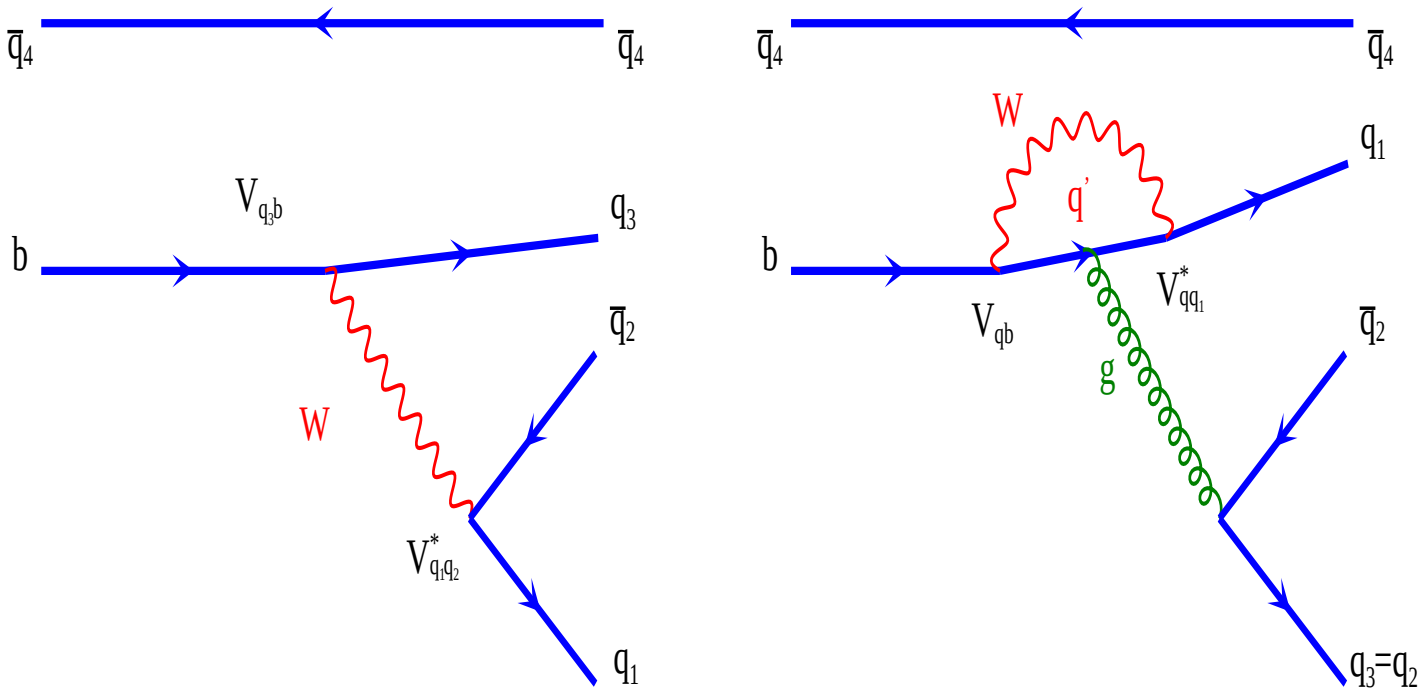


Figure 1: Tree diagram (left), and QCD-penguin diagram (right), for B decays.

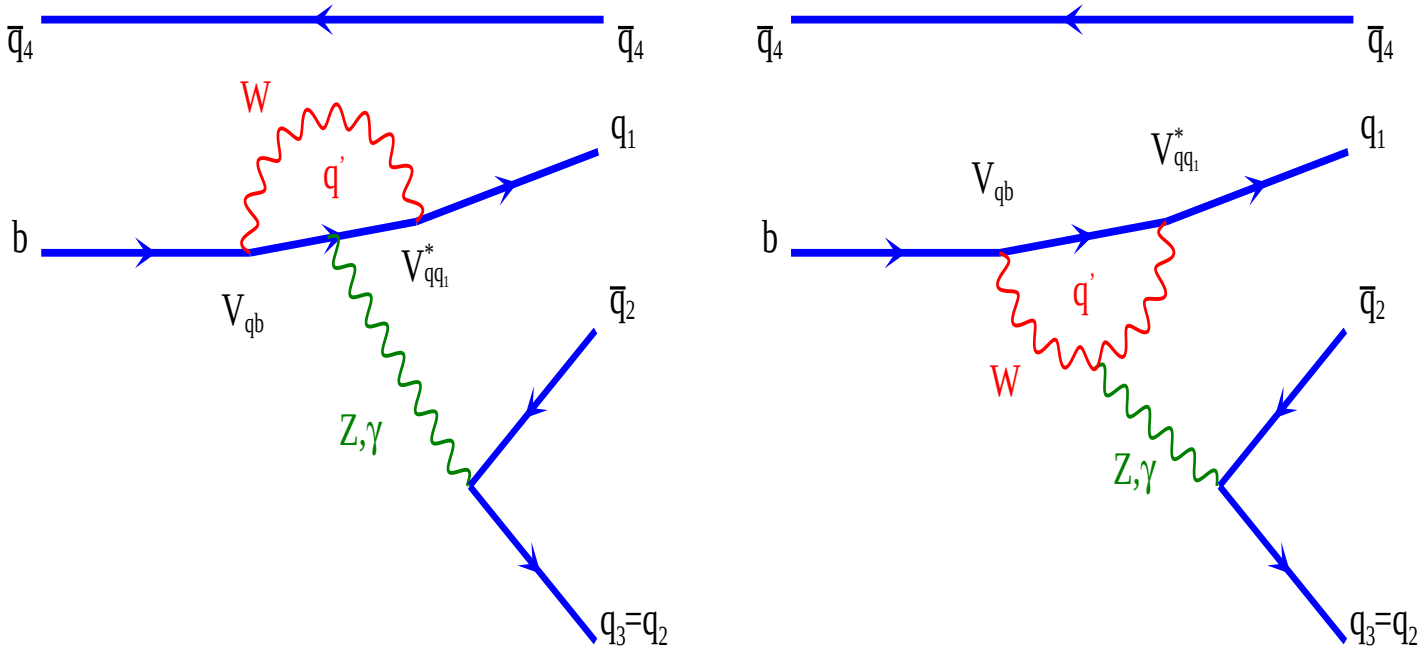
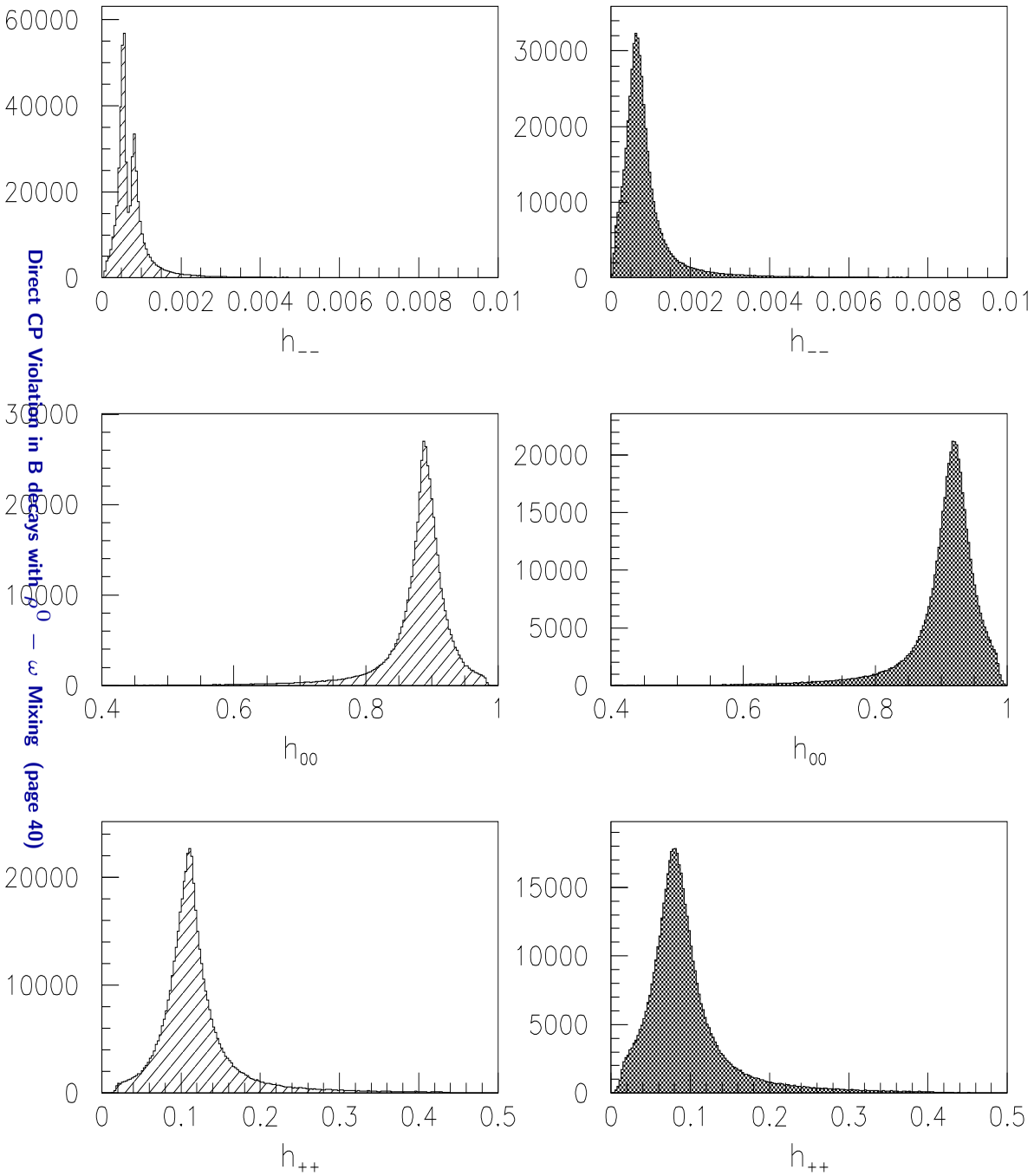
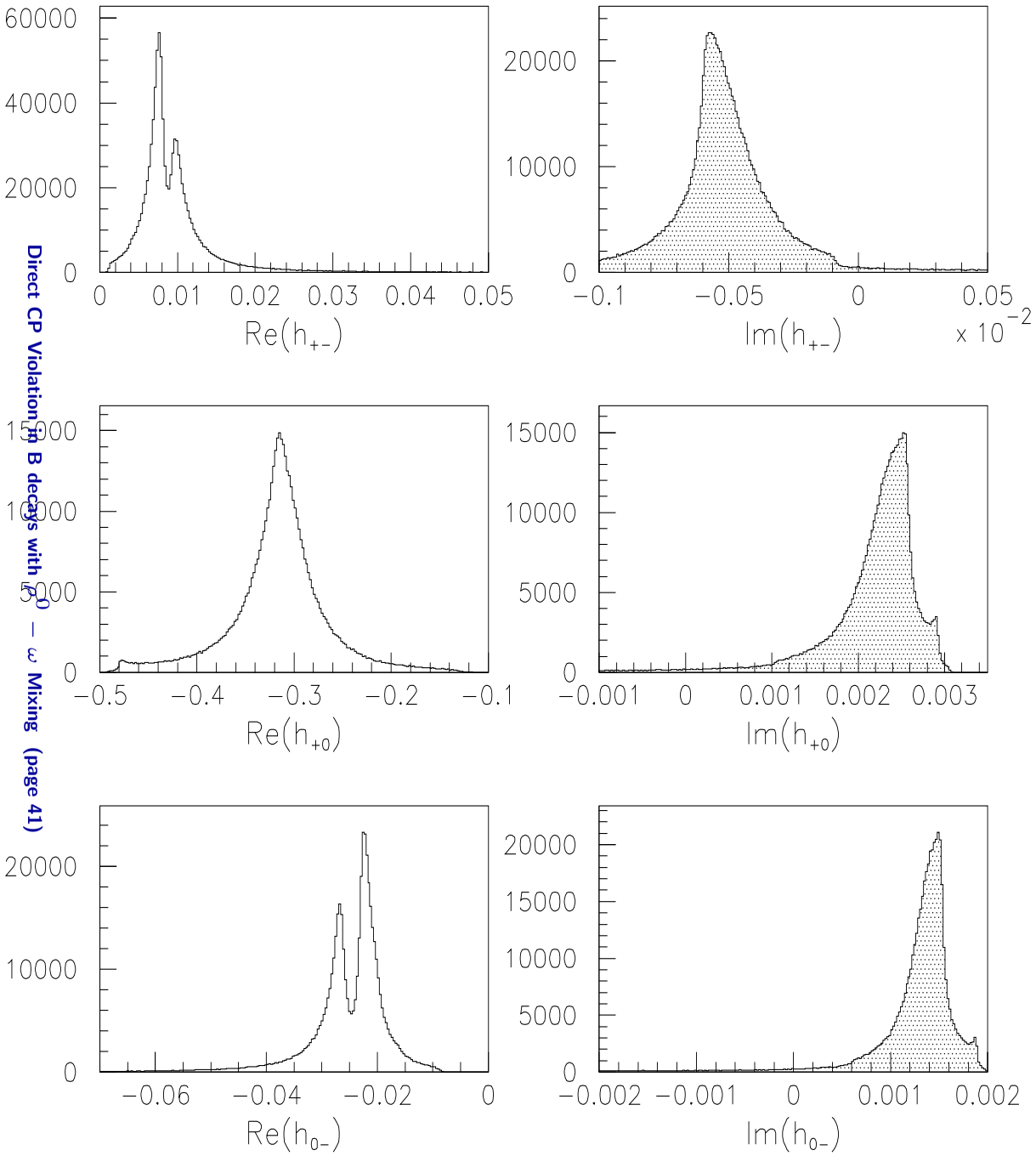
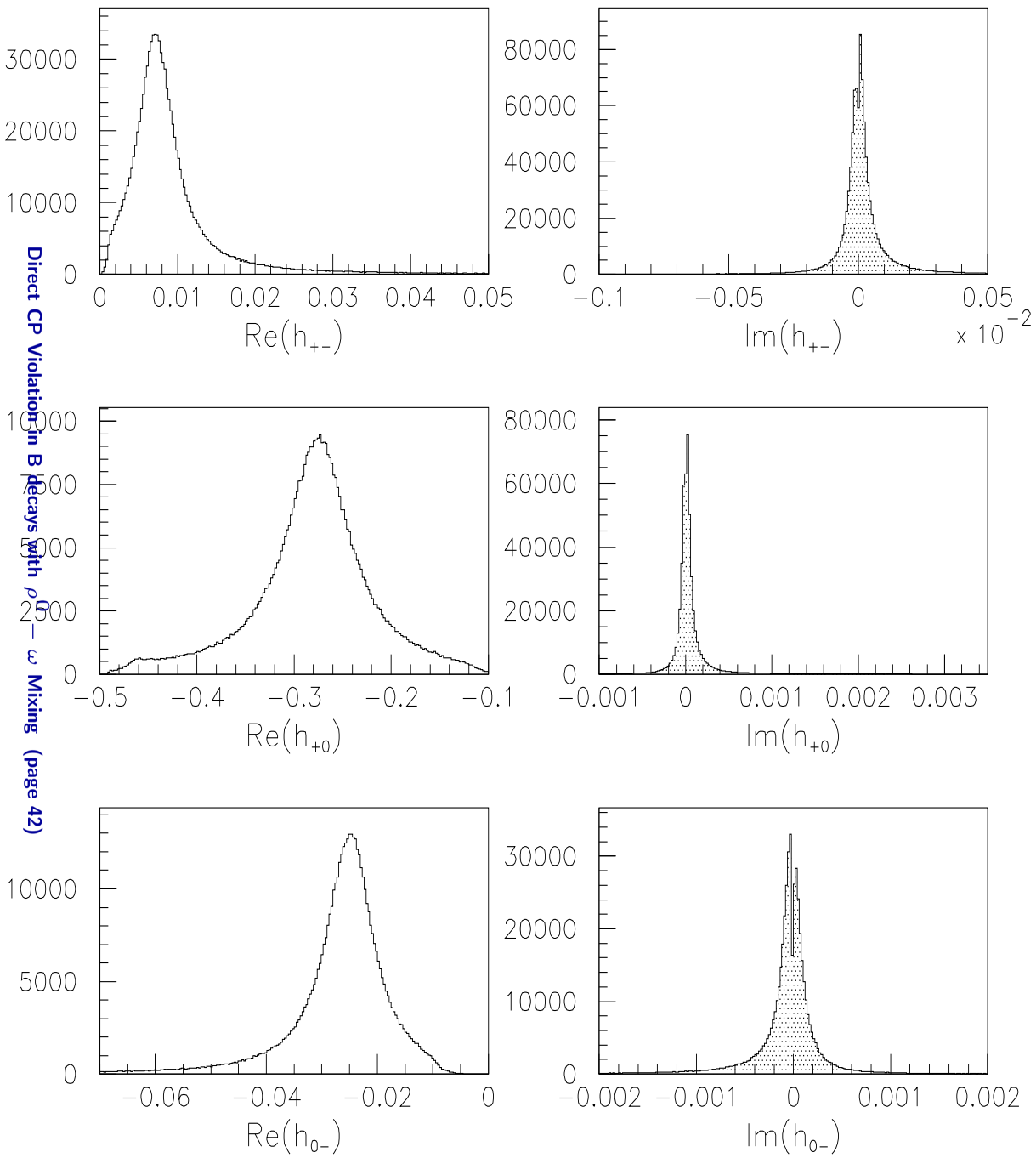
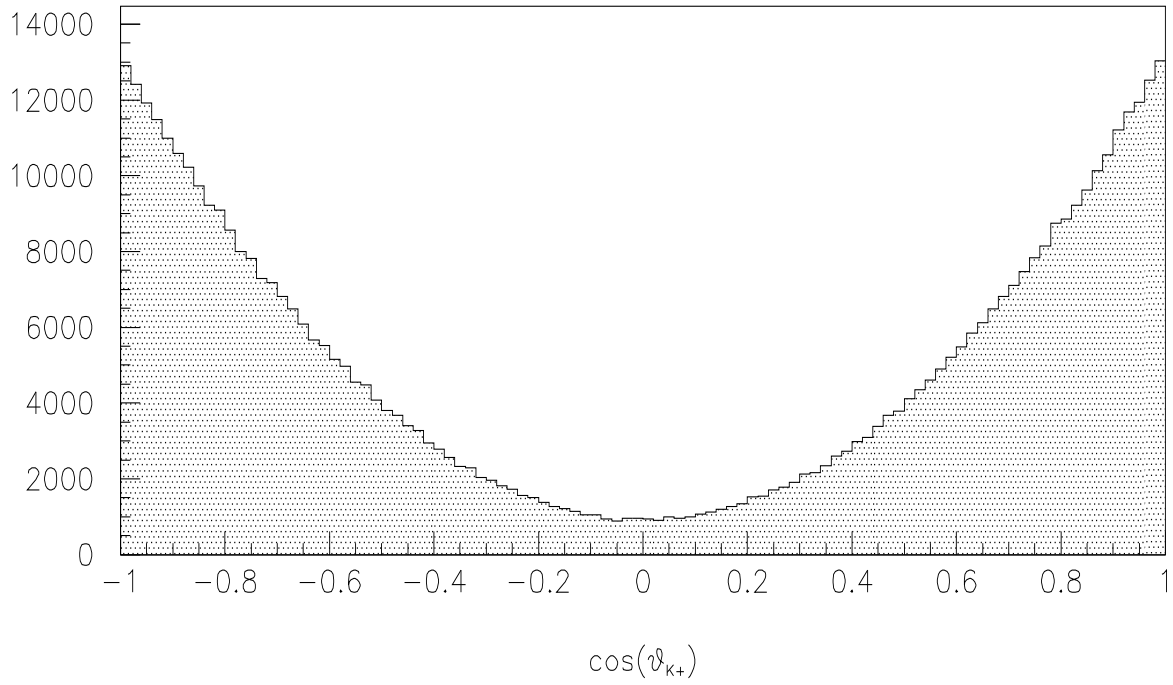


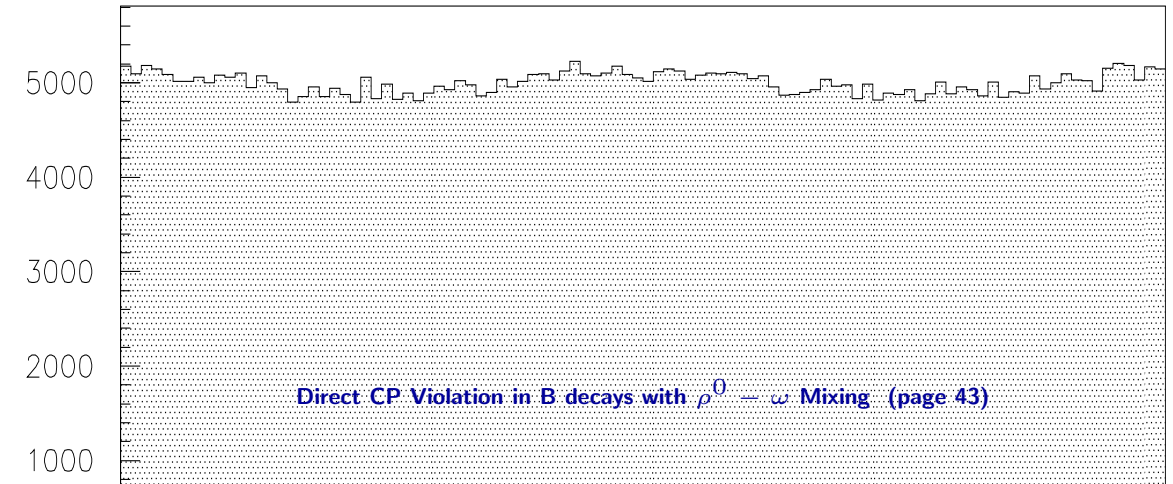
Figure 2: Electroweak-penguin diagram (left), and electroweak-penguin diagram with coupling between Z, γ and W (right), for B decays.


60000
40000
20000
0
0 0.002 0.004 0.006 0.008 0.01
h__
30000
20000
10000
0
0 0.002 0.004 0.006 0.008 0.01
h__
30000
20000
10000
0
0.4 0.6 0.8 1
h00
20000
15000
10000
5000
0
0.4 0.6 0.8 1
h00
20000
10000
0
0 0.1 0.2 0.3 0.4 0.5
h++
15000
10000
5000
0
0 0.1 0.2 0.3 0.4 0.5
h++

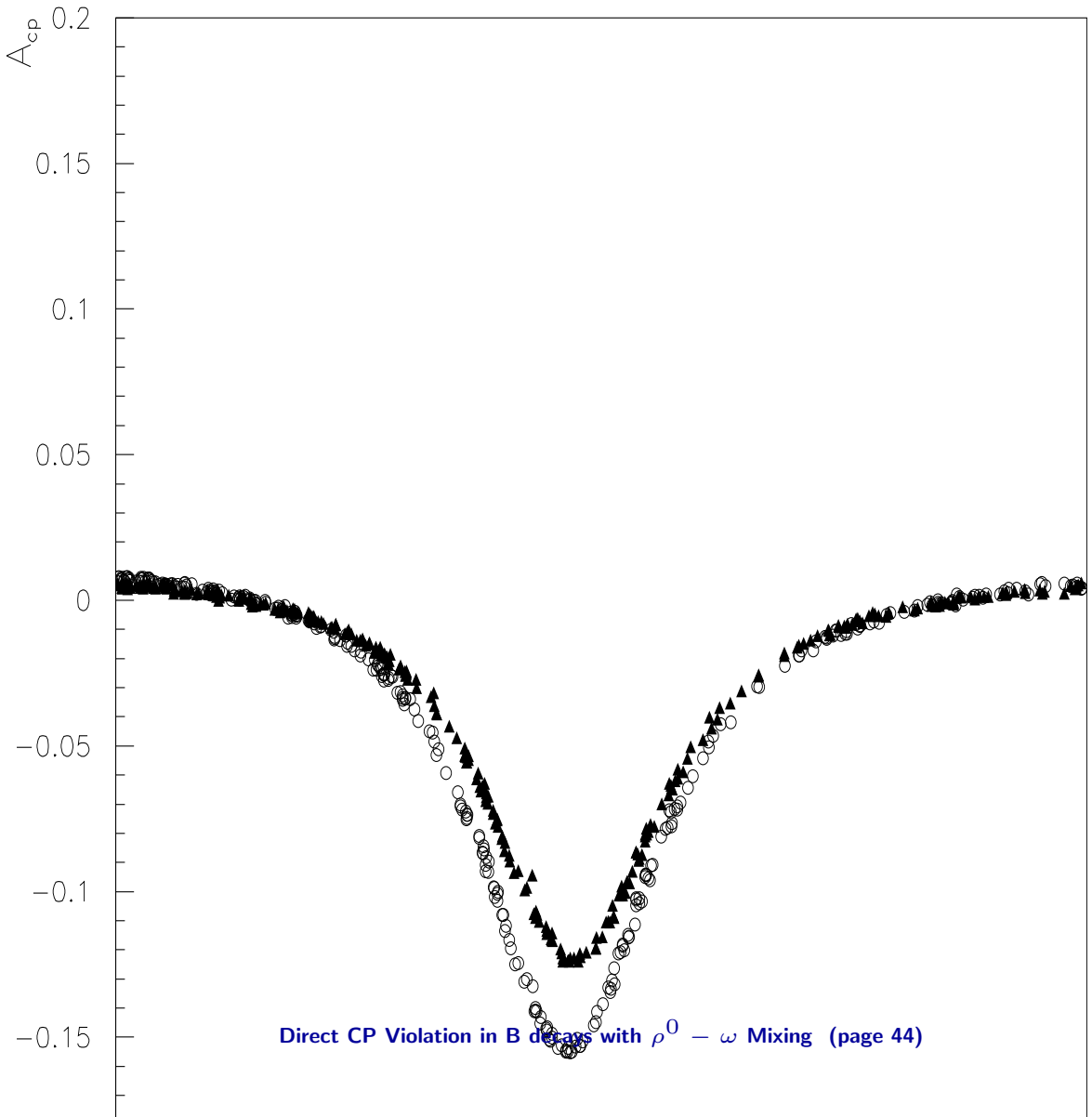

Re(h+-)
Im(h+-)
x 10^-2
Re(h+0)
Im(h+0)
Re(h0-)
Im(h0-)

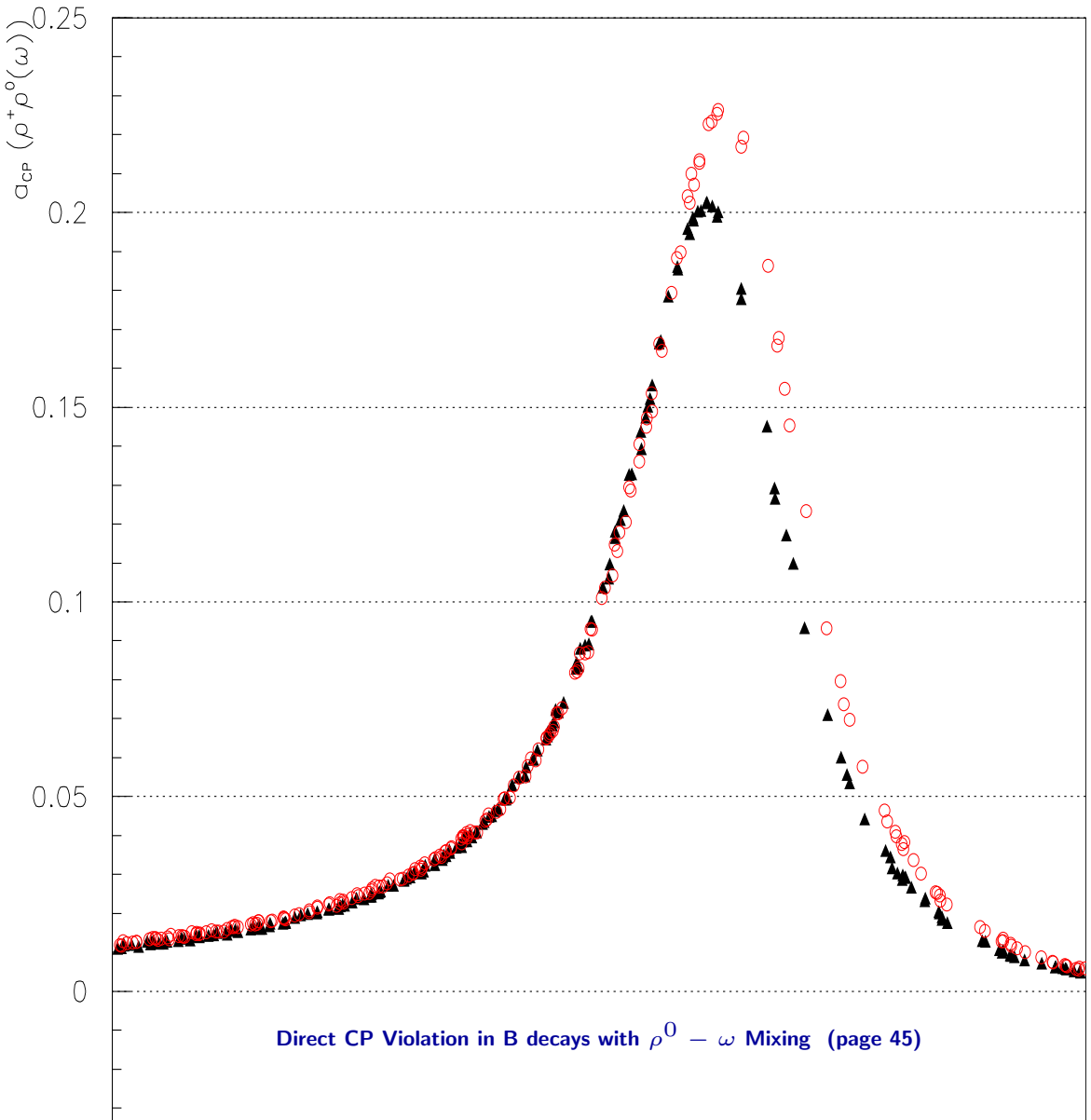

Re(h+-)
Im(h+-)
x 10^-2
Re(h+0)
Im(h+0)
Re(h0-)
Im(h0-)

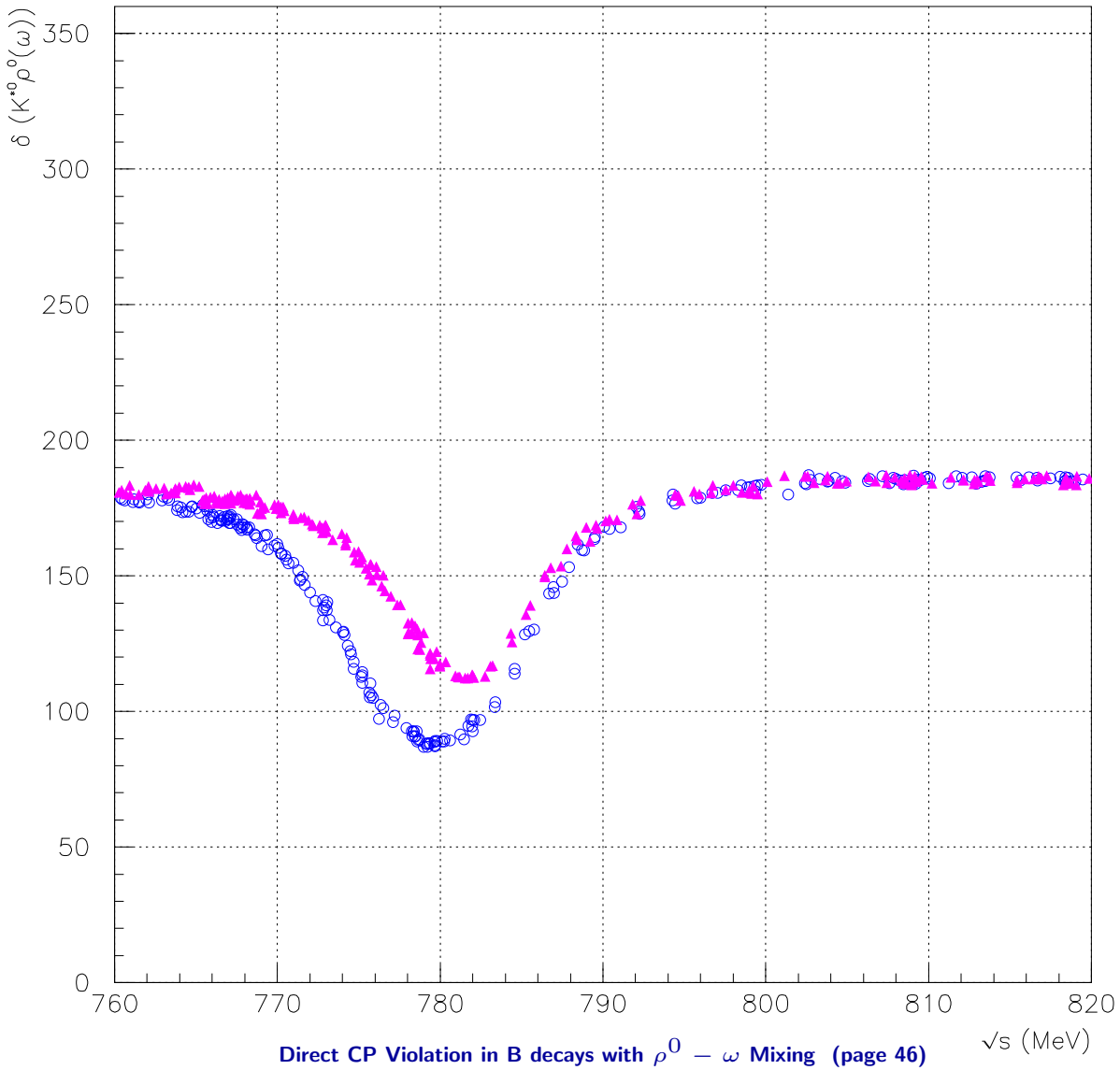


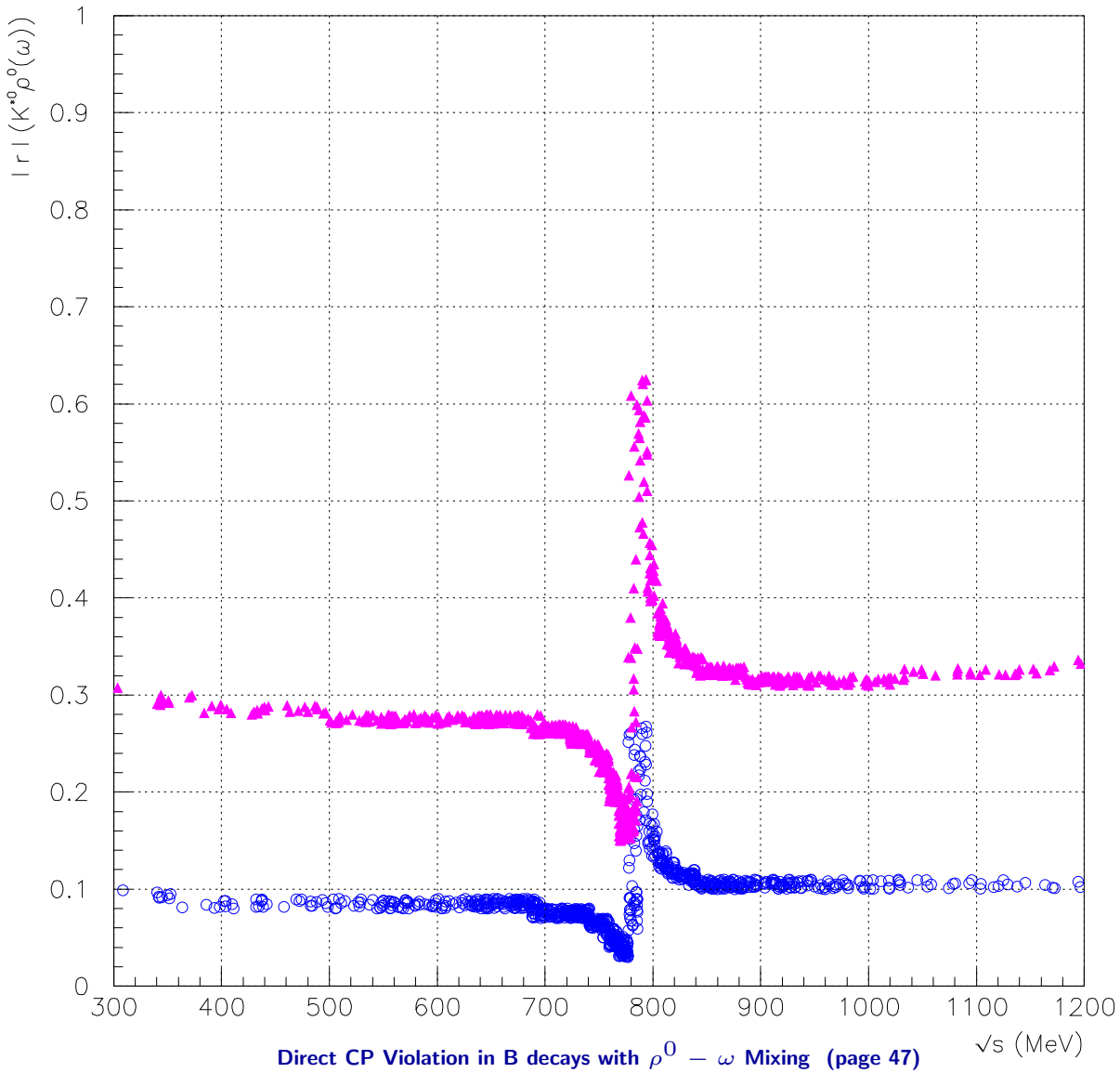


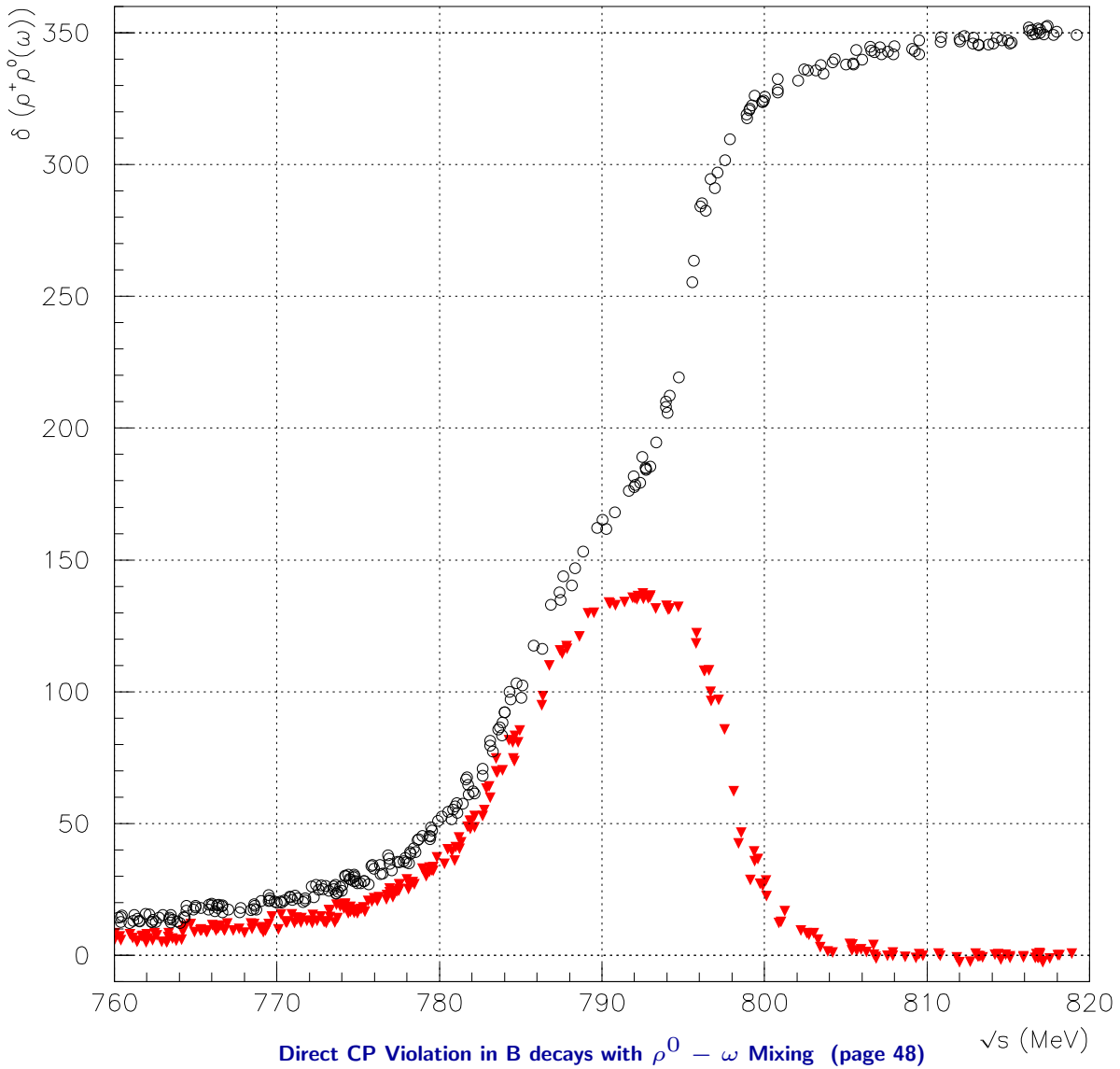
Direct CP Violation in B decays with $\rho^0 - \omega$ Mixing (page 43)

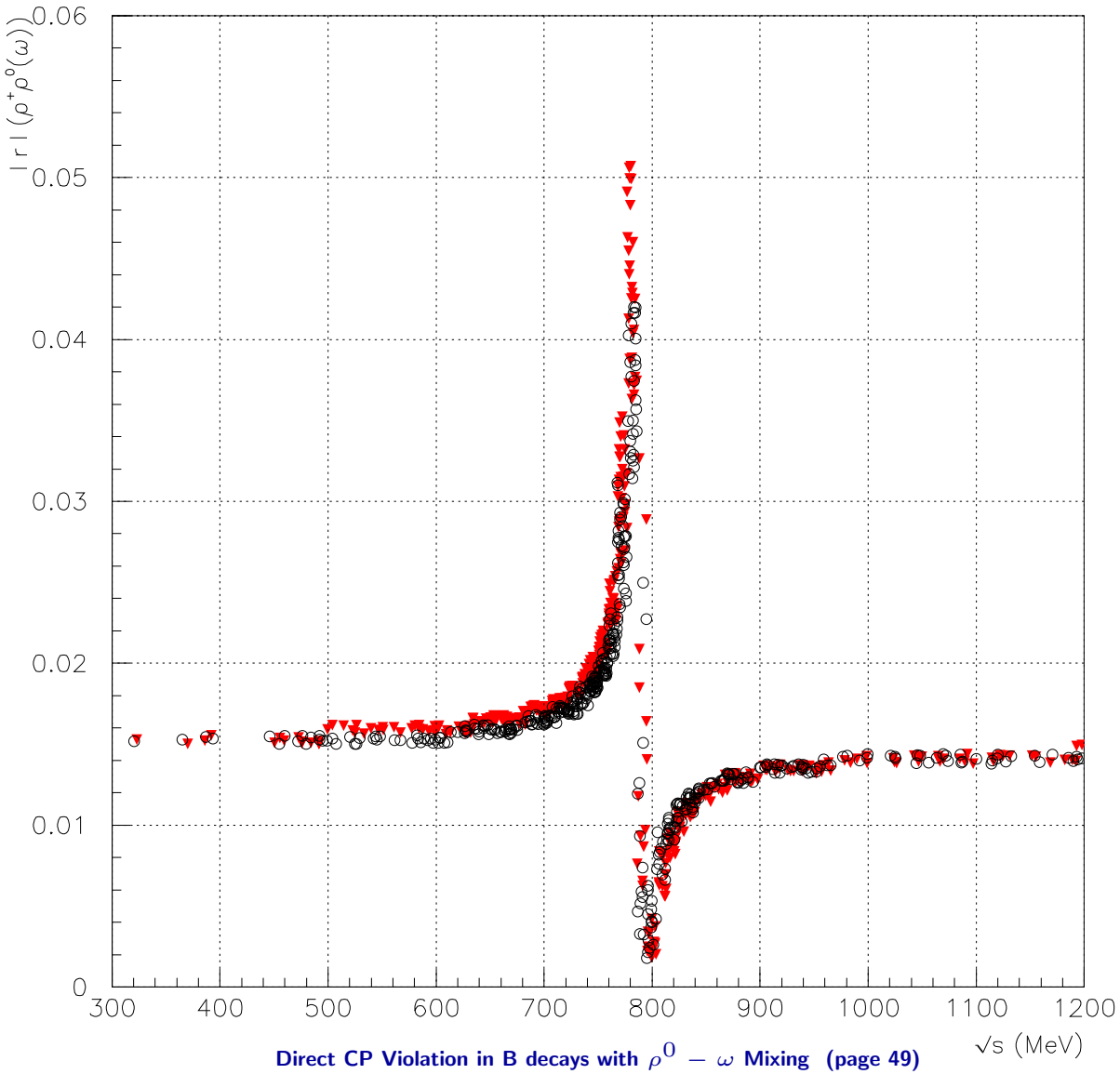

A_{cp}
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15


$a_{CP}(\rho^+\rho^0(\omega))$
0.25
0.2
0.15
0.1
0.05
0
Direct CP Violation in B decays with $\rho^0 - \omega$ Mixing (page 45)


δ (K*0ρ0(ω))
350
300
250
200
150
100
50
0
760
770
780
790
800
810
820
√s (MeV)


|r|(K*0ρ0(ω))
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
300
400
500
600
700
800
900
1000
1100
1200
√s (MeV)


δ (ρ⁺ρ⁰(ω))
350
300
250
200
150
100
50
0
760
770
780
790
800
810
820
√s (MeV)


0.06
0.05
0.04
0.03
0.02
0.01
0
300
400
500
600
700
800
900
1000
1100
1200
√s (MeV)
| r | (ρ⁺ρ⁰(ω))