The Description for the Spin Polarizabilities of Hadrons Based on the Covariant Lagrangian

S.A. Belousova, N.V. Maksimenko

Gomel State University, Sovetskay street, 102, Gomel, Belarus, 246699
E-mail:belousova@gsu.unibel.by

Abstract

On the basis of the correspondence principle between the relativistic moving medium electrodynamics and relativistic quantum field theory the covariant Lagrangian of the electromagnetic field interaction with the polarized spin particles have been obtained. This Lagrangian satisfies the main relativistic quantum field theory requirements and contains four independent covariant spin structures, which have particular physical meaning. It is shown that the spin polarizabilities give the contribution to the amplitude for Compton scattering on the spin-1/2 hadron in the $\mathcal{O}(\omega^3)$.

In the expansion of the amplitude for Compton scattering in powers of the incident photon energy ω, the terms of $\mathcal{O}(1)$ and $\mathcal{O}(\omega)$, depend only on the mass of the nucleon m, its electric charge and the anomalous magnetic moment μ (see, for example, Refs. [1, 2]). By virtue of the low-energy theorem in the $\mathcal{O}(\omega^2)$ the amplitude for spin-0 and -1/2 hadrons depends on their internal degrees of freedom, which are determined by the fundamental parameters such as the electric (α) and magnetic (β) polarizabilities [1, 2]. In turn, at the $\mathcal{O}(\omega^3)$ the effective Lagrangian describing the photon-nucleon interaction and, as the result the amplitude for Compton scattering are determined by gyrations, or so-called spin polarizabilities, the new electromagnetic characteristics [3, 4, 5, 6, 7, 8, 9]. These characteristics are immediately connected to the spin properties of hadrons as a composite particles.

The classic process for the investigation of these features of the photon-hadron interaction is the Compton scattering of photons whose energies are less than resonance region. Nevertheless the data on spin polarizabilities can be extracted from other electrodynamic processes (see, for example Ref. [10]).

The determination of the hadron polarizability contributions to the amplitudes of QED processes is sequentially carried out by the effective Lagrangians of the electromagnetic field interaction with the hadron as a composite particle. In the nonrelativistic electrodynamics the such kind of Lagrangian is rather well determined. On the other side in the relativistic QED when hadrons look like bound states, due to the kinematic relativistic effects, the interpretation of polarizabilities is ambiguous [2, 9].

In the present report the correspondence principle between relativistic moving medium electrodynamics and relativistic quantum field theory will be sequentially used for the covariant Lagrangian construction of the electromagnetic field interaction with polarized spin hadrons.

First of all we define the Lagrangian of the photon-hadron interaction taking into account the electric and magnetic polarizabilities.

In the nonrelativistic case, the Hamiltonian of the interaction of the isotropically-gyrotropic medium looks like [11, 12]:

$$H_I = -2\pi\left(\mathbf{PE} + \mathbf{MH}\right), \tag{1}$$

In this case vectors of polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ look like [11]

$$\mathbf{P} = \hat{\alpha}\mathbf{E}, \tag{2}$$

$$\mathbf{M} = \hat{\beta}\mathbf{H}, \tag{3}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the tensors of electric and magnetic polarizabilities, $\mathbf{E}$ and $\mathbf{H}$ are the strengths vectors of electric and magnetic fields. If the polarizability tensors in Eqs.(2) and (3) depend on the spin operator components $\hat{S}_i$, then due to commutation relations of these operators the Compton scattering amplitude for spin-0, -1/2 and -1 hadrons is defined by the following expression:

$$\begin{aligned}\frac{1}{8\pi m}T_{fi}^{NB} \quad = \quad & \omega^2\chi^{+}\{\alpha_0\ \mathbf{e}\ '^{*}\cdot\mathbf{e} + \mathbf{i}\boldsymbol{\alpha_1}\ \hat{\mathbf{S}}\cdot\mathbf{e}\times\mathbf{e}\ '^{*} + \\ & +\alpha_2\{\hat{\mathbf{S}}\cdot\mathbf{e}\ '^{*},\ \hat{\mathbf{S}}\cdot\mathbf{e}\} + \boldsymbol{\beta_0}\ \mathbf{s}\cdot\mathbf{s}\ '^{*} + \\ & +i\beta_1\ \hat{\mathbf{S}}\cdot\mathbf{s}\ '^{*}\times\mathbf{s} + \boldsymbol{\beta_2}\{\hat{\mathbf{S}}\cdot\mathbf{s},\ \hat{\mathbf{S}}\cdot\mathbf{s}\ '^{*}\}\}\chi. \end{aligned} \tag{4}$$

Here χ and χ^+ are the spin wave functions of the particles, ω is the photon energy, $\mathbf{n}$ ($\mathbf{e}$) and $\mathbf{n}\,'$ ($\mathbf{e}\,'$) are unit (polarization) vectors of incident and scattered photons respectively, $\mathbf{s} = \mathbf{n} \times \mathbf{e}$; $\mathbf{s}'^* = \mathbf{n}' \times \mathbf{e}'^*$. We assume $c = \hbar = 1$.

Due to the crossing symmetry requirement the amplitude (4) for the spin-0 and -1/2 hadrons are determined by the parameters α_0 and β_0. The scattering amplitude for vector hadrons together with the ordinary polarizabilities α_0 and β_0 contains spin ones α_2 and β_2, while the quantities α_1 and β_1 are equal to zero because of the crossing symmetry.

Let us define now the effective Lagrangian in the relativistic QED for spin-0, -1/2 and -1 hadrons taking into consideration their usual electric and magnetic polarizabilities. According to the relativistic moving medium electrodynamics the effective Lagrangian is [12]:

$$L_{eff}^{pol} = 2\pi \left\{ e^\mu \alpha_{\mu\nu} e^\nu + h^\mu \beta_{\mu\nu} h^\nu \right\}. \tag{5}$$

In this expression $e_\mu = F_{\mu\nu} U^\nu$, $h_\mu = \tilde{F}_{\mu\nu} U^\nu$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$, where $F^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are the electromagnetic field tensor and dual one respectively ($F^{0i} = -E^i, F^{ik} = -\varepsilon_{ikl}\, H^l$), $\alpha_{\mu\nu}$ and $\beta_{\mu\nu}$ are tensors, defining the polarizabilities of a medium at rest, U_μ is the 4-velocity of the medium and the Levi-Chevita antisymmetric tensor $\varepsilon_{\mu\nu\rho\sigma}$ is fixed by the condition $\varepsilon^{0123} = 1$.

Following to the non-relativistic quantum field theory we assumed that tensors $\alpha_{\mu\nu}$ and $\beta_{\mu\nu}$ depend on the momentum operator and Pauli-Lubansky's vector operator:

$$\begin{aligned} \alpha_{\mu\nu} &= \alpha_{\mu\nu}(\hat{W}_\mu, \hat{p}_\mu), \\ \beta_{\mu\nu} &= \beta_{\mu\nu}(\hat{W}_\mu, \hat{p}_\mu), \\ \hat{W}_\mu &= \frac{1}{2m} \varepsilon_{\mu\nu\rho\sigma}\; J^{[\nu\rho]}\; \hat{p}^\sigma. \end{aligned} \tag{6}$$

Here $J^{[\nu\rho]}$ are generators of the Lorentz group.

Using the commutation relations for the operators $\hat{W}_\mu$ and $\hat{p}_\mu$ in the framework of the quantum-mechanical Poincare group [13] the expansions for spin-1/2 particles read:

$$\alpha_{\mu\nu} = \alpha_0 g_{\mu\nu} + \alpha_1 \varepsilon_{\mu\nu\rho\sigma} \hat{W}^\rho \hat{p}^\sigma, \tag{7}$$

$$\beta_{\mu\nu} = \beta_0 g_{\mu\nu} + \beta_1 \varepsilon_{\mu\nu\rho\sigma} \hat{W}^\rho \hat{p}^\sigma. \tag{8}$$

From Eqs. (7) and (8) we can see that when α_1 and β_1 are equal to zero, the amplitude satisfies the crossing symmetry (*i.e.* at the $\mathcal{O}(\omega^2)$ the Lagrangian (5) is defined by α_0 and β_0).

Going from the Lagrangian (5) to the field theory Lagrangian by means of the correspondence principle, we obtain [8, 14]:

1. for spin-0 particles

$$\begin{aligned} L_{eff}^{pol} &= \frac{\pi}{m}[(\partial_\mu \varphi^+)(\partial_\nu \varphi) + (\partial_\mu \varphi)(\partial_\nu \varphi^+) - \\ &\quad -(\varphi^+ \partial_\mu \partial_\nu \varphi + \varphi \partial_\mu \partial_\nu \varphi^+)] K^{\mu\nu}, \end{aligned} \tag{9}$$

2. for spin-1/2 particles

$$L_{eff}^{pol} = -\frac{i\pi}{m} \left(\bar{\psi}\, \gamma^\mu\, \overleftrightarrow{\partial_\nu}\, \psi \right) K_\mu^\nu, \tag{10}$$

where $K_\mu^\nu = \alpha_0 F_{\mu\rho} F^{\rho\nu} + \beta_0 \tilde{F}_{\mu\rho} \tilde{F}^{\rho\nu}$, $\overleftrightarrow{\partial_\nu} = \overleftarrow{\partial_\nu} - \overrightarrow{\partial_\nu}$, φ and ψ are the wave functions of spin-0 and spin-1/2 particles. We have obtained the amplitude for Compton scattering in the $\mathcal{O}(\omega^2)$, taking into account the Lagrangian (10):

$$\frac{1}{8\pi m} T_{fi}^{NB,nospin} = \chi^+ \omega_1 \omega_2 [\alpha_E\; \mathbf{e}'^* \cdot \mathbf{e} + \beta_{\mathbf{M}}\; \mathbf{s}'^* \cdot \mathbf{s}] \chi, \tag{11}$$

where $\mathbf{s} = \mathbf{n} \times \mathbf{e}$; $\mathbf{s}'^* = \mathbf{n}' \times \mathbf{e}'^*$, ω_1 and ω_2 are the energies of the incident and scattered photons.

In order to define the effective Lagrangian with account of the nucleon spin polarizabilities, is used Eq. (1), where polarization and magnetization vectors ($\mathbf{P}$ and $\mathbf{M}$) are determined as follows

$$\mathbf{P} = \hat{\alpha}\mathbf{E} + \hat{\gamma}'_{\mathbf{E}}\,[\nabla\mathbf{E}], \tag{12}$$

$$\mathbf{M} = \hat{\beta}\mathbf{H} + \hat{\gamma}'_{\mathbf{M}}\,[\nabla\mathbf{H}]. \tag{13}$$

Then the following Lagrangian of the electromagnetic field interaction with the moving medium is obtained:

$$\begin{aligned} L_I{}^{eff} &= 2\pi\{[e_\mu\alpha^{\mu\nu}e_\nu + h_\mu\beta^{\mu\nu}h_\nu] - \\ &-[(\gamma'_E)_{\mu\nu}e^\mu(U\partial)h^\nu + (\gamma'_M)_{\mu\nu}h^\mu(U\partial)e^\nu]\}, \end{aligned} \tag{14}$$

where $(\gamma'_E)_{\mu\nu\rho}$ and $(\gamma'_M)_{\mu\nu\rho}$ are the gyration pseudotensors, $(U\partial) = U_\rho\partial^\rho$.

If pseudotensors $(\gamma'_E)_{\mu\nu}$ and $(\gamma'_M)_{\mu\nu}$ are determined via $g_{\mu\nu}$ as $(\gamma'_{E,M})_{\mu\nu} = (\gamma'_{E,M})g_{\mu\nu}$, then it reduce to violation of the spatial parity conservation low (*i.e.* spin of a composite particle is not taken into account).

From the expression (14) it follows that the usual polarizabilities (α, β) in a rest medium and the gyration give non-zero contributions to the Lagrangian, starting with the second and the third order in povers of frequency of an external electromagnetic field respectively.

Due to the correspondence principles [14] and the expression (14), the field theory effective Lagrangian of electromagnetic field interaction with spinless hadrons will not satisfy the parity conservation law when the components of pseudotensors $(\gamma'_E)_{\mu\nu}$ and $(\gamma'_M)_{\mu\nu}$ are not equal to zero. However, in the case of the spin particles it is easy to determine dependence of γ-structures from $(\bar{\psi}\,(\overleftrightarrow{\partial}_\alpha\overleftrightarrow{\partial}_\beta)\gamma^\mu\gamma^5\psi)$ and $(F^{\alpha\nu}\,\overleftrightarrow{\partial^\nu}\,\tilde{F}_{\sigma\mu})$.

Hence, the effective field Lagrangian discribing the electromagnetic field and the spin-1/2 hadron interaction is defined as follows:

$$L_{eff} = L^{pol}_{eff} + L^{Sp}_{eff}, \tag{15}$$

where L^{pol}_{eff} is determined by the expression (10) and L^{Sp}_{eff} looks like

$$\begin{aligned} L^{Sp}_{eff} &= \frac{\pi}{4m^2}(\bar{\psi}\,(\overleftrightarrow{\partial}_\alpha\overleftrightarrow{\partial}_\beta + \overleftrightarrow{\partial}_\beta\overleftrightarrow{\partial}_\alpha)\gamma^\mu\gamma^5\psi) \\ &\times\{-\frac{1}{2}\,\gamma_{E_1}\cdot\;F^{\alpha\nu}\,\overleftrightarrow{\partial^\beta}\,\tilde{F}_{\mu\nu} \\ &+\frac{1}{2}\,\gamma_{M_1}\cdot\;\tilde{F}^{\alpha\nu}\,\overleftrightarrow{\partial^\beta}\,F_{\mu\nu} \\ &-\gamma_{E_2}\cdot\;(F^{\alpha\nu}\,\overleftarrow{\partial_\mu}\,\tilde{F}^\beta_{\ \nu} - \tilde{F}^{\alpha\nu}\,\overrightarrow{\partial_\nu}\,F_\mu{}^\beta) \\ &+\gamma_{M_2}\cdot\;(\tilde{F}^{\alpha\nu}\,\overleftarrow{\partial_\mu}\,F^\beta_{\ \nu} - F^{\alpha\nu}\,\overrightarrow{\partial_\nu}\,\tilde{F}_\mu{}^\beta)\}, \end{aligned} \tag{16}$$

here $\gamma^5 = -\left(\begin{smallmatrix}0 & I\\ I & 0\end{smallmatrix}\right)$.

The effective interaction Hamiltonian of the electromagnetic field and the spin-1/2 hadron follows from the Lagrangian (16) [9]

$$\begin{aligned} H^{Sp}_{eff} &= -2\pi(\gamma_{E_1}\boldsymbol{\sigma}\cdot\mathbf{E}\times\dot{\mathbf{E}} + \gamma_{M_1}\boldsymbol{\sigma}\cdot\mathbf{H}\times\dot{\mathbf{H}} \\ &-2\gamma_{E_2}E_{ij}\sigma_iH_j + 2\gamma_{M_2}H_{ij}\sigma_iE_j), \end{aligned} \tag{17}$$

where $E_{ij} = \frac{1}{2}(\partial_iE_j + \partial_jE_i)$ and $H_{ij} = \frac{1}{2}(\partial_iH_j + \partial_jH_i)$.

The effective Lagrangian (15) satisfies both the crossing symmetry and all requirements of the relativistic quantum field theory. As we can see from (10), at the $\mathcal{O}(\omega^2)$ the effective Lagrangian of spin-1/2 hadrons are determined by the electric and magnetic polarizabilities only (α_0 and β_0).

Besides, as follows from expression (16), spin polarizabilities for the spin-1/2 hadrons give the contributions to the effective Lagrangian which are of order $\mathcal{O}(\omega^3)$. Using the Lagrangian (16) the Compton scattering amplitude for the spin-1/2 hadrons have been obtained:

$$\begin{aligned}
\frac{1}{4\pi m} T_{fi}^{NB,spin} = -\chi^{+}[-i \quad & \gamma_{E_1} \quad \boldsymbol{\sigma} \cdot \mathbf{e}'^{*} \times \mathbf{e}(\omega_1^2 \omega_2 + \omega_1 \omega_2^2) \\
-i \quad & \gamma_{M_1} \quad \boldsymbol{\sigma} \cdot \mathbf{s}'^{*} \times \mathbf{s}(\omega_1^2 \omega_2 + \omega_1 \omega_2^2) \\
-i \quad & \gamma_{E_2} \quad \{[\boldsymbol{\sigma} \cdot \mathbf{n}\mathbf{e} \cdot \mathbf{s}'^{*} + \boldsymbol{\sigma} \cdot \mathbf{e}\mathbf{n} \cdot \mathbf{s}'^{*}]\omega_1^2 \omega_2 \\
& \qquad -[\boldsymbol{\sigma} \cdot \mathbf{n}'\mathbf{e}'^{*} \cdot \mathbf{s} + \boldsymbol{\sigma} \cdot \mathbf{e}'^{*}\mathbf{n}' \cdot \mathbf{s}] \, \omega_1 \omega_2^2\} \\
+i \quad & \gamma_{M_2} \quad \{[\boldsymbol{\sigma} \cdot \mathbf{n}\mathbf{e}'^{*} \cdot \mathbf{s} + \mathbf{n} \cdot \mathbf{e}'^{*}\boldsymbol{\sigma} \cdot \mathbf{s}]\omega_1^2 \omega_2 \\
& \qquad -[\boldsymbol{\sigma} \cdot \mathbf{n}'\mathbf{e} \cdot \mathbf{s}'^{*} + \mathbf{n}' \cdot \mathbf{e}\boldsymbol{\sigma} \cdot \mathbf{s}'^{*}]\omega_1 \omega_2^2\}]\chi.
\end{aligned} \tag{18}$$

The amplitude (18) for Compton scattering contains the structures typical for the spin polarizabilities and satisfies both the gauge invariance and the crossing symmetry.

To summarize, on the basis of the correspondence principle between the relativistic moving medium electrodynamics and relativistic quantum field theory the covariant Lagrangian of the electromagnetic field interaction with the polarized spin particles have been obtained. This Lagrangian satisfies the main relativistic quantum field theory requirements (are cross-invariant, P-, T- and gauge invariant). The obtained Lagrangian can be used for description of spin polarizabilites in two-photon processes (Compton scattering, scattering of electrons on nucleones, and others).

References

[1] V.A. Petrun'kin, Phys At. Nucl. 12 (1981) 692

[2] A.J.L'vov, FIAN preprint N344 (1987)

[3] S. Ragusa, Phys. Rev. D47 (1993) 3757

[4] S. Ragusa, Phys. Rev. D47 (1994) 3157

[5] K.Y. Lin, Nuovo Cimento A2 (1971) 695

[6] K.Y. Lin, Phys. Rev. D24 (1981) 1014

[7] M.I.Levchuk, L.G.Moroz, Vesti AN BSSR *Ser.: fiz.-mat. nauk 1.* (1985) 49

[8] N.V.Maksimenko, L.G.Moroz, *In Proc. 11 Intern. School on High Energy Physics and Relativistic Nucl. Phys.* Dubna JINR D2-11707 (1979) 533

[9] D.Babusci, J.Jiordano, A.J.L'vov, J. Matone , A.N.Nathan, Phys. Rev. C58 (1998) 1013

[10] S.Scherer, A. Yu Korchin, J.H.Koch, Phys. Rev. C54 (1996) 904

[11] F.I.Fedorov Gyrotropy theory. Minsk: Nauka i tehnika, (1973)

[12] B.V.Bokut', A.N.Serdyukov, Journal of Applied Spectroscopy, 11 (1969) 704

[13] Yu.V.Novojilov, Introduction in theory of elementary particles.– Nauka. . (1972).

[14] ..Bogush, L.G.rz, Introduction in classic fields theory.– Nauka i tehnika.– Minsk. (1968).