Casimir Energy for Models with Confined Fermion Field

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Abstract

The Casimir energy of confined fermion fields in $(1{+}1)D$-bag is renormalized within the framework of a procedure based on the extraction of singularities from the infinite sum of the Hamiltonian eigenvalues and subsequent compensation of them by means of the redefinition of "bare" constants. A finite ground state energy of the 2-phase hybrid chiral fermion bag model is evaluated and studied in detail.

1 Introduction

Nowadays a number of methods is developed for evaluation of the Casimir energy for systems with quantized fields under nontrivial boundary conditions [1]. The most known of them are the Green function method, the zeta function approach, the application of contour integration, the multiple scattering method, and the direct mode summation with thermal regularization [2] (for recent discussion see [3]). An important field of application for these techniques is an investigation of ground state energy for the quark bag models with account of one-loop corrections due to the filled Dirac's negative energy sea [4]. In this paper we propose an approach for explicit calculation of ground state energy for the models with confined fermion field coupled to the boson (scalar) field in a certain spatial region, so called hybrid bag models [5]. This method allows to extract the singular terms from the divergent sums of eigenvalues ω_n and obtain the finite Casimir energy of fermionic sea as the result of absorbing singularities into the "dressed" model parameters. The realization of such a strategy requires, as a rule, an including of contact terms into the "classical" expression for the energy of system. The form of these terms is determined by their dependence on the geometric parameters [6]. By means of this method, we renormalize the ground state energy of the hybrid chiral bag model in $(1+1)D$ and study it as a function of the bag's size. It's shown that this function has unique minimum what means that there exists a stable configuration for the considered model.

2 Renormalization

The ground state energy of fermion field is defined as the vacuum expectation value of the Hamiltonian:

$$\langle 0|H|0\rangle \equiv E_0 = -\tfrac{1}{2}\left(\sum_{\omega_n>0}\omega_n - \sum_{\omega_k<0}\omega_k\right). \tag{1}$$

Provided that the spectrum ω_n possesses the symmetry $\omega_n \to -\omega_n$, the vacuum energy is

$$E_0 = -\sum_{\omega_n>0}\omega_n. \tag{2}$$

This sum diverges. On of the ways to regularize it is to use the exponential cutoff [7, 8] what yields

$$E_0^{exp}(\tau) = \lim_{\tau\to 0}\left(-\sum_{\omega_n>0}\omega_n \mathrm{e}^{-\tau\omega_n}\right) = \lim_{\tau\to 0}\frac{d}{d\tau}\sum_{\omega_n>0}\mathrm{e}^{-\tau\omega_n}, \tag{3}$$

where $\tau = \tilde{\tau}\mu^{-1}$ and μ has the dimension of mass. This regularization can be applied for calculation of the Casimir energy in several simple situations [1, 7, 8], but usually only for systems with an explicitly known spectrum ω_n. In this paper we'll try to show that one can succeed in calculations even if this is not the case. Suppose that an asymptotical expansion of ω_n for $n \geq 1$ [5] can be found for a model with an unknown spectrum. This expansion reads

$$\omega_n = \sum_{i=1}^{-\infty} \Omega_i n^i = \Omega_1 n + \Omega_0 + \frac{\Omega_{-1}}{n} + O\left(\frac{1}{n^2}\right) \ . \tag{4}$$

These are the three leading terms in (4) which determine the divergencies in (2). To begin, consider the case when the expansion (4) contains only two terms (what really takes place, e. g. for the free massless fermion field in (1+1)-bag):

$$\omega_n^{(1)} = \Omega_1 n + \Omega_0 \ . \tag{5}$$

Then the regularized energy is [7]

$$E_1^{exp}(\tau) = \lim_{\tau \to 0} \frac{d}{d\tau} \sum_{n=1} \mathrm{e}^{-\tau(\Omega_1 n + \Omega_0)} - \omega_0 = \lim_{\tau \to 0} \frac{d}{d\tau}\left(\mathrm{e}^{-\tau\Omega_0}\sigma_1(\tau)\right) - \omega_0 \ , \tag{6}$$

$$\sigma_1(\tau) = \sum_{n=1} \mathrm{e}^{-\tau\Omega_1 n} = \frac{1}{\mathrm{e}^{\tau\Omega_1} - 1} \ . \tag{7}$$

The term ω_0 in (6) is written separately because the expansion (4) is not valid for it. By virtue of

$$\frac{1}{\mathrm{e}^x - 1} = \sum_{k=0} \frac{B_k}{k!} x^{k-1} \ , \tag{8}$$

where B_k are the Bernoulli numbers, one gets

$$E_1^{exp}(\tau) = -\frac{1}{\tau^2\Omega_1} + \frac{\Omega_1}{12} + \frac{\Omega_0}{2} + \frac{\Omega_0^2}{2\Omega_1} - \omega_0 \ . \tag{9}$$

This expression contains the quadratic divergence $E^{quad}(\tau) = -\frac{1}{\tau^2\Omega_1} = -\frac{\mu^2}{\tilde{\tau}^2\Omega_1}$ depending on the arbitrary mass μ and the geometric parameters of the bag from Ω_1.

Consider now the case when

$$\omega_n^{(2)} = \Omega_1 n + \Omega_0 + \frac{\Omega_{-1}}{n} \ . \tag{10}$$

In this situation the regularized energy is

$$E_2^{exp}(\tau) = \lim_{\tau \to 0} \frac{d}{d\tau}\sigma_2(\tau) - \omega_0 \ , \tag{11}$$

$$\sigma_2(\tau) = \sum_{n=1} \mathrm{e}^{-\tau\left(\Omega_1 n + \Omega_0 + \frac{\Omega_{-1}}{n}\right)} = \mathrm{e}^{-\tau\Omega_0} \sum_{n=1} \mathrm{e}^{-\tau\Omega_1 n}\mathrm{e}^{-\tau\Omega_{-1}/n} \ . \tag{12}$$

One easily see that only the two leading terms in the expansion of $\mathrm{e}^{-\tau\Omega_{-1}/n}$ in powers of τ yield a non-vanishing contribution to $E_2^{exp}(\tau)$ as $\tau \to 0$. Then we get

$$\sigma_2(\tau) = \mathrm{e}^{-\tau\Omega_0} \sum_{n=1} \mathrm{e}^{-\tau\Omega_1 n}\left(1 - \frac{\tau\Omega_{-1}}{n}\right) + O(\tau^2) =$$

$$= \sigma_1(\tau) + \tau\Omega_{-1}\mathrm{e}^{-\tau\Omega_0} \sum_{n=1} \frac{1}{n}\mathrm{e}^{-\tau\Omega_1 n} + O(\tau^2) \ , \tag{13}$$

where $\sigma_1(\tau)$ is already obtained in (7). Using the known relation

$$\sum_{n=1} \frac{1}{n}\mathrm{e}^{-\alpha n} = -\ln(1 - \mathrm{e}^{-\alpha})$$

we find

$$\sigma_2(\tau) = \sigma_1(\tau) + \tau\Omega_{-1}\mathrm{e}^{-\tau\Omega_0}\left(\ln\tau\Omega_1 - \frac{\tau\Omega_1}{2}\right) + O(\tau^2) \ . \tag{14}$$

So the regularized energy reads

$$E_2^{exp}(\tau) = -\frac{1}{\tau^2\Omega_1} + \Omega_{-1}\ln\tau\mu + \Omega_{-1}\ln\frac{\Omega_1}{\mu} + \frac{\Omega_1}{12} + \frac{\Omega_0}{2} + \frac{\Omega_0^2}{2\Omega_1} + \Omega_{-1} - \omega_0. \tag{15}$$

The contribution of the terms of order $O(1/n^2)$ (E^{fin}) is finite and can be found in any particular case. The divergent parts

$$E^{quad}(\tau) = -\frac{1}{\tau^2\Omega_1} \quad \text{and} \quad E^{log}(\tau) = \Omega_{-1}\ln\tau\mu$$

is to be compensated using a certain renormalization scheme, which will be discussed later.

Let us compare the result obtained above with another method of regularization — the ζ-function approach. In this case the energy is regularized as:

$$E_1^{zeta}(s) = -\sum_n \omega_n \to -\lim_{s\to -1}\mu^{1+s}\sum_n \omega_n^{-s} \ , \tag{16}$$

where the arbitrary mass μ is included in order to restore the correct dimension. We assume that this mass equals μ in (16). For $\omega_n^{(1)} = \Omega_1 n + \Omega_0$ one has

$$E_1^{zeta}(s) = -\lim_{s\to -1}\mu^{s+1}\sum_{n=1}(\Omega_1 n + \Omega_0)^{-s} - \omega_0 = -\lim_{s\to -1} z_1(s) - \omega_0 \ , \tag{17}$$

where

$$z_1(s) = \mu^{s+1}\sum_1 (\Omega_1 n + \Omega_0)^{-s} =$$

$$= \sum_1 \frac{1}{(\Omega_1 n)^s}\left(1 - \frac{s\Omega_0}{\Omega_1 n} + \frac{s(s+1)}{2!}\left(\frac{\Omega_0}{\Omega_1 n}\right)^2 + O(1/n^3)\right) \ . \tag{18}$$

It is clear that the contribution of terms $O(1/n^3)$ vanishes as $s \to -1$, so the regularized sum $E_1^{zeta}(s)$ is determined by the leading three terms in the expansion (18). Then one finds:

$$z_1(s) = \mu^{s+1}\left(\Omega_1^{-s}\zeta(s) - \frac{s\Omega_0}{\Omega_1^{s+1}}\zeta(s+1) + \frac{\Omega_0^2}{2\Omega_1^{s+2}}s(s+1)\zeta(s+2)\right) . \tag{19}$$

The values of $\zeta(z)$ analytically continued to the total real axis are known [9]:

$$\zeta(0) = -\frac{1}{2} \ , \quad \zeta(-1) = -\frac{B_2}{2} = -\frac{1}{12} \ . \tag{20}$$

For $\zeta(z)$ in the vicinity of 1 we have

$$\lim_{z\to 1}\zeta(z) = \frac{1}{z-1} + C \ , \tag{21}$$

where $C = 0.5772156649...$ is the Euler constant. Thus taking the limit $s \to -1$ one obtains

$$E_1^{zeta}(s) = \frac{\Omega_1}{12} + \frac{\Omega_0}{2} + \frac{\Omega_0^2}{2\Omega_1} - \omega_0 \ . \tag{22}$$

This expression reproduces explicitly the finite part of (9). The absence of the divergent term is due to the analytical continuation for $\zeta(z)$. Taking into account the next term of the expansion in powers of $1/n$ (10) we find

$$E_2^{zeta}(s) =$$

$$= - \lim_{s\to -1} \mu^{s+1} \sum_{n=1} \left(\Omega_1 n + \Omega_0 + \frac{\Omega_{-1}}{n} \right)^{-s} - \omega_0 = - \lim_{s\to -1} z_2(s) - \omega_0 \ , \tag{23}$$

where

$$z_2(s) = z_1(s) - \frac{s\Omega_{-1}\mu^{s+1}}{\Omega_1^{s+1}} \zeta(s+2) \tag{24}$$

(we write down only the terms which yield a non-vanishing contribution). The second term in (24) can be found using (21) and the expansion $x^\varepsilon = 1 + \varepsilon \ln x + O(\varepsilon^2)$, $\varepsilon = s+1$, $\varepsilon \to 0$. Then

$$\lim_{s\to -1} s\Omega_{-1} \left(\frac{\mu}{\Omega_1} \right)^{s+1} \zeta(s+2) =$$

$$= \Omega_{-1} \lim_{\varepsilon\to 0} (-1+\varepsilon) \left(1 - \varepsilon \ln \frac{\Omega_1}{\mu} \right) \left(\frac{1}{\varepsilon} + C \right) = -\Omega_{-1} \left(\frac{1}{\varepsilon} - 1 + C - \ln \frac{\Omega_1}{\mu} \right) \ . \tag{25}$$

Therefore the regularized energy is of the form

$$E_2^{zeta}(s) = \frac{\Omega_1}{12} + \frac{\Omega_0}{2} + \frac{\Omega_0^2}{2\Omega_1} - \omega_0 + \Omega_{-1} \left(\ln \frac{\Omega_1}{\mu} + 1 \right) - \Omega_{-1} \left(\frac{1}{\varepsilon} + C \right) \ , \tag{26}$$

what coincides with (15) (except of the quadratic singularity E_2^{quad}) provided that

$$\frac{1}{\tau} = \mu\gamma \mathrm{e}^{1/\varepsilon} \ , \ \ln\gamma = C. \tag{27}$$

If one needs to take into account the contribution of terms of order $O\left(\frac{1}{n^2}\right)$, then their sum $(-E^{fin})$ should be added to (15) and (26). Therefore it is shown that the exponential and ζ-function regularizations provide the equivalent results for regularized Casimir energy in $(1+1)D$.

The singular part is:

$$E^{div}(\tau) = -\frac{1}{\tau^2\Omega_1} + \Omega_{-1} \ln \tau\mu \ . \tag{28}$$

It can be removed, for example, by means of the redefinition of the "bare" constants from the initial Lagrangian. It is very important to note that the way of renormalization is prescribed by the dependence of the extracted singularities on the geometric parameters (in our case the only geometric parameter is the bag's size R). We collect all singularities with the similar dependence from R, and then find the "contact term" to be redefined [6].

Let us describe this procedure in the simplest case, i. e., MIT bag model with massive fermions in $(1+1)D$ [7, 10]. The Lagrangian reads

$$\mathcal{L}_{MIT} = \theta(|x|<R)(i\bar\psi\hat\partial\psi - m_F\bar\psi\psi - B) + \theta(|x|>R)(i\bar\psi\hat\partial\psi - M_F\bar\psi\psi) \ , \tag{29}$$

where B is the so-called bag constant which characterizes an excess of the energy density inside a hadron compared to the energy of the nonperturbative vacuum. Taking the limit $M_F \to \infty$ in the exterior region $|x| > R$ one gets the "bag", which is just the segment of real axis $[-R,\ R]$. The boundary conditions

$$(\pm i\gamma^1 + 1)\psi(\pm R) = 0 \tag{30}$$

lead to the spectrum

$$\omega_n = \sqrt{\left(\frac{\pi}{2R} n + \frac{\pi}{4R} \right)^2 + m_F^2} \ . \tag{31}$$

Assume that the fermion mass m_F is small and expand the energy in powers of it. Then (with the accuracy up to m_F^4) one has

$$\omega_n = (\Omega_1 n + \Omega_0) + \frac{m_F^2}{2(\Omega_1 n + \Omega_0)} + O(m_F^4) \ , \tag{32}$$

$\Omega_1 = \frac{\pi}{2R}$, $\Omega_0 = \frac{\pi}{4R}$. For $n \geq 1$ the expansion (4) can be found, where

$$\Omega_{-1} = \frac{m_F^2}{2\Omega_1} = \frac{m_F^2 R}{\pi} \tag{33}$$

and for $n = 0$ we have

$$\omega_0 = \Omega_0 + \frac{m_F^2}{2\Omega_0} = \Omega_0 + 2\Omega_{-1} \ . \tag{34}$$

The divergent part of the energy (according to (28)) is

$$E^{div}(\tau|R) = \left(-\frac{1}{\tau^2\pi} + \frac{m_F^2}{2\pi}\ln\tau\mu\right)2R = 2B'(\tau)R \ . \tag{35}$$

In accordance with the approach of [6, 7], the renormalization is performed by means of the redefinition of the bare bag constant B in the Lagrangian (29):

$$B = B_0 - B'(\tau) \ . \tag{36}$$

The remaining parts of energy (15) are finite and can be found explicitly. The contribution of terms $O\left(n^{-2}\right)$ in the expansion (4) is determined by

$$E^{fin}(R) = -\frac{m_F^2}{2}\sum_{n=1}\left(\frac{1}{\Omega_1 n + \Omega_0} - \frac{1}{\Omega_1 n}\right) =$$

$$= \frac{m_F^2}{2\Omega_1}\sum_{n=1}\frac{1}{n(2n+1)} = \frac{2m_F^2 R}{\pi}(1 - \ln 2) \ . \tag{37}$$

Therefore, the renormalized energy of the fermionic sea for $(1+1)D$ massive MIT bag model as the function of the bag's size R and renormalized bag constant B_0 reads (up to the terms of order m_F^4):

$$E_{MIT}(R) = 2B_0 R - \frac{\pi}{48R} + \frac{m_F^2 R}{\pi}\left(1 - 2\ln 2 + \ln\frac{\pi}{2R\mu}\right) + O(m_F^4) \ . \tag{38}$$

Taking the limit $m_F \to 0$ (massless MIT bag model), one gets from (38) the well-known result of [11]. Note, that this configuration is unstable and tends to $R \to 0$. It is possible to make it stable by adding one valence fermion into the lowest level $n = 0$. Then the energy reads

$$\tilde{E}_{MIT}(R) = 2B_0 R + \frac{11}{48}\frac{\pi}{R} + \frac{m_F^2 R}{\pi}\left(3 - 2\ln 2 + \ln\frac{\pi}{2R\mu}\right) + O(m_F^4) \ . \tag{39}$$

Now we are ready to study the hybrid bag model, in which the fermion ("quark") field interacts with scalar ("meson") field. Compared to the models considered in [5], here is no phase of the massless fermions. The Lagrangian reads

$$\mathcal{L} = i\bar{\psi}\hat{\partial}\psi + \tfrac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi - \theta(|x|<R)\left(\tfrac{1}{2}\ m_F[\bar{\psi}, \mathrm{e}^{i\gamma_5\varphi}\psi] - B\right) -$$

$$-\theta(|x|>R)(V(\varphi) + \tfrac{1}{2}\ M_F[\bar{\psi}, \mathrm{e}^{i\gamma_5\varphi}\psi]\ ). \tag{40}$$

The commutator in the fermionic forms provides the charge-conjugation symmetry. For the sake of simplicity, we consider the scalar field $\varphi(x,t)$ in the mean-field approximation (MFA) [12], i. e., assume it to be a $c-$number function of the space-time variables; suppose also that it is independent of the temporal coordinate: $\varphi(x,t) = \varphi(x)$. Besides this, we will take into account only the leading terms of the perturbative expansion in the chiral coupling constant [12]. In other words, the fermionic mass m_F is assumed to be small inside the segment $|x| < R$ while the mass M_F in the exterior region $|x| > R$ is infinitely large. Thus the fermion field at $|x| > R$ vanishes and the equations of motion for $|x| < R$ are

$$(i\hat{\partial} - m_F\mathrm{e}^{i\gamma_5\varphi})\psi(x,t) = 0, \tag{41}$$

$$\varphi'' = i\frac{m_F}{2}\langle[\bar{\psi}, \gamma_5 \mathrm{e}^{i\gamma_5\varphi}\psi]\rangle_{sea}, \tag{42}$$

where in r. h. s. of (42) the v. e. v. of the axial current is taken accordingly to MFA. For $|x| > R$, the scalar field is determined by the non-linear equation

$$-\varphi''(x) = V'_\varphi(\varphi) \ . \tag{43}$$

The solution of the equations (41, 42) have been firstly found by Sveshnikov and Silaev and studied in detail in [5], so we will discuss it here only briefly. Let us assume that the scalar field is an odd soliton function and for $|x| > R$ it has the form:

$$\varphi(x) = \pi\left(1 - A\mathrm{e}^{-mx}\right), \ x > 0 \tag{44}$$

where m is the meson mass, and $\varphi(x)$ for $x < 0$ is determined by the oddness. The boundary conditions are

$$\begin{gathered}(\pm i\gamma^1 + \mathrm{e}^{i\gamma_5\varphi(x)})\psi(\pm R) = 0 \ , \\ \varphi(\pm R \pm 0) = \varphi(\pm R \mp 0) \ , \\ \varphi'(\pm R \pm 0) = \varphi'(\pm R \mp 0) \ . \end{gathered} \tag{45}$$

The fermionic spectrum possesses the symmetry $\nu \to -\nu$, where $\nu^2 = k^2 + m_F^2$. The corresponding unitary transformations of the wave function are $\chi \to i\gamma_1\chi$ (here $\chi = \mathrm{e}^{i\gamma_5\varphi/2}\psi$), therefore the v. e. v. of the axial current in the r. h. s. of (42) is equal to zero and the solution of (42) for $|x| < R$ appears to be the linear function $\varphi(x) = 2\lambda x$ (for detailed discussion of the self-consistent solution for this system see [5]). The eigenvalues ω_n can be obtained from the equation

$$\begin{aligned}&\left(1 - \mathrm{e}^{2ikR}\frac{m_F + i(\nu+k)}{m_F + i(\nu-k)}\right)\left(1 - \mathrm{e}^{-2ikR}\left(\frac{\nu-k}{\nu+k}\right)\frac{m_F - i(\nu+k)}{m_F - i(\nu-k)}\right) = \\ &= \left(1 - \mathrm{e}^{-2ikR}\frac{m_F - i(\nu+k)}{m_F - i(\nu-k)}\right)\left(1 - \mathrm{e}^{-2ikR}\left(\frac{\nu-k}{\nu+k}\right)\frac{m_F + i(\nu+k)}{m_F + i(\nu-k)}\right),\end{aligned} \tag{46}$$

where $\nu = \omega - \lambda$. Provided that the signatures of ω_n and ν_n are the same for all n, the fermionic Casimir energy is determined by (2) with the replacement $\omega_n \to \nu_n$. For the sake of being definite we regularize this expression by means of the exponential cutoff (3):

$$E_0^{exp}(\tau) = \lim_{\tau\to 0}\Big(-\sum_{\nu_n>0}\nu_n\mathrm{e}^{-\tau\nu_n}\Big) = \lim_{\tau\to 0}\frac{d}{d\tau}\sum_{\nu_n>0}\mathrm{e}^{-\tau\nu_n} \ .$$

Evaluating (46) one gets the finite fermionic energy in the form

$$E_F = E_{Cas} + 2BR + \Lambda \ , \tag{47}$$

where B and Λ are the bag and cosmological constants, and E_{Cas} is the divergent fermionic Casimir energy. Hence the renormalization requires the redefinition of B (36) and Λ:

$$\Lambda = \Lambda_0 - \frac{m_F}{\pi}\ln\tau\mu \ . \tag{48}$$

The contribution E^{fin} of the non-singular terms $O\left(\frac{1}{n^2}\right)$ reads

$$\begin{aligned}E^{fin} &= \Omega_{-1}\sum_{n=1}\frac{1}{n(2n+1)} + \frac{16Rm_F^2}{\pi^3}\sum_{n=0}\frac{1}{(2n+1)^3} = \\ &= \Omega_{-1}2(1-\ln 2) + \frac{16Rm_F^2}{\pi^3}A \ ,\end{aligned} \tag{49}$$

where $A = 1.051799....$ Therefore the finite energy of the fermionic sea is (provided that $\Lambda_0 = 0$):

$$\begin{aligned}E(R) = 2B_0R - \frac{\pi}{48R} + \\ + \frac{m_F}{\pi}(1 + m_FR)\left[\ln\frac{\pi}{2\mu R} + 1 - 2\ln 2\right] + \frac{16Rm_F^2}{\pi^3}A \ .\end{aligned} \tag{50}$$

The total energy of this system contains also the contribution of the scalar field, but in is out of interest for our purposes.

3 Conclusion

The present approach to renormalization of the infinite ground state energy of the models with confined fermion fields in $(1+1)D$ is based on the analytical regularization of the divergent sums [13] (e. g., the exponential cutoff, or ζ-function regularization), the extraction of the singular terms in the form of poles at the point of the physical value of regularization parameter, and subsequent absorbing of these singularities by means of redefinition of bare constants from the initial Lagrangian. In this framework the singularities are isolated unambiguously using the scheme analogous to MS in QFT. The remaining parts are finite and demonstrate the non-trivial dependence on the geometrical parameters of the model. The generalization of the proposed framework to the higher dimensions appears to be a separate problem and will be the subject of the future investigations [14].

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