On the Stability Criteria for Hierarchical Three-Body Systems

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It is often important to decide if a given hierarchical triple star system is stable over an extended period of time. Here, we test a stability criterion, modified from earlier work, where we use the closest approach ratio Q of the third star to the inner binary centre of mass in their initial osculating orbits. We study by numerical integration the orbits of over 100,000 triple systems varying masses, outer and inner eccentricities, and inclinations i. The definition of the instability is either the escape of one of the bodies, or the exchange of the members between the inner and outer systems. The dependence of Q_{st} (the smallest Q value which allows the system to be stable over $N = 10,000$ revolutions of the initial outer orbit) on the mass values and on the outer orbit eccentricity e_{out} is briefly explored, and it is also found to agree with the analytical theory. The final stability limit formula is

$$Q_{st} = 10^{1/3} A[(f \cdot g)^2/(1-e_{out})]^{1/6}$$

where the coefficient $A = 1$ should be used in N-body experiments, and $A = 2$ when the absolute long term stability is required. The functions $f(e_{in}, cosi)$ and $g(m_1, m_2, m_3)$ are

$$f(e_{in}, \cos i) = \left\{1 - \frac{2}{3}e_{in}\left[1 - \frac{1}{2}e_{in}^2\right] - 0.3\cos i\left[1 - \frac{1}{2}e_{in} + 2\cos i\left(1 - \frac{5}{2}e_{in}^{3/2} - \cos i\right)\right]\right\}.$$

$$g(m_1, m_2, m_3) = \left(1 + \frac{m_3}{m_1 + m_2}\right).$$

At the limit of $e_{in} = i = m_3 = 0$, $f \cdot g = 1$.