Radiative Corrections to $e^-e^+ \to W^-W^+$ in MSSM with GRACE/SUSY/1LOOP System[1]

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Abstract

The complete 1-loop corrections to $e^-e^+ \to W^-W^+$ in the minimum supersymmetric standard model (MSSM) have been calculated by means of `GRACE/SUSY/1LOOP`, the program package for evaluating the Feynman amplitudes of the MSSM processes at 1-loop level. The deviation from the standard model(SM) is small in the energy region of LEP2 experiments and of the future linear colliders.

The process of e^+e^- annihilation into W pairs has played one of the most important roles in LEP2 experiments. The precision of this measurement is reaching a 1% level. Corresponding to such a measurement, the great efforts have been made for the theoretical prediction of the cross section in the Standard Model(SM)[1]. The similar precision measurement will be expected at the future linear e^+e^- colliders, because still it is one of the dominant processes. Is there any possibility of obtaining information on the physics beyond the SM from this process? In Refs.[2] and [3], the 1-loop corrections in spontaneously broken supersymmetry and sfermion-loop effects in the MSSM have been calculated, respectively. Recently the complete 1-loop corrections in the MSSM have been computed by means of `FeynArts/FormCalc` [4]. In this report, we also present the magnitude of the 1-loop corrections in the MSSM evaluated by means of `GRACE/SUSY/1LOOP` system. The results are consistent with [4], which reports that the deviation of the MSSM value from the SM one is rather small.

The calculation was done with the usual Feynman diagram technique. The `GRACE/SUSY`[5], which is a system for the automatic calculation of the Feynman diagrams, equips with the full Feynman rules in the MSSM in which 55 particles and 3,553 vertices appear. The implementation of these Feynman rules in the system have been checked varying the gauge parameters of the covariant gauges. More than 30 digits agreement is achieved in the quadruple precision computation for different values of gauge parameters in all the 583,102 two-to-four processes. In the case of processes which have photons or gluons as external particles, we have also checked the gauge invariance by substituting the momentum of the photon or the gluon for its polarization vector.

Based on this confirmation, the `GRACE/SUSY` has been applied to calculate several processes with the multi-final states in the tree level[6].

In order to proceed to 1-loop calculations, the `GRACE/SUSY` system was upgraded in the following points:

1. Introduction of the dimensional reduction scheme for the treatment of the ultraviolet divergence.
2. Creation of a file of the counterterm interactions in a semi-automatic way with `REDUCE`[7].
3. Necessary 2-point functions have been prepared by means of `GRACE/SUSY/1LOOP` system itself.
4. Evaluation of the renormalization constants in the on-shell renormalization scheme with above 2-point functions.

In the gauge boson sector, both of the following on-shell renormalization schemes are alternatively used in order to check the results: 1)the renormalization is done *before* the mixing[8]. 2)the renormalization is done *after* the mixing[9]. Performance of the 1-loop calculations with the `GRACE/1LOOP` for the SM is explained elsewhere[10].

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The first application of GRACE/SUSY/1LOOP was done for the $O(\alpha)$ and $O(\alpha_s)$ corrections to the charged Higgs boson decay into top and bottom quarks, $H^+ \to t\bar{b}$[11].

The GRACE/SUSY/1LOOP supplies us 916 1-loop diagrams for the process $e^-e^+ \to W^-W^+$ in the MSSM, in which each diagram with the renormalized self-energy and with a counterterm interaction is prepared as a single diagram. The $O(\alpha)$ corrected cross section is given by

$$d\sigma^{O(\alpha)}(k_c) \propto \left|T^{(0)}\right|^2 (1+\delta_{soft}(k_c)) + 2\Re T^{(0)\dagger}T^{(1)}, \tag{1}$$

where $T^{(0)}$ and $T^{(1)}$ are the sum of the amplitudes of the tree level and the 1-loop level, respectively. $\delta_{soft}(k_c)$ is the QED soft photon factor, which depends on the cut-off parameters, k_c, separating the soft photon emission from the hard one. The $O(\alpha)$ corrected cross section in the SM was also calculated but in the dimensional reguralization scheme. The magnitude of the Higgs mass used in the SM is chosen to be equal to the mass of the lighter CP-even Higgs in the MSSM. The SM cross section agrees well with the previous calculation[12]. Since we are interesting in the deviation of the MSSM cross section from the SM one, we will introduce the following deviation factor δ,

$$\delta(k_c) = \left(\sigma^{O(\alpha)}_{MSSM}(k_c) - \sigma^{O(\alpha)}_{SM}(k_c)\right)/\sigma^{O(\alpha)}_{SM}(k_c), \tag{2}$$

where σ_{MSSM}, σ_{SM} are $O(\alpha)$ corrected total cross sections in the MSSM and in the SM, respectively.

Following Ref.[13], we choose as independent parameters the following 10 quantities;

$$e,\quad g_s,\quad M_W,\quad M_Z,\quad \tan\beta,\quad M_A,\quad \mu,\quad M_1,\quad M_2,\quad M_3, \tag{3}$$

plus 20 sfermion parameters Eqs.4 and 5 under two constraints Eq.6 (for the third generation),

$$\begin{array}{lllll} m^2_{\tilde{u}_L} = m^2_{\tilde{d}_L}, & m^2_{\tilde{\nu}e_L} = m^2_{\tilde{e}_L}, & m^2_{\tilde{u}_R}, & m^2_{\tilde{d}_R}, & m^2_{\tilde{e}_R}, \\ m^2_{\tilde{c}_L} = m^2_{\tilde{s}_L}, & m^2_{\tilde{\nu}\mu_L} = m^2_{\tilde{\mu}_L}, & m^2_{\tilde{c}_R}, & m^2_{\tilde{s}_R}, & m^2_{\tilde{\mu}_R}, \end{array} \tag{4}$$

$$\begin{array}{llllll} m^2_{\tilde{b}_1}, & m^2_{\tilde{b}_2}, & \theta_b, & m^2_{\tilde{t}_1}, & m^2_{\tilde{t}_2}, & \theta_t, \\ m^2_{\tilde{\tau}_1}, & m^2_{\tilde{\tau}_2}, & \theta_\tau, & m^2_{\tilde{\nu}\tau_L}, & & \end{array} \tag{5}$$

$$\begin{aligned} \cos^2\theta_t m^2_{\tilde{t}_1} + \sin^2\theta_t m^2_{\tilde{t}_2} - m_t^2 &= \cos^2\theta_b m^2_{\tilde{b}_1} + \sin^2\theta_b m^2_{\tilde{b}_2} - m_b^2 + M_W^2\cos 2\beta, \\ m^2_{\tilde{\nu}\tau} &= \cos^2\theta_\tau m^2_{\tilde{\tau}_1} + \sin^2\theta_\tau m^2_{\tilde{\tau}_2} - m_\tau^2 + M_W^2\cos 2\beta. \end{aligned} \tag{6}$$

In the Higgs boson sector, the on-shell condition is applied to the CP-odd Higgs particle, A^0. In order to get the masses of the other Higgs bosons, the tree level relations are used.

For the analysis we use the following values for the input parameters:

$$\begin{gathered} M_Z = 91.1867 \text{ GeV},\ M_W = 80.35 \text{ GeV},\ m_t = 174 \text{ GeV},\ m_b = 4.7 \text{ GeV}, \\ m_e = 0.51099906 \text{ MeV},\ m_\mu = 105.658389 \text{ MeV},\ m_\tau = 1.7771 \text{ GeV}, \\ m_u = 58 \text{ MeV},\ m_d = 58 \text{ MeV},\ m_s = 92 \text{ MeV},\ m_c = 1.5 \text{ GeV}, \\ \tan\beta = 15,\ \mu = -300 \text{ GeV},\ M_2 = 200 \text{ GeV}, \\ m_{\tilde{e}_1} = m_{\tilde{e}_2} = 300 \text{ GeV},\ \ \theta_e = 0.1,\ \ m_{\tilde{\nu_e}} = 289.14 \text{ GeV}, \\ m_{\tilde{\mu}_1} = m_{\tilde{\mu}_2} = 300 \text{ GeV},\ \ \theta_\mu = 0.1,\ \ m_{\tilde{\nu_\mu}} = 289.14 \text{ GeV}, \\ m_{\tilde{\tau}_1} = m_{\tilde{\tau}_2} = 300 \text{ GeV},\ \ \theta_\tau = 0.1,\ \ m_{\tilde{\nu_\tau}} = 289.14 \text{ GeV}, \\ m_{\tilde{u}_1} = m_{\tilde{u}_2} = m_{\tilde{d}_1} = 300 \text{ GeV},\ \ \theta_u = \theta_d = 0.1,\ \ m_{\tilde{d}_2} = 396.25 \text{ GeV}, \\ m_{\tilde{c}_1} = m_{\tilde{c}_2} = m_{\tilde{s}_1} = 300 \text{ GeV},\ \ \theta_c = \theta_s = 0.1,\ \ m_{\tilde{s}_2} = 396.25 \text{ GeV}, \\ m_{\tilde{t}_1} = 200 \text{ GeV},\ \ m_{\tilde{t}_2} = 400 \text{ GeV},\ \ m_{\tilde{b}_1} = 200 \text{ GeV},\ \ \theta_t = 0.85, \theta_b = 0.95,\ \ m_{\tilde{b}_2} = 221.8 \text{ GeV}. \\ m_{h_0} = 90.258 \text{ GeV},\ \ m_{H_0} = 250.337 \text{ GeV},\ \ m_{H_A} = 250 \text{ GeV},\ \ m_{H_\pm} = 262.595 \text{ GeV}, \\ m_{\tilde{\chi}^0_1} = 94.66 \text{ GeV},\ \ m_{\tilde{\chi}^0_2} = 184.23 \text{ GeV},\ \ m_{\tilde{\chi}^0_3} = 309.89 \text{ GeV},\ \ m_{\tilde{\chi}^0_4} = 326.97 \text{ GeV}, \\ m_{\tilde{\chi}^+_1} = 184.41 \text{ GeV},\ \ m_{\tilde{\chi}^+_2} = 330.01 \text{ GeV}, \end{gathered}$$

This set was chosen based on the following criteria: 1) The value of $\tan\beta$ is not too small to have already been experimentally excluded. 2) the lighter chargino can be pair-produced at $\sqrt{s}$ =500 GeV, 3) all sfermions are less than $\sim$ 1 TeV but larger than 250 GeV, so that they are not pair-produced at $\sqrt{s}$ = 500 GeV. 4) All the Higgs bosons except for the lighter CP-odd Higgs boson are not pair-produced at $\sqrt{s}$ =500 GeV.

The calculation has been confirmed through the following checks:

- The UV divergence is canceled among the 1-loop diagrams.
- The infrared divergence is canceled between the 1-loop diagrams and the soft photon corrections.
- The results are independent of the renormalization schemes [8] and [9] adopted in the computation.

$\sqrt{s}$(GeV)	$\sigma_{\rm tree}$(pb)	$\sigma^{O(\alpha)}_{\rm SM}$(pb)	$\sigma^{O(\alpha)}_{\rm MSSM}$(pb)	δ(%)
200	18.001	15.928	15.785	-0.897
500	6.765	5.789	5.700	-1.69
1000	2.110	1.725	1.666	-3.37

Table 1 : The total cross section at tree level σ_{tree}, $O(\alpha)$ corrected total cross sections in the SM($\sigma^{O(\alpha)}_{SM}$) and in the MSSM($\sigma^{O(\alpha)}_{MSSM}$), respectively, at $\sqrt{s}$ =,200, 500 and 1000GeV. δ shows the deviation of the MSSM results from the SM one in %. k_c is set $0.05\sqrt{s}$.

The results are shown in Table.1. The soft photon cut parameter k_c is set $0.05\sqrt{s}$. For the set of the above input parameters, the deviation from the SM value is less than 1% in the LEP2 energy region, and still small at $\sqrt{s}$ =500 GeV and 1000 GeV in the linear collider energy region. Figs. 1, 2 and 3 show the angular dependence of the differential cross sections with $k_c = 0.05\sqrt{s}$ and of the deviation, δ. The solid and dashed lines of the differential cross sections corresponding to the MSSM case and to the SM case, respectively. At $\sqrt{s}$ =200 GeV, two curves are visually indistinguishable.

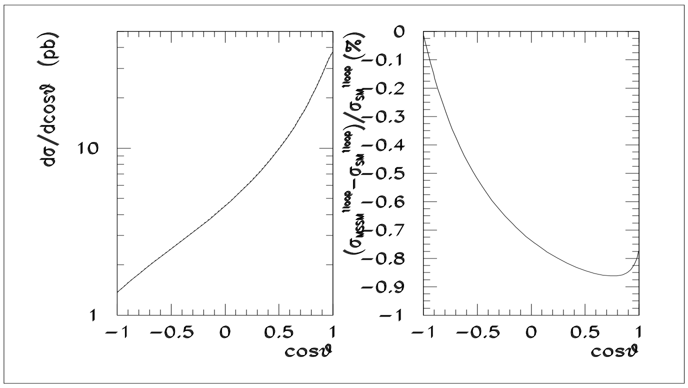

Figure 1: $O(\alpha)$ corrected $d\sigma/d\cos\theta$ in the SM(the dashed line) and in the MSSM(the solid line) and the deviation δ in % at $\sqrt{s}$ =200 GeV with $k_c = 0.05\sqrt{s}$.

On the other hand, at $\sqrt{s}$ =500 GeV and 1000 GeV, the deviation becomes large in the backward region. It might be, however, not so easy to see it in experiments because of its small cross sections in that angular region.

Fig.4 shows the energy dependence of the deviation δ with the above set of the input parameters. Up to $\sqrt{s}$ =1000 GeV, the deviation is still small and is contained in around 3%.

In order to confirm the statement that the deviation is small, it is necessary to calculate δ over the possible region of the parameter set. Such a parameter scan performed in Ref.[4] concluded that the maximum deviation from the SM value is 2.7%. As the scheme of the set of independent parameter of Ref.[4] is different from ours, the direct comparison is not made here. The comparison of our result with Ref.[3] is also not easy because it is nontrivial for `GRACE` to switch off interactions not containing sfermion.

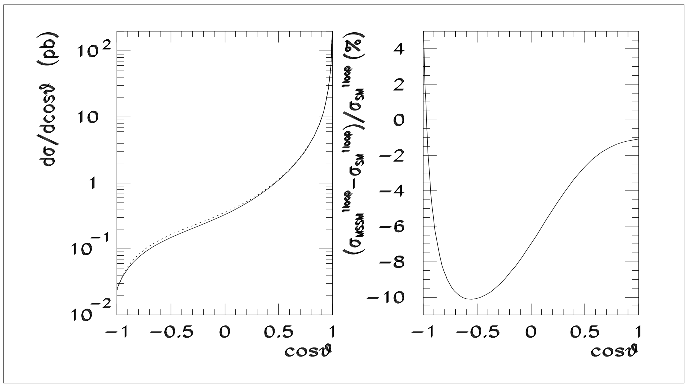

Figure 2: $O(\alpha)$ corrected $d\sigma/d\cos\theta$ in the SM(the dashed line) and in the MSSM(the solid line) and the deviation δ in % at $\sqrt{s}$ =500 GeV with $k_c = 0.05\sqrt{s}$.

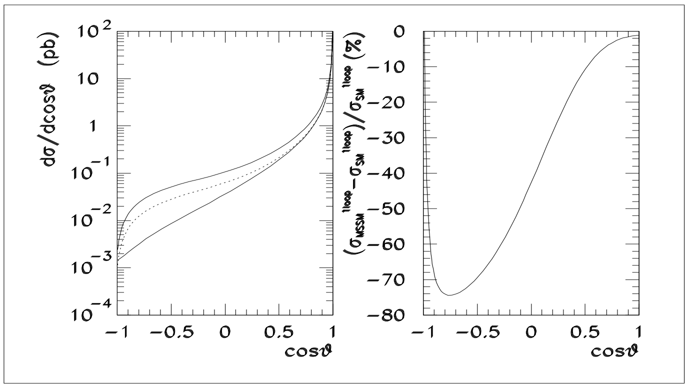

Figure 3: $d\sigma/d\cos\theta$ of the tree level(the upper solid line), $O(\alpha)$ corrected ones in the SM(the dashed line) and inn the MSSM(the lower solid line) and the deviation δ in % at $\sqrt{s}$ =1000 GeV with $k_c = 0.05\sqrt{s}$.

As a summary, we have developed the automatic computation system, **GRACE/SUSY/1LOOP**, of 1loop Feynman-diagrams in the MSSM. It was applied to calculate $O(\alpha)$ corrections to $e^-e^+ \to W^-W^+$. For the set of the input parameters of the MSSM studied in this report, the deviation of the MSSM value from the SM value is a few % up to $\sqrt{s}$ =1000 GeV. It is necessary to repeat the analysis for other values of the set of the input parameters. We are going to analyze other $2 \to 2$ MSSM processes, which are almost beyond our ability to calculate by hand.

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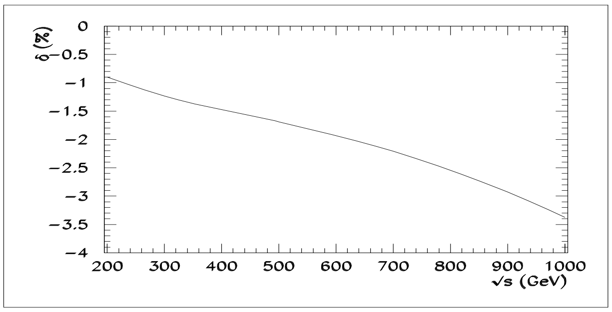

Figure 4: The center of energy dependence of the deviation δ in % with $k_c = 0.05\sqrt{s}$.

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