Analysis of degenerate solutions with variational equations

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Abstract

We suggest a new method of analysis of degenerate solutions in families of periodic solutions to nonlinear ODEs depending on parameters. The analysis exploits geometric properties of families of solutions as analytical manifolds. The method is based upon the computation of variational equations of higher order. These computations can be done effectively only with the use of a computer algebra system. We demonstrate on some examples that such phenomena as bifurcations of periodic solutions, generating solutions, loss of stability etc. are expressed in terms of solutions to variational equations is simple relations, although intermediate computations can be forbidding. We use computer algebra to generate systems of variational equations, to analyze and solve them when possible, and to transform them into computational programs for further numerical analysis.