Vector Mesons in Quasilocal Quark Models

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Abstract

We consider the Quasilocal Quark Model of NJL type including vector and axial-vector four-fermion interaction with derivatives. In the presence of a strong attraction in the scalar channel the chiral symmetry is spontaneously broken and as a consequence the composite meson states are generated in the vector and axial-vector channels. The appropriate set of Chiral Symmetry Restoration Sum Rules in these channels are imposed as matching rules to QCD at intermediate energies. The mass spectrum and some decay constants for ground and excited meson states are calculated.

1. Introduction

The QCD-inspired quark models with four-fermion interaction are often applied for the effective description of low-energy QCD in the hadronization regime. The local four-fermion interaction is involved to induce the dynamical chiral symmetry breaking (DCSB) due to strong attraction in the scalar channel. As a consequence, the dynamical quark mass m_{dyn} is created, as well as an isospin multiplet of pions, massless in the chiral limit, and a massive scalar meson with mass $m_\sigma = 2m_{dyn}$ arise. However it is known from experiment that there are series of meson states with equal quantum numbers and heavier masses, in particular, $0^{-+}(\pi, \pi', \pi'', ...)$; $0^{++}(\sigma(f_0), \sigma', \sigma'', ...)$; $1^{--}(\rho, \rho', \rho'', ...)$. Due to confinement, one expects an infinite number of such excited states with increasing masses. Therefore in order to describe the physics of those resonances at intermediate energies one can extend the quark model with local interaction of the Nambu-Jona-Lasinio (NJL) type [1-10] taking into account higher-dimensional quark operators with derivatives, i.e. quasilocal quark interactions [11]. For sufficiently strong couplings the new operators promote the formation of additional meson states. Such a quasilocal approach (see also [12-16]) represents a systematic extension of the NJL-model towards the complete effective action of QCD where many-fermion vertices with derivatives possess the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. In the effective action of QuasilocalModels of NJL type (QNJLM) the low-energy gluon effects are hidden in the coupling constants. The alternative schemes including the condensates of low-energy gluons can be found in [6,17].

At intermediate energies the correlators of QNJLM can be matched [18] to the O.P.E. of QCD correlators [19]. This matching realizes the correspondence to QCD and improves the predictivity of QNJLM. It is based on the large-N_c approach which is equivalent to planar QCD [20, 21]. In this approximation the correlators of color-singlet quark currents are saturated by narrow meson resonances. In particular, the two-point correlators of scalar, pseudoscalar, vector and axial-vector quark currents are represented by the sum of related meson poles in Euclidean space-time:

$$\Pi^C(p^2) = \int d^4x \exp(ipx)\langle T(\bar{q}\Gamma q(x)\bar{q}\Gamma q(0))\rangle_{planar} = \sum_n \frac{Z_n^C}{p^2 + m_{C,n}^2} + D_0^C + D_1^C p^2, \qquad (1)$$

$$C \equiv S, P, V, A; \qquad \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = const.$$

The last two terms both in the scalar-pseudoscalar and in the vector-axial-vector channel represent a perturbative contribution with D_0 and D_1 being contact terms required for the regularization of infinite sums. On the other hand the high-energy asymptotics is provided [19] by the perturbation theory and the operator product expansion due to asymptotic freedom of QCD. Therefrom the above correlators increase at large p^2,

$$\Pi^C(p^2) \mid_{p^2\to\infty} \sim p^2 \ln\frac{p^2}{\mu^2}. \qquad (2)$$

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When comparing the two expressions one concludes that the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotics.

Meantime the differences of correlators of opposite-parity currents rapidly decrease at large momenta [18, 22]:

$$(\Pi^P(p^2) - \Pi^S(p^2))_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + O(\frac{1}{p^6}), \qquad \Delta_{SP} \simeq 24\pi\alpha_s < \bar{q}q >^2 \tag{3}$$

and [19, 23, 24]

$$(\Pi^V(p^2) - \Pi^A(p^2))_{p^2\to\infty} \equiv \frac{\Delta_{VA}}{p^6} + O(\frac{1}{p^8}), \qquad \Delta_{VA} \simeq -16\pi\alpha_s < \bar{q}q >^2 \tag{4}$$

where we have defined in the V,A channels

$$\Pi^{V,A}_{\mu\nu}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\mu p_\nu)\Pi^{V,A}(p^2), \tag{5}$$

and the vacuum dominance hypothesis [19] in the large-N_c limit is adopted.

Therefore the chiral symmetry is restored at high energies and the two above differences represent a genuine order parameter of CSB in QCD. As they decrease rapidly at large momenta one can perform the matching of QCD asymptotics by means of few lowest lying resonances that gives a number of constraints from Chiral Symmetry restoration. They may be used both for obtaining some additional bounds on the model parameters and for calculating of some decay constants (see, some examples in [23, 26] and references therein). In the present work the QNJL model is considered with two channels where two pairs of vector and axial-vector mesons are generated. Respectively it is expected to reproduce the lower part of QCD meson spectrum in the planar limit and the leading asymptotics of chiral symmetry restoration for higher energies.

2. Quasilocal Quark Model of NJL-type

The low energy effective action of QCD in the quark sector has a qualitatively different structure depending on whether it is built at the CSB scale Λ by means of perturbation theory [11, 25] or below the CSB scale when the formation of light pseudoscalar meson is implemented manifestly [22, 26, 27]. We will discuss only the first case where the QNJL model [11] extends the NJL one with chiral symmetry broken due to strong attractive four-fermion forces in the color-singlet scalar channel. When dynamical quark masses are supposed to be sufficiently smaller than the CSB scale (cutoff) the minimal n-channel lagrangian of the QNJLM has [11, 14] the following form,

$$L = \bar{q}i\hat{\partial}q + L^I_{SP}, \tag{6}$$

where indices S, P denote the scalar and pseudoscalar case and

$$L^I_{SP} = \frac{1}{4N_f N_c \Lambda^2}\sum_{k,l=1}^{n} a_{kl}[\bar{q}f_k q \cdot \bar{q}f_l q + \bar{q}f_k i\gamma_5 q \cdot \bar{q}f_l i\gamma_5 q]. \tag{7}$$

a_{kl} represents here a symmetric matrix of real coupling constants. We will restrict ourselves by the case $n = 2$ and describe the ground meson states and their first excitations only. Let us choose the polynomial formfactors as being orthogonal on the unit interval,

$$\int_0^1 f_k(\tau)f_l(\tau)d\tau = \delta_{kl}\,. \tag{8}$$

We select out here:

$$f_1(\tau) = 2 - 3\tau; \qquad f_2(\tau) = -\sqrt{3}\tau; \qquad \tau \equiv -\frac{\partial^2}{\Lambda^2}. \tag{9}$$

As this model interpolates the low-energy QCD action it is supplied with a cutoff Λ (of order of the CSB scale) for virtual quark momenta in quark loops. For simplicity we neglect the isospin effects encoded in quarks of different flavor, $N_f = 1$. Moreover the chiral limit $m_q = 0$ is implied throughout this paper.

Let us now incorporate the vector and axial-vector interaction with appropriate matrix of couplings b_{kl},

$$L^I_{SPVA} = \frac{1}{4N_c\Lambda^2}\sum_{k,l=1}^{2}\{a_{kl}[\bar{q}f_k q\cdot\bar{q}f_l q + \bar{q}f_k i\gamma_5 q\cdot\bar{q}f_l i\gamma_5 q]$$

$$+b_{kl}[\bar{q}f_k i\gamma_\mu q\cdot\bar{q}f_l i\gamma_\mu q + \bar{q}f_k i\gamma_\mu\gamma_5 q\cdot\bar{q}f_l i\gamma_\mu\gamma_5 q]\}. \qquad (10)$$

Following the standard procedure we introduce auxiliary scalar, pseudoscalar, vector and axial-vector fields,

$$L^I_{aux} = \sum_{k=1}^{2} i\bar{q}(\sigma_k + i\gamma_5\pi_k + i\rho_{k,\mu}\gamma_\mu + ia_{k,\mu}\gamma_\mu\gamma_5)f_k q$$

$$+N_c\Lambda^2\sum_{k,l=1}^{2}\{\sigma_k a^{-1}_{kl}\sigma_l + \pi_k a^{-1}_{kl}\pi_l + \rho_{k,\mu}b^{-1}_{kl}\rho_{l,\mu} + a_{k,\mu}b^{-1}_{kl}a_{l,\mu}\}. \qquad (11)$$

The observables should not depend on the cutoff Λ. The scale invariance is achieved by appropriate prescription of cutoff dependence for effective coupling constants a_{kl}, b_{kl}. Namely, we require the cancellation of quadratic divergences and parametrize the matrices of coupling constants in the vicinity of polycritical point as follows,

$$8\pi^2 a^{-1}_{kl} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}; \qquad \Delta_{kl} \ll \Lambda^2$$

$$16\pi^2 b^{-1}_{kl} = \delta_{kl} - \frac{4}{3}\frac{\bar{\Delta}_{kl}}{\Lambda^2}; \qquad \bar{\Delta}_{kl} \ll \Lambda^2 \qquad (12)$$

The last inequalities provide the masses to be essentially less then the cutoff.

For strong four-fermion coupling constants $a_{kl} \geq 8\pi^2\delta_{kl}$ in the scalar channel the above interaction induces the dynamical chiral symmetry breaking. In the language of the theory of critical phenomena it is equivalent to developing our model around a critical or scaling point (a critical surface in our case) where the quantum system undergoes the second-order phase transition. The parameters Δ_{kl} just describe the deviation from a critical point and determine the physical mass of scalar mesons. The CSB is generated by the dynamic quark mass function corresponding to nontrivial v.e.v. of scalar fields σ_1, σ_2,

$$M(\tau) = \sigma_1 f_1(\tau) + \sigma_2 f_1(\tau); \qquad M_0 \equiv M(0) = 2\sigma_1. \qquad (13)$$

This CSB is characterized by the quark condensate,

$$<\bar{q}q> \simeq -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1 - \sqrt{3}\sigma_2). \qquad (14)$$

We remark that this quark condensate in general is not defined solely by the dynamical mass M_0 as in the conventional (one-channel) NJL model but depends also from the second v.e.v. σ_2. Only if the dynamical mass function is strictly constant, $\sigma_2 = -\sqrt{3}\sigma_1$, one reproduces the NJL relation

$$<\bar{q}q> \simeq -\frac{N_c\Lambda^2 M_0}{4\pi^2}. \qquad (15)$$

When integrating out the quark fields one comes to the bosonic effective action where the quadratic parts in boson field fluctuations are retained only to describe the meson mass spectrum,

$$S_{eff} \simeq \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\sum_{k,l=1}^{2}[\sigma_k\Gamma^{\sigma\sigma}_{kl}\sigma_l + \pi_k\Gamma^{\pi\pi}_{kl}\pi_l$$

$$+\rho_{k,\mu}\Gamma^{\rho\rho,\mu\nu}_{kl}\rho_{l,\nu} + a_{k,\mu}\Gamma^{aa,\mu\nu}_{kl}a_{l,\nu} + 2\pi_k\Gamma^{\pi a,\mu}_{kl}a_{l,\mu}]. \qquad (16)$$

Herein Γ_i (except the last one - $\Gamma^{\pi a}$, which corresponds to the $\pi - a$ mixing) have the structure:

$$\hat{\Gamma} = \hat{A}p^2 + \hat{B}, \qquad (17)$$

where the two symmetric matrices - the kinetic term $\hat{A}$ and the momentum independent part $\hat{B}$ - have been introduced. Several comments are in order.

(i) We neglect terms $O(\frac{1}{\Lambda^2})$ in all calculations.

(ii) The large-log approximation $\ln \frac{\Lambda^2}{M_0^2} \gg 1$ is adopted, where M_0 denotes the dynamical mass at zero external momentum. It is compatible with confinement.

(iii) In order that the physical masses were insensitive to Λ, the effective coupling constants should be weakly dependent on the cutoff, Δ_{kl}, $\bar{\Delta}_{kl} \sim M_0^2 \ln \frac{\Lambda^2}{M_0^2}$.

The physical mass spectrum can be found from solutions of the corresponding secular equation,

$$det(\hat{A}p^2 + \hat{B}) = 0; \qquad m_{phys}^2 = -p_0^2. \tag{18}$$

The consistency of mass equation in the vector and axial-vector channel imposes the similar scale condition on parameters $\bar{\Delta}_{kl}$ as in the scalar and pseudoscalar case:

$$\bar{\Delta}_{kl} = O(\ln \frac{\Lambda^2}{M_0^2}). \tag{19}$$

Let us display the mass-spectrum for ground meson states and their first excitations. We introduce the notations,

$$\sigma^2 \equiv \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2 > 0, \tag{20}$$

$$d \equiv 3\bar{\Delta}_{11} + 2\sqrt{3}\bar{\Delta}_{12} + \bar{\Delta}_{22}, \tag{21}$$

and take into account the consistency inequalities

$$\Delta_{22} < 0, \qquad \bar{\Delta}_{22} < 0. \tag{22}$$

Spectrum for scalar and pseudoscalar mesons

$$m_\sigma = 4\sigma_1 = 2M_0; \qquad m_\pi = 0; \tag{23}$$

$$m_{\pi'}^2 \simeq -\frac{4}{3}\Delta_{22} + \sigma^2; \qquad m_{\sigma'}^2 - m_{\pi'}^2 \simeq 2\sigma^2 > 0. \tag{24}$$

Spectrum for vector and axial-vector mesons

$$m_\rho^2 \simeq -\frac{\det\bar{\Delta}_{kl}}{2\bar{\Delta}_{22}\ln\frac{\Lambda^2}{M_0^2}}; \qquad m_a^2 \simeq m_\rho^2 + 6M_0^2; \tag{25}$$

$$m_{\rho'}^2 \simeq -\frac{4}{3}\bar{\Delta}_{22} - \frac{d}{6\ln\frac{\Lambda^2}{M_0^2}} - m_\rho^2; \qquad m_{a'}^2 - m_{\rho'}^2 \simeq \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2) \simeq 3\sigma^2 > 0. \tag{26}$$

The prime labels everywhere the corresponding excited meson state.

Let us comment on these spectra.

(i) The formula for m_σ is a well-known Nambu relation.

(ii) The expression for m_a is also a known result (see, for example, [7]-[10]). Combining it with the Nambu relation we arrive to the remarkable equality:

$$m_a^2 - m_\rho^2 \simeq \frac{3}{2}m_\sigma^2, \tag{27}$$

which has the same form as the relation for excited meson states (26) for $m_\pi = 0$.

(iii) In both spectra the radial excitations are logarithmically heavier than the corresponding ground meson states.

(iv) The value of mass splitting between excited states in both spectra is logarithmically suppressed as compared with the mass values. Therefore the masses of first excited meson states both in the scalar-pseudoscalar channel and in the vector-axial-vector one are approximately equal.

(v) Another prediction is that a mass of σ'-meson is a little more than that of π'-meson. The situation in the vector-axial-vector spectrum is opposite - the a'-meson is a little heavier than ρ'-one.

(vi) When taking into account the $\pi - a_1$ mixing the pion decay constant is determined by the physical meson masses,

$$f_\pi^2 \simeq \frac{N_c m_\sigma^2 m_\rho^2}{16\pi^2 m_{a_1}^2} \ln\frac{\Lambda^2}{M_0^2}\,, \tag{28}$$

wherefrom it follows that the logarithm of the cutoff is uniquely obtained from this relation in terms of physical parameters.

We identify σ with $f_0(400-1200)$, σ' with $f_0(1370)$, π' with $\pi(1300)$, ρ with $\rho(770)$, ρ' with $\rho(1450)$ and a with $a_1(1260)$ [28]. The first excitation of the a_1 - the particle a' - is not found yet. This second axial-vector state, perhaps, could be found from hadronic τ decays, although there are the strong phase space limitations [23].The experimental data give us:

$$m_\sigma = 400 \div 1200 \text{ MeV}; \qquad m_{\sigma'} = 1200 \div 1500 \text{MeV}; \qquad m_{\pi'} = 1300 \pm 100 \text{ MeV};$$

$$m_\rho = 770 \pm 0.8 \text{ MeV}; \qquad m_{\rho'} = 1465 \pm 25 \text{ MeV}; \qquad m_a = 1230 \pm 40 \text{ MeV}. \tag{29}$$

The prediction for mass of σ-meson is then

$$m_\sigma \simeq 800 \text{ MeV}, \tag{30}$$

which is close to the averaged experimental value. Furthermore we have the following prediction for the mass of a'-particle,

$$m_{a'} \cong 1465 \div 1850 \text{ MeV}. \tag{31}$$

The large range for a possible mass of a'-meson is accounted for by a big experimental uncertainty for the mass of σ' and π' mesons. If we accept the averaged values for them, then $m_{a'} - m_{\rho'} \approx 30$ MeV.

Let us compare the paremeters of the NJL and QNJL models. First we notice that in both models the Nambu relation (23) together with (25) leads to the large value of the dynamical mass $M_0 \simeq 400$ MeV for physical masses of ρ and a_1 mesons. Then the physical value of the pion decay constant (28) $f_\pi \simeq 90$ MeV can be achieved for a cutoff $\Lambda \simeq 1$ GeV. In turn, in the NJL it leads to a unacceptably high values of the quark condensate (15), $<\bar{q}q> \simeq -(310)\text{MeV}^3$ whereas in the QNJL one can easily adjust the second v.e.v. to reproduce the condensate in agreement with the estimations from Chiral Perturbation Theory [29, 30] (see also,[31]). In particular, the value $<\bar{q}q> \simeq -(250)\text{MeV}^3$ corresponds to $\sigma_1 \simeq -\sqrt{3}\sigma_2$ (see the discussion of this point in the next section).

3. Chiral symmetry restoration constraints

Let us exploit the constraints based on chiral symmetry restoration at QCD at high energies. Expanding the meson correlators (1) in powers of p^2 one arrives to the CSR Sum Rules. In the scalar-pseudoscalar case (3) they read:

$$\sum_n Z_n^S - \sum_n Z_n^P = 0;$$

$$\sum_n Z_n^S m_{S,n}^2 - \sum_n Z_n^P m_{P,n}^2 = \Delta_{SP}, \tag{32}$$

and in the vector-axial-vector one (4) one obtains:

$$\sum_n Z_n^V - \sum_n Z_n^A = 4f_\pi^2;$$

$$\sum_n Z_n^V m_{V,n}^2 - \sum_n Z_n^A m_{A,n}^2 = 0,$$

$$\sum_n Z_n^V m_{V,n}^4 - \sum_n Z_n^A m_{A,n}^4 = \Delta_{VA}. \tag{33}$$

The first two relations are the famous Weinberg Sum Rules, with f_π being the pion decay constant. The residues in resonance pole contributions in the vector and axial-vector correlators have the structure,

$$Z_n^{(V,A)} = 4 f_{(V,A),n}^2 m_{(V,A),n}^2, \tag{34}$$

with $f_{(V,A),n}$ being defined as corresponding decay constants.

In the scalar-pseudoscalar case it has been obtained [25, 32] that:

(i) the residues in poles are of different order of magnitude,

$$\frac{Z_{\sigma,\pi}}{Z_{\sigma',\pi'}} = O(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}). \tag{35}$$

Moreover, due to $m_{\pi'}^2 \simeq m_{\sigma'}^2$ it happens to be

$$Z_\pi \simeq Z_\sigma; \qquad Z_{\pi'} \simeq Z_{\sigma'} \simeq Z_0 \equiv \frac{N_c\Lambda^4}{2\pi^2}. \tag{36}$$

(ii) The remarkable relation takes place:

$$Z_\sigma \simeq 4\frac{<\bar{q}q>^2}{f_\pi^2}. \tag{37}$$

(iii) The second CSR Sum Rules constraint results in the estimation for splitting between the σ'- and π'-meson masses,

$$Z_0(m_{\sigma'}^2 - m_{\pi'}^2) \simeq 24\pi\alpha_s <\bar{q}q>^2 . \tag{38}$$

If the optimal fit for f_π and the quark condensate is performed then $\sigma_1 \simeq -\sqrt{3}\sigma_2$ (see the end of the previous section) and,

$$m_{\sigma'}^2 - m_{\pi'}^2 \simeq \frac{1}{6}m_\sigma^2. \tag{39}$$

(iv) The chiral coupling constant L_8 [29, 33] are calculated:

$$L_8 = \frac{f_\pi^4}{64<\bar{q}q>^2}(\frac{Z_\sigma}{m_\sigma^2} + \frac{Z_{\sigma'}}{m_{\sigma'}^2} - \frac{Z_{\pi'}}{m_{\pi'}^2}) \simeq \frac{f_\pi^2}{16m_\sigma^2}(1 - \frac{6\alpha_s\pi f_\pi^2 m_\sigma^2}{m_{\pi'}^4}). \tag{40}$$

Its value $L_8 = (0.9 \pm 0.4)\cdot 10^{-3}$ from [29, 30, 34] accepts $m_\sigma \simeq 800$ MeV.

In the vector-axial-vector case all residues are found to be of the same order of magnitude in contrast to the scalar-pseudoscalar channel. When using the notation (21) they read:

$$Z_\pi = 4f_\pi^2 \simeq -\frac{N_c\Lambda^4(m_{a_1}^2 - m_\rho^2)(6m_\rho^2\ln\frac{\Lambda^2}{M_0^2} + d)}{32\pi^2 m_\rho^2 m_{a_1}^2 m_{a_1'}^2 \ln\frac{\Lambda^2}{M_0^2}},$$

$$Z_\rho \simeq \frac{m_{a_1}^2}{m_{a_1}^2 - m_\rho^2}Z_\pi; \qquad Z_{a_1} \simeq \frac{m_\rho^2}{m_{a_1}^2 - m_\rho^2}Z_\pi;$$

$$Z_{\rho'} \simeq \frac{Z_1}{m_{\rho'}^2}; \qquad Z_{a_1'} \simeq \frac{Z_1}{m_{a_1'}^2}; \qquad Z_1 \equiv \frac{3N_c\Lambda^4}{16\pi^2}. \tag{41}$$

The relation for Z_π is considered a constraint on effective coupling constants of the QNJLM $\bar{\Delta}_{kl}$ (see (28)). The first and the second Sum Rules are fulfilled identically in the large-log approach. The third one takes the form:

$$Z_1(m_{a_1'}^2 - m_{\rho'}^2) \simeq 16\pi\alpha_s <\bar{q}q>^2 .$$

It is instructive to compare this sum rule with the similar one, (38) in the scalar channel. In terms of (14) and (20) and taking into account eqs.(24), (26) one obtains:

$$\sigma^2 \simeq \frac{4N_c\alpha_s}{9\pi}(\sigma_1 - \sqrt{3}\sigma_2)^2, \quad \text{vector SR} \quad \textbf{vs.} \quad \sigma^2 \simeq \frac{3N_c\alpha_s}{8\pi}(\sigma_1 - \sqrt{3}\sigma_2)^2, \quad \text{scalar SR.}$$

The minor discrepancy between 4/9 and 3/8 is about 15% and can be referred to the quality of two-resonance approximation. Thus the above sum rules consistently bound v.e.v.'s of scalar fields, i.e. control CSB.

The structure of $Z_{\rho'}$ and $Z_{a'}$ shows that if $m_{a'} \simeq m_{\rho'}$ then $Z_{a'} \simeq Z_{\rho'}$ and therefore $f_{a'} \simeq f_{\rho'}$. As a consequence these residues approximately cancel each other in Sum Rules and the one-channel results for f_ρ, f_a holds,

$$f_\rho \simeq \frac{f_\pi m_a}{m_\rho\sqrt{m_a^2 - m_\rho^2}}; \qquad f_a \simeq \frac{f_\pi m_\rho}{m_a\sqrt{m_a^2 - m_\rho^2}}. \tag{42}$$

After evaluating we get $f_\rho \approx 0.15$ and $f_a \approx 0.06$ to be compared with the experimental values [35] from the decay $\rho^0 \to e^+e^-$, $f_\rho = 0.20 \pm 0.01$, and from the decay $a_1 \to \pi\gamma$, $f_a = 0.10 \pm 0.02$. We have also a reasonable prediction for the chiral constant L_{10} [29]

$$L_{10} = \frac{1}{4}(\sum_n f_{A,n}^2 - \sum_n f_{V,n}^2) \simeq \frac{1}{4}(f_a^2 - f_\rho^2) \approx -5.1 \cdot 10^{-3}, \tag{43}$$

which confronts the phenomenologically accepted estimation [34] $L_{10} = -(5.5 \pm 0.7) \cdot 10^{-3}$.

4. Summary

We have shown that the QNJL model truncating low energy QCD effective action in vector sector can serve to describe the physics of meson resonances. The matching to non-perturbative QCD based on chiral symmetry restoration at high energies improves the predictability of the model. The CSR Sum Rules are well saturated by four resonances. Let us summarize the results obtained in the present work.

(i) The mass of the second axial-vector particle with $I = 1$ is predicted. It is comparable with the mass of the vector counterpartner: $m_{a'} = 1465 \div 1850$ MeV and the most plausible value of the mass difference is $m_{a'} - m_{\rho'} \approx 30$ MeV.

(ii) The estimation on the mass of the σ-meson does not contradict to existing experimental data [28]: $m_\sigma \simeq 800$ MeV.

(iii) The couplings f_ρ, f_a and the chiral constant L_{10} are evaluated from CSR Sum Rules as matching rules for our model to QCD at intermediate energies, with the result being $f_\rho \approx 0.15, f_a \approx 0.06, L_{10} \approx -5.1 \cdot 10^{-3}$;

Finally we would like to mention possible applications of QNJLM. Firstly, such models are thought of as relevant for investigations of behaviour of hadron matter at high temperatures and nuclear densities in the region near the restoration of chiral symmetry. One could expect that for increasing quark densities the mass-splitting is collapsing and therefore excitations become lighter and more important in hadron kinetics. Secondly, QNJLM can be used to describe Higgs particles in extensions of the Standard Model (SM). The models of such extension can be found in [32, 36].

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