# Gauge-Invariant Description of W-pair Production in the NLO Approximation

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**Abstract**

The processes $e^+e^- \to 4f(\gamma)$ mediated by W-pair (and single) production are considered in the framework of the modified perturbation theory based on the expansion of probability, instead of the amplitude, in powers of the coupling constant. The full set of one-loop corrections to the cross-section and two-loop corrections to self-energies of unstable particles are completely taken into consideration. It is shown that the manifestly non-factorizable corrections do not make contributions to the next-to-leading-order approximation. Moreover, the colour reconnection does not make contributions within this precision either. The remaining corrections, together with the leading-order contribution, provide a gauge-invariant description of the processes.

**1°**. The current requirement for precision of description of the processes mediated by production and decay of W-bosons (as well as Z-bosons) implies the necessity of direct inclusion of the effects of their instability [1, 2, 3]. A conventional way to take into account this instability consists in Dyson resummation of self-energies of unstable particles. This procedure makes possible to satisfy the unitarity condition [4] and to avoid the nonintegrable phase-space singularities that arise while calculating the probability in the framework of the standard perturbation theory (PT). However, in the case of pair production of unstable particles the Dyson resummation violates the Ward identities (WI) [5, 6, 7]. This fact creates a danger of an uncontrolled error in description of particular processes.

The way to solve this problem consists in making use of the pole scheme which provides gauge-invariance from the start [8]. In the case of W-pair production, the pole scheme has been applied within the double pole approximation (DPA) [9, 10, 11]. At present, DPA remains the sole scheme, where the radiative corrections to this process have been actually evaluated. However, at the Born level DPA turns out to be untenable, because its direct application leads to a numerical error of several percent (see, for instance, section 3.5 of Ref. [11]). Therefore, the practical use of DPA at the Born level is realized with the substitution of the non-gauge-invariant CC03 off-shell cross-section, instead of the intrinsic gauge-invariant Born DPA. Thereby, it is supposed that this substitution leads to a reasonable result, at least in the 't Hooft-Feynman gauge.

It is possible that the latter substitution is numerically reasonable, at least at LEP energies. Nevertheless, the gauge-invariant description of the W-pair production, which would be free from the difficulties of DPA, should be found. Firstly, it should be found from a conceptual point of view. Secondly, in order to have an opportunity to estimate the error of the above-mentioned implementation of DPA. Thirdly, to extend the range of validity of description. From the viewpoint of future prospects, the gauge-invariant description should be found as a starting point to reach a greater precision of the description.

In fact, such gauge-invariant description has been already proposed [12]. Also, a more effective way, in comparison with DPA, for evaluaton of radiative corrections has been found [12]. The point is that within the next-to-leading-order (NLO) approximation the manifestly non-factorizable corrections actually do not contribute to the cross-section of W-pair production. (Throughout the paper we consider the order of contributions in the sense of powers of electroweak coupling constant $\alpha$.) Earlier, it was known that contributions of the soft photons to non-factorizable corrections were cancelled in probability, together with the infrared (IR) divergences [13]. However, it remained unknown whether the same property took place for the intermediate-energy photons. In the DPA approach, the fact of cancellation of the manifestly non-factorizable corrections to the total cross-section and the pure angular distributions was confirmed by explicit calculations [14]. Nevertheless, the nature of this phenomenon was not made clear. Consequently, it remained unclear how general this phenomenon was, and whether it occurred in other cases.

The purpose of the present paper is twofold. First, I present the general proof of the absence of contributions of the manifestly non-factorizable corrections to the total cross-section in the NLO approximation, whence the validity of the result in other cases becomes clear. Then, I propose a further development of the method. Namely, I show that within the NLO precision, the effect of colour reconnection does not make contribution to the cross-section of W-pair production.

**2°**. Let us start with discussion of the basic concepts of the approach. Its primary idea [15] consists in expanding the probabilities, instead of amplitudes, in powers of the coupling constant. Actually this change of order of operations affects the unstable-particle propagators only. Really, while calculating the probability of the completely expanded amplitude, one faces the divergences in the phase space caused by the presence of nonintegrable singularities in the free propagators squared of unstable particles. On the other hand, the probability is finite, in spite of phase-space integration, when it is calculated on the basis of the (renormalized) amplitude with the Dyson resummed propagators. Consequently, when the probability is calculated in this latter way, the result of the expansion should be a series with finite coefficients.

The latter result may be strictly proved by analyzing the expansion of the Dyson resummed propagator squared. The first term of this expansion is, in fact, well-known in view of the following formula for Breit-Wigner resonance,

$$\frac{1}{|s-M^2+\mathrm{i}M\Gamma|^2} \xrightarrow{\Gamma\to 0} \frac{\pi}{M\Gamma}\,\delta(s-M^2)\,. \tag{1}$$

Unlike the function $1/|s-M^2|^2$ that appears as a result of the expansion before the squaring procedure, the delta-function $\delta(s-M^2)$ is integrable with respect to $s$. So, it produces the finite contribution to the probability while integrated over the phase space. To obtain the next terms of the expansion in formula (1) is a nontrivial problem, but solvable in the framework of the theory of generalized functions (distributions) [16] with the use of asymptotic operation [17]. Under the assumption of the absence of contributions of zero mass particles (photons), the next terms of the expansion have been derived in [15].[1]

Below we present the corresponding result within the NLO precision. We use the following notation for the *renormalized* self-energy of the unstable particle:

$$\Sigma(s;\alpha) = \alpha\Sigma_1(s) + \alpha^2\Sigma_2(s) + \cdots\,. \tag{2}$$

In on-mass-shell (OMS) scheme of UV renormalization (where $\mathrm{Re}\Sigma_1(M^2)=\mathrm{Re}\Sigma_1'(M^2)=0$), the result looks as follows:

$$\frac{1}{|s-M^2+\Sigma(s;\alpha)|^2} = \frac{\pi}{\alpha\mathrm{Im}\Sigma_1(M^2)}\,\delta(s-M^2) - \pi\frac{\mathrm{Im}\Sigma_2(M^2)}{\mathrm{Im}\Sigma_1^2(M^2)}\,\delta(s-M^2) + \mathrm{VP}\frac{1}{(s-M^2)^2} + O(\alpha)\,. \tag{3}$$

Here VP means the principal-value prescription for the functions non-integrable in the conventional sense. Owing to the unitarity relation $\alpha\mathrm{Im}\Sigma_1(M^2) = M\Gamma$, where $\Gamma$ is the lowest-order width, which was verified by direct calculations [18], the first term in the r.h.s. of (3) is exactly the r.h.s in formula (1). The next terms in (3) describe the NLO corrections. It is worth noticing that in the OMS scheme the real part of the self-energy does not contribute within the given precision. Moreover, the self-energy contributes on mass shell only, and without derivatives. The latter fact means that the soft-photon problem is not actually present within the given precision in the propagator squared. However, it arises while considering the exchanges by soft photons between different charged particles of the process. Moreover, in the next orders of the expansion, the problem arises even at level of the propagator squared, raising thereby a question about the validity of the expansion.

In fact, the problem of IR divergences can be simply solved by introducing the mass for soft photons. Then, at the end of calculations, it is necessary to show that the soft-photon mass is cancelled, and the gauge invariance is restored. The problem can be simplified by performing the (secondary) Dyson resummation of the one-loop corrections. The latter operation is equivalent to an incomplete expansion in the coupling constant of the full unstable propagator squared. The general investigation of the problem has been carried out in [12]. For our purposes the following resultant formula is relevant:

$$\frac{1}{|s-M^2+\Sigma(s;\alpha)|^2} = \frac{1}{|s-M^2+\alpha\Sigma_1^{\mathrm{fer}}(s)|^2} - \pi\frac{\mathrm{Im}\Sigma_2(M^2)}{\mathrm{Im}\Sigma_1^2(M^2)}\,\delta(s-M^2) + O(\alpha)\,. \tag{4}$$

[1] Although in Ref. [15] it was claimed that was found a non-trivial generalization of the method for the case of presence of the soft photons, the result was only supposed, not really proved, because the actual cancellation of the IR divergences had not been shown in any physically significant case. The same applies to the status of the declared curing [15] of difficulties in gauge invariance.

Here $\Sigma_1^{\rm fer}(s)$ is the fermionic one-loop correction to the self-energy of W-boson. Notice, the delta function in the second term in (4) may be regularized by means of formula (1).

Let us discuss now the properties of the formula (4). First of all, it should be noted that its first term is explicitly gauge-invariant. This fact follows immediately from the explicit gauge invariance of $\Sigma_1^{\rm fer}(s)$. Moreover, the first term in (4) is exactly the quantity which was used in the framework of fermion-loop scheme [5, 19], proposed for the gauge-invariant approximate description of the W-pair production. The two-loop corrections, that are important in the resonance region within the NLO precision [6, 7], but missed in the fermion-loop scheme, are collected in the second term in (4). Since this term, originating in probability, cannot be obtained from the analysis of the amplitude, it has been called the *anomalous* additive term [12].

Actually, the anomalous term is gauge invariant, as well. This follows from the equality $\mathrm{Im}\Sigma_1(M^2) = \mathrm{Im}\Sigma_1^{\rm fer}(M^2)$ which may be derived, in particular, from the corresponding explicit expressions [3], and this follows from the gauge invariance of $\mathrm{Im}\Sigma_2(M^2)$ in the OMS-like schemes [20]. Notice, the two-loop-level gauge-dependent part in the OMS mass $M^2$ [21] may be neglected in the anomalous term. Indeed, the second term in (4) describes the *highest*-order correction within the given precision. So, any higher-order variation in the mass actually manifests itself as a correction $O(\alpha)$ in formula (4).

The last, but not least property which should be noted, and which is common for both formulas (3) and (4), is the change of the natural order of individual contributions from the viewpoint of the standard PT. Really, the second term in (4) involves a two-loop correction. However, it describes the NLO correction, but not the NNLO one. The importance of this property is great — it enables one to take into account selectively only those one-loop and two-loop corrections which are really needed in the NLO. Fortunately, the sum of these corrections turns out to be gauge-invariant. The rest of the one-loop and two-loop corrections, that contribute beyond the NLO, are gauge-dependent.

**3$^{\rm o}$**. Let us proceed now to description of the W-pair-mediated four-fermion production in $e^+e^-$ annihilation with taking into account the complete set of NLO corrections. The proposed construction may be presented as the generalization of the fermion-loop scheme. Remember, the latter scheme consists in including all fermionic one-loop corrections in the tree-level amplitudes and Dyson resumming the self-energies. As a result, the amplitude becomes gauge-independent, in spite of the Dyson resummation [5, 19]. However, the fermion-loop scheme does not include certain corrections that are really important within the NLO precision [6, 7]. Namely, the fermion-loop scheme misses the two-loop correction to the self-energy in the Dyson resummed propagators, which is needed in the resonance region, and all the one-loop bosonic corrections, both to the numerators in the amplitude and to the denominators in the Dyson resummed propagators. The idea of the proposed generalization consists in taking into consideration these corrections in probability but not in amplitude, i.e. in the framework of the modified PT.

So, in our approach, let the first-step approximation be the cross-section obtained in the fermion-loop scheme. Actually it includes completely the leading-order contribution and a part of the NLO correction. In order to describe another part, let us fully use the property that it is a correction. Consequently, for its build-up, it is enough to know only the leading-order contribution to the cross-section, but not the full result of the fermion-loop scheme. It is obvious, that the leading-order contribution includes the double propagator squared, since only it includes the lower power of $\alpha$. Namely, it includes $\alpha^{-2}$, with $\alpha^{-1}$ twice arising from the leading-order term in the unstable propagator squared, cf. formula (3). (Notice, only the transverse parts of the unstable propagators should be Dyson resummed and squared. The longitudinal parts of the unstable propagators are to be cancelled at the end of calculations, by virtue of the expected gauge invariance, by contributions of the unphysical states.) Due to the presence of the delta function in the leading-order terms in the unstable propagators squared, the leading-order contribution to the cross-section should be the cross-section of the on-shell W-pair production, multiplied by the corresponding branchings. In fact, this is the cross-section for the CC03 on-shell process.

Let us discuss now the corrections that are missed in the fermion-loop scheme. As has been mentioned above, there are two types of these corrections — the two-loop corrections to self-energy in the Dyson resummed propagators, and the one-loop bosonic corrections. The former ones can be taken into consideration in a quite simple way; one needs only to take into account the anomalous term in formula (4). Since the anomalous term contributes additively and with the delta function — like the leading-order term in (3) — its total effect is reduced to the additive correction of the form of the cross-section of the on-shell W-pair production multiplied by the corresponding branchings, and multiplied by the "anomalous" factor. The latter one is determined as follows:

$$\delta^{\,\mathrm{anom}} = \left[\frac{\pi}{\alpha\,\mathrm{Im}\Sigma_1(M^2)}\right]^{-1} \times 2 \times \left[-\pi\frac{\mathrm{Im}\Sigma_2(M^2)}{\mathrm{Im}\Sigma_1^2(M^2)}\right] = -2\alpha\frac{\mathrm{Im}\Sigma_2(M^2)}{\mathrm{Im}\Sigma_1(M^2)}\,. \tag{5}$$

Here in the middle expression the first multiplier in square brackets cancels the coefficient at the delta-function in the leading-order term in (3), factor 2 is due to the presence of two W's in the CC03 process and to the additive character of the anomalous term. The last multiplier in (5) is the coefficient at the delta function in the anomalous term in (4). Remember, the anomalous factor is completely gauge invariant due to the gauge invariance of its ingredients. Actually, without loss of precision, the anomalous factor may be considered as a factor in the full fermion-loop-scheme cross-section, but not necessary in its leading-order contribution only, which is the cross-section of the CC03 on-shell process.

The problem of the one-loop bosonic corrections may be easily solved as well. Let us group these corrections in two classes. To the first class we refer the corrections to self-energies of unstable particles. To the second class we refer the corrections to the vertex factors, the corrections due to the exchange processes, and due to the real (soft) photons.

In fact, the corrections of the first class have already been taken into consideration by making use of the formula (4). Indeed, in the OMS scheme, the bosonic one-loop correction to the W-boson self-energy does not contribute to the propagator squared within the NLO precision. This is because only the imaginary part of the one-loop on-shell self-energy in the OMS scheme is relevant within the NLO precision. However, the bosonic part of this quantity is zero.

The bosonic corrections of the second class have not yet been taken into consideration. To do this, let us make use of the fact that within the NLO approximation and in presence of *corrections*, the Dyson resummed unstable propagators squared may be considered in the leading-order approximation only. As a result, the bosonic corrections of the second class may be taken into consideration as the corrections to the CC03 on-shell process. Examples of the diagrams of unitarity which generate contributions of this class are shown in Fig.1. In presence of a *finite* non-zero mass of photons, all these diagrams include exactly *two* pairs of unstable propagators of the identical mass and momenta on both sides of the cut. (The only exception, the configuration of Fig.1i, is discussed below.) Therefore, they include *two* delta-functions of the corresponding kinematic variables, which make these configurations on-shell. The sum of all such configurations may be represented as the sum of two groups of terms. The first group includes the one-loop cross-section of the on-shell pair production (Fig.1a,c,e), or the tree-level cross section of the W-pair plus one real photon on-shell production (Fig.1b,d,f) times the lowest-order decay blocks of W-bosons. Another group includes the Born cross-section of the pair on-shell production times their one-loop decay blocks, or the tree-level decay blocks with one real photon (Fig.1g,h,j).

It should be noted that among the diagrams of Fig.1 there are configurations of both factorizable and non-factorizable types. Nevertheless, at an intermediate step, when the photons are supplied with a finite mass, the non-factorizable configurations of Fig.1 become factorizable. All other non-factorizable corrections that do not possess this property produce configurations that contain *less* than two pairs of the unstable propagators squared of identical mass and momenta on both sides of the unitarity cut. Therefore, they do not include the leading-order factor $1/\alpha^2$. Examples of configurations of this type are shown in Fig.2. Owing to presence of an additional factor $\alpha$ which is due to the bosonic exchange, all of them contribute beyond the NLO. Really, the configurations of Fig.2a-f contain one propagator squared only, which includes the factor $1/\alpha$. Consequently, they contribute the NNLO. The configurations of Fig.2g,h do not at all contain the propagators squared. Consequently, they contribute NNNLO.

Let us note that behaviour in $\alpha$ of each of the configuration discussed above does not depend on the value of photon mass. So, this behaviour in $\alpha$ of the sum of the corresponding groups of diagrams should be the same in the zero-mass limit. The zero-mass limit operation can always be done in view of the continuity property of the above-mentioned cross-sections and decay blocks, considered as the functions of the photon mass. Notice, that after taking the zero-mass limit, the gauge invariance must be restored, if it has been broken by the photon mass. The above-mentioned properties of the continuity in the photon mass and of the gauge invariance after the limiting procedure follow from the well-known theorems for the standard on-shell cross-sections with contributions of the real soft photons.

Now let us discuss the above-mentioned exceptional configuration of Fig.1i. Strictly speaking, it should not be considered among the diagrams of unitarity, since it describes the self-energy correction to

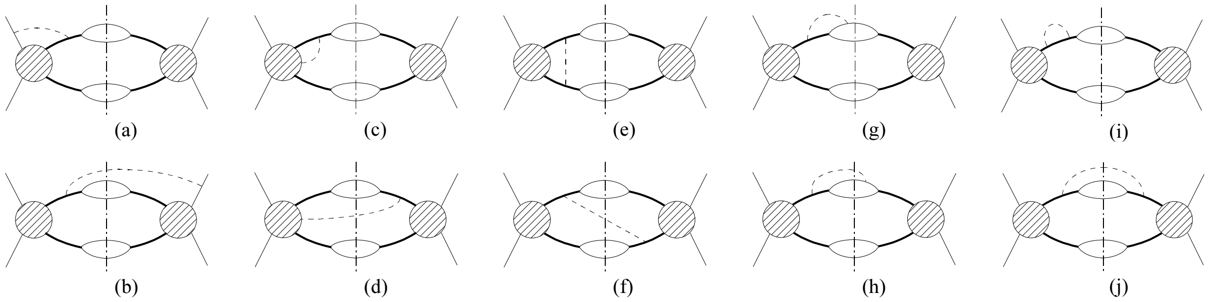

Figure 1: Examples for factorizable and (quasi) non-factorizable corrections to the W-pair production which have to be taken into consideration in the NLO approximation. (The dashed lines denote the massive bosons or the photons. The thin and thick lines represent the initial/final fermions and W-bosons. The vertical dot-dashed lines mean the cut of unitarity. The hatched areas denote the corresponding lowest-order Green functions.)

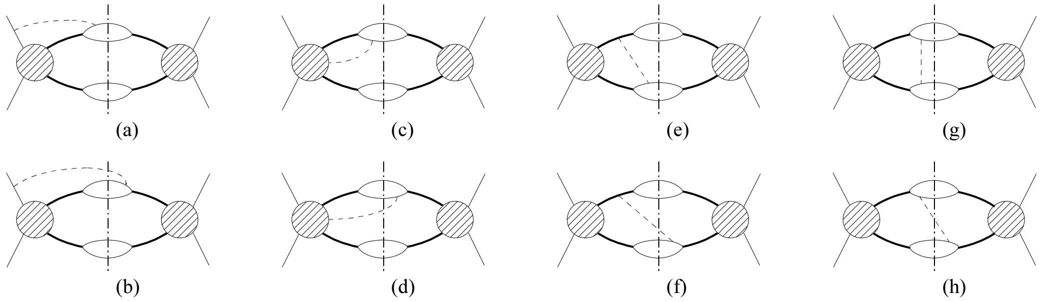

Figure 2: Examples for manifestly non-factorizable corrections to the W-pair production.

the unstable propagator, which has already been taken into consideration in formula (4). Nevertheless, while considering the cross-section of the on-shell pair production, one has to take into account the virtual soft-photon insertions to the external legs, which are due to the wave function renormalization. The configuration of Fig.1i was added to the list of diagrams of Fig.1 only in order to indicate this fact.

The above discussion leads us to the following resultant formulas for the cross-section:

$$\sigma(s) = \int_0^1 \mathrm{d}z \; \phi(z;s)\, \hat{\sigma}(zs)\,, \qquad \hat{\sigma}(s) = \int_0^s \mathrm{d}s_+ \int_0^{(\sqrt{s}-\sqrt{s_+})^2} \mathrm{d}s_- \; \hat{\sigma}_0(s\,; s_+, s_-). \tag{6}$$

Here $\sigma(s)$ is the experimentally measured total cross-section at the center-of-mass energy squared $s$. $\phi(z;s)$ is the "flux" function describing the contributions of the initial-state and final-state photon radiations with large IR and collinear logarithms. Quantity $\hat{\sigma}(s)$ is the hard scattering cross-section at the reduced center-of-mass energy squared (see Refs. [1, 2, 3] and references therein). It should be noted that $\hat{\sigma}(s)$ contributes to $\sigma(s)$ as distribution, because $\hat{\sigma}(s)$ is smeared by the flux function over the allowed range of kinematic variable ($\phi(z;s)$ is peaked at $z=1$ and has a tail up to the cut at lower values of $z$). The second formula in (6) represents $\hat{\sigma}(s)$ in the form of the explicit phase-volume integration (over the virtualities of the unstable particles, which are the invariant masses of the corresponding final states). The $\hat{\sigma}_0(s)$ is the quantity that we have discussed above. It reads as follows:

$$\begin{aligned}
\hat{\sigma}_0(s\,; s_+, s_-) &= \hat{\sigma}_0^{\text{fermion-loop-scheme}}(s\,; s_+, s_-) \times (1+\delta^{\,\text{anom}}) \\
&+ \hat{\sigma}_0^{\text{CC03-on-shell, boson-one-loop+real-photon}}(s\,; M_+, M_-) \times \prod_{\kappa=\pm} \delta(s_\kappa - M_\kappa^2) \times BR_\kappa^{\,\text{tree}} \\
&+ \hat{\sigma}_0^{\text{CC03-on-shell,tree}}(s\,; M_+, M_-) \times \prod_{\kappa=\pm} \delta(s_\kappa - M_\kappa^2) \times BR_\kappa^{\,\text{tree/boson-one-loop+real-photon}}\,.
\end{aligned} \tag{7}$$

Here the first term (without the $\delta^{\,\text{anom}}$) represents the result of the conventional fermion-loop scheme. All other terms describe the corrections. The quantities $BR_{\pm}^{\rm tree}$ mean the lowest-order on-shell branchings of the $W^{\pm}$-bosons. In the third term, one of *BR*'s is determined as the sum of the bosonic one-loop correction to the partial width of W-boson and the width with one real photon, divided by the total Born width. (In fact, the third term includes the sum of four subterms with the modified *BR* for one of two unstable particle.) All the above ingredients are IR-finite and gauge invariant. Moreover, except for $\mathrm{Im}\Sigma_2(M^2)$ in $\delta^{\rm anom}$, all of them have already been calculated [22, 23].

**4°**. In the above discussion we have omitted the corrections caused by the gluon exchanges. However, they contribute to the real processes and, so, they have to be taken into consideration. The gluon exchanges occur between the final-state quarks, between the final-state quarks and W-bosons in the quark loops, and between the W-bosons and quark loops. Examples of the corresponding configurations are shown in Fig.1c-j and Fig.2c-h with a modified specification. Namely, by the dashed lines we imply now a group of gluons (two or more), and by the points of intersection between the dashed lines and the W-boson lines, or the vertices, we imply the presence of the fermionic loop. Correspondingly, these points of intersection are supplied with the additional factor $\alpha$. (We do not take into account the strong coupling constant $\alpha_s$, assuming that it is of order $O(1)$. The latter assumption means that we consider the soft-gluon exchanges. The hard-gluon exchanges are additionally suppressed by the smallness of $\alpha_s$.)

In order to avoid the collinear and IR divergences, we consider the gluons at the intermediate stage to be massive (it is enough to make the groups of the gluons to be effectively massive). Simultaneously, this permits to delete from the list of diagrams that contribute to the NLO the following configurations: Fig.1c-f and Fig.1i,j as including the additional $\alpha^2$, Fig.2c-f as suppressed by one additional $\alpha$ and switching off the propagator squared, and Fig.2g,h as suppressed by switching off two unstable propagator squared. All the above-mentioned configurations considered with the new specification make contributions to the NNLO.

So, the configurations which are responsible for the phenomenon of color reconnection make contributions beyond the NLO approximation. The only configurations surviving in the NLO are those which contribute to the decay factors of W-bosons. Among the listed above configurations they are of Fig.1g,h. They have to be taken into consideration while calculating the corresponding *BR*'s in formula (7).

**5°**. The above results may be easily generalized to the cases of the differential cross-sections. In these cases one must replace the expressions that stand behind the above-discussed diagrams of unitarity by the corresponding amplitudes squared, saving the necessary integrations ( configurations remain the same). Then, in the case of pure angular distributions, the only modification in formulas (6) and (7) consists in replacing the cross-sections (and branchings, if necessary) by the corresponding angular distributions. A generalization to the cases of the invariant-mass distributions is possible as well, although it is not so straightforward. Really, it should necessarily involve a serious modification in the structure of the integrals in (6). Namely, at least one of the two phase-volume integrals must disappear in the second formula in (6). As a result, the convolution becomes acting directly on the corresponding hard-scattering distribution. This rather sharp modification may have nontrivial consequences. In particular, it may result in the change to the worse of the convergence properties of the series in powers of $\alpha$ in the modified PT. In practice, exactly this change seems to take place, because the manifestly non-factorizable corrections become non-negligible in the case of the invariant-mass distributions [14]. Nevertheless, in view of the sharp decreasing of the effect with the increasing $s$ [7, 14], the property of convergence seems to be rapidly improved with the increased energy. Since the invariant-mass distributions are not important for the practical use, at least near the threshold, the above-discussed effect should not have important consequences.

In the cases of the total cross-section and pure angular distributions, the property of convergence of the modified PT remains quite satisfactory almost everywhere, except for the range in the direct vicinity of the threshold, which is numerically less than the width of W-boson. This observation follows from the well-known property of closeness in this range of the on-shell CC03 total cross-section to the off-shell CC03 total cross-section [1]. Actually, the difference between these two cross-sections is a characteristic of the accuracy of the description of the W-pair production in the leading-order approximation. The numerical accuracy of the description within the NLO approximation is controlled by this difference as well. The more detailed discussion of this problem is the subject of the forthcoming paper.

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