String Models of Mesons and Baryons and Regge Trajectories

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Abstract

For the relativistic string with massive ends (the meson model) and four various string baryon models (quark-diquark, linear, three-string and triangle) classical rotational and quasirotational motions are considered. The rotational motions or planar uniform rotations are used to describe orbitally excited hadron states on the Regge trajectories. Analyzing small disturbances of these rotations we obtained two types of oscillatory motions for the relativistic string with massive ends. They have the form of stationary waves in the rotational plane and in the orthogonal direction. These solutions may be used for describing larger spectrum of hadron excitations. Analysis of small disturbances allows to solve the stability problem for the rotational motions for the studied string configurations and shows that these motions on the classical level are unstable for the three-string (Y) and linear (q-q-q) models, whereas for the meson-like and the triangle model they are stable.

Introduction

The considered string meson model [1, 2] (see Fig. 1 a) includes two massive points connected by the relativistic string that simulates the QCD confinement mechanism. String models of the baryon were suggested in Ref. [3] soon after the meson ones. In Fig. 1 (b) – (e) four variants with different topologies of spatial junctions of the three massive pointlike quarks by relativistic strings are represented: (a) the meson-like quark-diquark model q-qq [4], (b) the linear configuration q-q-q [5], (c) the "three-string" model or Y configuration [6, 7], and (d) the "triangle" model or Δ configuration [8, 9].

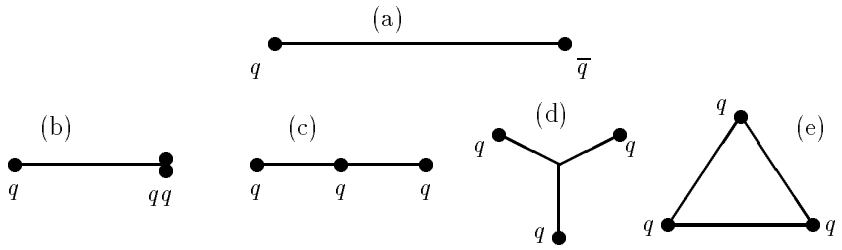

Figure 1: String models of the meson (a) and the baryon (b) – (e).

The problem of choosing the most adequate string baryon model among the four mentioned ones has not been solved yet. All these models can work in particle physics, and under certain assumptions describe the linearly growing Regge trajectories for orbitally excited baryon states [10] (Sect. 1). All models have some degree of the QCD motivation, but different authors give some arguments in favour of the Y configuration [11] or the "triangle" model [12] in the framework of the QCD-based baryon Wilson loop operator approach.

The quark-diquark configuration was more popular (in comparison with other string baryon models), because of its simplicity and similarity with the meson model. Any approach suggested for the meson string model q-$\overline{q}$ may be applied to the baryon model q-qq with minimal alterations. The important advantage of the quark-diquark string baryon configuration (this concerns also the q-q-q model) is connected with the equality of the predicted Regge slopes for mesons and baryons, if the string tensions are the same.

The linear q-q-q configuration was supposed to be unstable with respect to the transformation into the quark-diquark one [4]. However this problem was studied quantitatively only recently in Ref. [5], where we showed that the classical rotational motions of this model are indeed unstable, but the system does not

transform into the q-qq configuration. For the three-string model or Y configuration the massless variant was better investigated [3, 6, 7] in comparison with the q-q-q or Δ ones (for other models the massless case is trivial non-baryon open or closed string), but even for the massless three-string the dynamics remains too complicated. The delay with the study of the "triangle" baryon model was also connected with some difficulties in its dynamics.

For choosing the most adequate string baryon configuration, it is very important to solve the stability problem for the classical rotations for all the abovementioned models. This problem was studied in Ref. [13]. It was surprising that rotations of the Y configuration appeared to be unstable too. The latter instability is proved (even in the linear approximation) in Sect. 3 with the help of analysis of the quasirotational motions of the three-string model. This result in linear approximation is compared with the stability of the rotational motions for the relativistic string with massive ends that was obtained in Sect. 2. The latter result gives a possibility for quantization of this nonlinear system in the linear vicinity of the rotational solution (this is possible only for a stable solution).

1. Rotational states and Regge trajectories

For all mentioned string hadron models, the classical solutions describing the rotational motion (planar uniform rotations of the system) are known and widely used for modelling the orbitally excited hadron states on the main Regge trajectories [4, 10, 14, 15]. The rotational motion of the meson model or the baryon configuration q-qq is a rotation of the rectilinear string segment [1, 4] (with the middle quark at the rotational center for the model q-q-q). The world surfaces $X^\mu(\tau,\sigma)$ of these rotational motions may be represented in the form [10, 13]

$$X^\mu(\tau,\sigma) = X^\mu_{rot}(\tau,\sigma) = \Omega^{-1}\big[\omega\tau e^\mu_0 + \cos(\omega\sigma+\phi_0)\cdot e^\mu(\tau)\big]. \tag{1}$$

Here Ω is the angular velocity, e^μ_0 is the unit time-like velocity vector of c.m. in Minkowski space with signature $+,-,-,\ldots$, $e^\mu(\tau) = e^\mu_1\cos\omega\tau + e^\mu_2\sin\omega\tau$ is the unit ($e^2=-1$) space-like rotating vector directed along the string, $\sigma\in[\sigma_1,\sigma_2]$, where without loss of generality $\sigma_1=0$, $\sigma_2=\pi$ [2]. The parameter ω (dimensionless frequency) is connected with the constant speeds v_1, v_2 of the ends through the relations $v_1=\cos\phi_0$, $v_2=-\cos(\pi\omega+\phi_0)$, $m_i\gamma^{-1}\Omega=(1-v_i^2)/v_i$, γ is the string tension, m_i are the masses of the endpoints.

For the three-string model the rotational motion is also well known [3, 7]. The form of rotating Y configuration is three rectilinear string segments with the world surfaces similar to Eq. (1) joined in a plane at the angles 120°.

The rotational motion of the baryon 'triangle" model describes an uniformly rotating closed string (curvilinear triangle) composed of three segments of a hypocycloid. The corresponding world surface is [8, 10]

$$X^0 = \tau - \tfrac{T}{D}\sigma, \qquad X^1 + iX^2 = u(\sigma)\cdot e^{i\omega\tau}. \tag{2}$$

Here $u(\sigma) = A_i\cos\omega\sigma + B_i\sin\omega\sigma$, $\sigma\in[\sigma_i,\sigma_{i+1}]$ (for the whole world surface $\sigma_0<\sigma<\sigma_3$), the six complex constants A_i, B_i and the real constants σ_i, $D=\sigma_3-\sigma_0$, V_i^2, $\tau^*-\tau=T$ are connected by the set of relations [8, 10, 14].

Expressions (1) and (2) are exact solutions of the classical equations of motion for these systems satisfying all boundary conditions. Both the equations of motion and the boundary conditions result from the action that for all the configurations may be presented in the form [2, 10]

$$S = -\gamma\int\limits_G \sqrt{-g}\,d\tau\,d\sigma - \sum_{i=1}^N m_i\int\sqrt{\dot{x}_i^2(\tau)}\,d\tau, \tag{3}$$

where the number N of material points with masses m_i (quarks, antiquarks or diquarks) is $N=2$ for the meson-like models q-$\overline{q}$, q-qq and $N=3$ for others, $g=\dot{X}^2X'^2-(\dot{X},X')^2$, $\dot{X}^\mu=\partial_\tau X^\mu$, $X'^\mu=\partial_\sigma X^\mu$, the domain G mapping into the world surface is bounded by the inner lines $\sigma=\sigma_i(\tau)$ of the material points, G is different for different configurations [10, 13], $\dot{x}_i^\mu=\frac{d}{d\tau}X^\mu(\tau,\sigma_i(\tau))$, $c=1$.

The geometry of the systems Y and Δ should be taken into account: for the triangle configuration the string is closed (but it is not smooth), so we use the general form of the closure condition [8] $X^\mu(\tau,\sigma_0(\tau)) = X^\mu(\tau^*,\sigma_3(\tau^*))$. Here $\sigma=\sigma_0(\tau)$ and $\sigma=\sigma_3(\tau)$ describe the trajectory of the same

quark. For the three-string baryon model in three parametrizations $X_i^\mu(\tau_i,\sigma)$ of the three world sheets the different "time-like" parameters τ_i [13] are connected at the junction world line $\tau_2=\tau_2(\tau)$, $\tau_3=\tau_3(\tau)$, $\tau_1\equiv\tau$. So the general form of the junction condition is

$$X_1^\mu\big(\tau,0\big)=X_2^\mu\big(\tau_2(\tau),0\big)=X_3^\mu\big(\tau_3(\tau),0\big). \tag{4}$$

For all models, the equations of motion and the boundary conditions at the trajectories of massive points resulting from the action (3) take the simplest form under the orthonormality conditions

$$\dot{X}^2+X'^2=0, \qquad (\dot{X},X')=0, \tag{5}$$

which may be used for all the models [2, 8, 13]. If conditions (5) are satisfied, the equations of motion become linear

$$\ddot{X}^\mu-X''^\mu=0, \tag{6}$$

but the boundary conditions for the massive point at an end

$$m_i\frac{d}{d\tau}U_i^\mu(\tau)\pm\gamma\big[X'^\mu+\sigma_i'(\tau)\,\dot{X}^\mu\big]\Big|_{\sigma=\sigma_i}=0, \quad U_i^\mu(\tau)=\frac{x_i^\mu(\tau)}{x_i^\mu(\tau)}=\frac{\dot{X}^\mu+\sigma_i'X'^\mu}{|\dot{X}+\sigma_i'X'|}\Big|_{\sigma=\sigma_i} \tag{7}$$

or the middle points (for the models q-q-q or Δ)

$$m_i\frac{d}{d\tau}U_i^\mu(\tau)-\gamma\big[X'^\mu+\sigma_i'(\tau)\,\dot{X}^\mu\big]\Big|_{\sigma=\sigma_i+0}+\gamma\big[X'^\mu+\sigma_i'(\tau)\,\dot{X}^\mu\big]\Big|_{\sigma=\sigma_i-0}=0, \tag{8}$$

remain essentially nonlinear. These massive points make the models much more realistic, but they bring additional nonlinearity and (hence) a lot of problems with quantization of these models.

The energy E and angular momentum J of the states (1) and (2) are [8, 10]

$$E=E_{st}+\sum_{i=1}^N\frac{m_i}{\sqrt{1-v_i^2}}+\Delta E, \quad J=\frac{1}{2\Omega}\left[E_{st}+\sum_{i=1}^N\frac{m_iv_i^2}{\sqrt{1-v_i^2}}\right]+S, \tag{9}$$

where $E_{st}=\gamma\Omega^{-1}\arcsin v_i$ for the motions (1) and $E_{st}=\gamma D(1-T^2/D^2)$ for the triangle states (2). The quark spins with projections s_i ($S=\sum_{i=1}^N s_i$) are taken into account, in particular, as the spin-orbit correction $\Delta E=\Delta E_{SL}=\sum\limits_i\beta(v_i)(\vec{\omega}\vec{s}_i)$ to the energy of the classical motion. Here we use $\beta(v_i)=1-(1-v_i^2)^{1/2}$ for this correction [10, 14]. The expression (9) for all string hadron models describes quasilinear Regge trajectories with the similar ultrarelativistic behavior:

$$J\simeq\alpha'E^2-\nu E^{1/2}\sum_{i=1}^N m_i^{3/2}+\sum_{i=1}^N s_i\big[1-\beta(v_i)\big], \quad v_i\to 1.$$

Here the slopes are different: $\alpha'=(2\pi\gamma)^{-1}$ for the meson-like models, $\alpha'=\frac{2}{3}(2\pi\gamma)^{-1}$ for the Y and $\alpha'=\frac{3}{8}(2\pi\gamma)^{-1}$ for the so called simple states (2) of the triangle configuration [9, 10, 14].

All mentioned string baryon models can work in the baryon spectroscopy, if we (with the described type of the spin-orbit correction) suppose that $\gamma=\gamma_{q-qq}=0.175$ GeV^2, the effective tension for the Y and "triangle" is to be different $\gamma_Y=\frac{2}{3}\gamma$, $\gamma_\Delta=\frac{3}{8}\gamma$, the effective quark masses $m_u=m_d=130$ MeV, $m_s=300$ MeV, $m_c=1600$ MeV. Under these assumptions, all these baryon models describe orbital excitations on the following Regge trajectories: $N(938)$ (with $J^P=1/2^+,5/2^+,9/2^+,\ldots$), $N(1520)$ ($J^P=3/2^-,7/2^-,\ldots$), $N(1675)$ ($J^P=5/2^-,7/2^+,9/2^-$), $\Delta(1232)$ ($J^P=3/2^+,7/2^+,\ldots$), $\Delta(1930)$ ($J^P=5/2^+,9/2^+,\ldots$), $\Delta(1700)$ ($J^P=3/2^-,5/2^+,7/2^-,\ldots$); in the strange sector Λ ($J^P=1/2^+,3/2^-,5/2^+,7/2^-,\ldots$), $\Lambda(1405)$ ($J^P=1/2^-,3/2^+$), $\Sigma(1193)$ ($J^P=1/2^+,3/2^-,5/2^+,\ldots$), $\Xi(1315)$ ($J^P=1/2^+,3/2^-,5/2^+$), and the charmed trajectory with $\Lambda_c^+(2625)$ [10, 14]. Under the same assumptions, in the framework of the string meson model shown in Fig. 1 (a), the Regge trajectories for the light unflavored ($\rho-a$) and strange mesons K^* are also well described. Thus, the results in the meson spectroscopy in comparison with the baryon sector confirm the adequacy of the considered string models.

2. Quasirotational states for the meson string model

After this brief review of the rotational motions, we consider the quasirotational states of all these systems. They are small disturbances of the rotational motions (1) and (2). They are interesting due to the following three reasons: (a) we are looking for the motions describing the hadron states, which are usually interpreted as higher radially excited states, and other states in the potential models [14], in other words, we must describe the daughter Regge trajectories; (b) the quasirotational states are the basis for quantization of these nonlinear problems in the linear vicinity of the solutions (1), (2) (if they are stable); (c) the quasirotational motions are necessary for solving the important problem of stability of rotational states (1), (2) for all discussed string models (that has not been solved yet).

For the meson string model, the quasirotational motions of a slightly curved string with massive ends were studied in Refs. [16]. But in these papers the authors used very specific ansatz for the search of these disturbances and a complicated nonlinear form of the string equations of motion beyond the conditions (5). Besides, they neglected some important dependencies and the boundary conditions (7), so these solutions in Refs. [16] were not correct (details are in Ref. [13]).

In Ref. [13] another approach for obtaining the quasirotational solutions was suggested. It includes the orthonormality conditions (5) and, hence, the linear equations of motion (6) with the general solution

$$X^{\mu}(\tau,\sigma) = \frac{1}{2}\big[\Psi^{\mu}_{+}(\tau+\sigma) + \Psi^{\mu}_{-}(\tau-\sigma)\big]. \tag{10}$$

So the problem is reduced to the system of ordinary differential equations with shifted arguments resulting from the boundary conditions (7). As the unknown function may be taken $\Psi^{\mu}_{+}(\tau)$, $\Psi^{\mu}_{-}(\tau)$, or unit velocity vectors of the endpoints $U^{\mu}_{1}(\tau)$ or $U^{\mu}_{2}(\tau)$ — this is equivalent due to relations of Ref. [17, 18]

$$\Psi'^{\mu}_{\pm}(\tau \pm \sigma_i) = m_i \gamma^{-1}\big[\sqrt{-U_i'^{2}(\tau)}\, U^{\mu}_{i}(\tau) \mp (-1)^{i} U'^{\mu}_{i}(\tau)\big]. \tag{11}$$

After the substitution of the slightly disturbed rotational motion (1)

$$U^{\mu}_{1}(\tau) = U^{\mu}_{1rot}(\tau) + u^{\mu}(\tau), \qquad |u^{\mu}| \ll 1. \tag{12}$$

(U^{μ}_{1rot} corresponds to X^{μ}_{rot}) into the boundary conditions (7), (11) we obtain in the first linear approximation the linearized system of equations describing the evolution of small arbitrary disturbances u^{μ}. Solutions of this system may be expanded in the Fourier series $u^{\mu}(\tau) = \sum\limits_{n=-\infty}^{+\infty} u^{\mu}_{n} \exp(-i\omega_n \tau)$, and the resulting arbitrary quasirotational motions for the string with massive ends were obtained in the form [13]

$$\begin{aligned} X^{\mu}(\tau,\sigma) = X^{\mu}_{rot}(\tau,\sigma) + \sum_{n=-\infty}^{\infty} \Big\{ & e^{\mu}_{3}\alpha_n \cos(\omega_n\sigma + \phi_n)\exp(-i\omega_n\tau) \\ & + \beta_n\big[e^{\mu}_{0} f_0(\sigma) + e^{\mu}_{\perp}(\tau) f_{\perp}(\sigma) + i e^{\mu}(\tau) f_r(\sigma)\big]\exp(-i\tilde{\omega}_n\tau)\Big\}. \end{aligned} \tag{13}$$

Here X^{μ}_{rot} is the pure rotational motion (1), $e^{\mu}_{\perp}(\tau) = \omega^{-1}\frac{d}{d\tau}e^{\mu}(\tau)$; e_0, e_1, e_2, e_3 is the orthonormal tetrad, $\omega = \omega_1$. Each term in Eq. (13) describes the string oscillation that looks like the stationary wave with n nodes. There are two types of these stationary waves: (a) orthogonal oscillations along z or e_3-axis with the frequencies proportional to the roots ω_n of equation[1]

$$(\omega^2 - Q_1 Q_2)/\big[(Q_1 + Q_2)\,\omega\big] = \cot \pi\omega, \tag{14}$$

where $Q_i = \omega_1 v_i/\sqrt{1-v_i^2} = \text{const}$, ω_1 is equal to the parameter ω in Eq. (1); and (b) planar oscillations (in the rotational plane e_1, e_2) with the dimensionless frequencies $\tilde{\omega}_n$ satisfying the equation

$$\frac{(\tilde{\omega}^2 - q_1)(\tilde{\omega}^2 - q_2) - 4Q_1Q_2\tilde{\omega}^2}{2\tilde{\omega}\big[Q_1(\tilde{\omega}^2 - q_2) + Q_2(\tilde{\omega}^2 - q_1)\big]} = \cot \pi\tilde{\omega}, \tag{15}$$

[1] It is interesting that the same equation (14) describes the spectrum of states for the relativistic string with massive ends with the linear boundary conditions [18].

Here $q_i = Q_i^2(1+v_i^{-2})$.

The frequencies ω_n and $\tilde{\omega}_n$ from Eqs. (14) and (15) are real numbers, so the rotations (1) of the string with massive ends are stable in the linear approximation. One may consider the expansion (13) for an arbitrary quasirotational motion as the basis for further quantization of this system in the linear vicinity of the solution (1).

3. Quasirotational states and instability of the three-string model

Let us compare the picture for the quasirotational states of the string with massive ends with the similar motions for the string baryon models q-q-q, Y and Δ. These motions are more complicated, but for the three-string model they may be investigated by using the above mentioned method of small disturbances (12). For the Y configuration [6, 7] three world sheets (swept up by three segments of the relativistic string) are parametrized by three functions $X_i^\mu(\tau_i,\sigma)$ with different "time-like" parameters τ_1, τ_2, τ_3 connected on the junction world line ($\sigma = 0$) in the following way: $\tau_1 \equiv \tau$, $\tau_2 = \tau_2(\tau)$, $\tau_3 = \tau_3(\tau)$ [13, 19].

The rotational motions for the three-string are parametrized similar to (1)

$$X^\mu_{i(rot)}(\tau_i,\sigma) = \Omega^{-1}\left[\omega\tau_i e_0^\mu + \sin\omega\sigma\cdot e^\mu(\tau_i+\Delta_i)\right], \quad \Delta_i = 2\pi(i-1)/(3\omega_1). \tag{16}$$

Substituting small disturbances (12) $U_i^\mu(\tau_i) = U^\mu_{i(rot)}(\tau_i) + u_i^\mu(\tau_i)$ in the boundary conditions and looking for solutions of this linearized system in the form

$$u_i^\mu(\tau_i) = \left[A_i^0 e_0^\mu + A_i e^\mu(\tau_i+\Delta_i) + v_i^{-1}A_i^0 e_\perp^\mu(\tau_i+\Delta_i) + A_i^z e_3^\mu\right]\exp(-i\tilde{\omega}\tau_i)$$

with $\tau_i(\tau) = \tau + \delta_i\exp(-i\tilde{\omega}\tau)$, we obtain the linear homogeneous system with respect to amplitudes A_i^0, A_i, A_i^z, δ_i. Its nontrivial solutions similar to Eq. (13) describe separately quasirotational motions in e_3 or z-direction and planar motions. In the first case, the equations for amplitudes A_i^z are

$$(\omega\cot\pi\omega + Q_1)(A_1^z + A_2^z + A_3^z) = 0, \quad (Q_1\cot\pi\omega - \omega)(A_1^z - A_i^z) = 0, \quad i = 2,3. \tag{17}$$

Here for simplicity we consider the system with equal masses $m_i = m_1$ and $Q_i = \omega_1 v_i/\sqrt{1-v_i^2} = Q_1$. Nontrivial solutions of Eq. (17) are possible only if $\omega = \omega_n$ are roots of the equations

$$\omega/Q_1 = \cot\pi\omega, \qquad \omega/Q_1 = -\tan\pi\omega. \tag{18}$$

The first equation coincides with Eq. (14) in the case $m_2 \to \infty$ or $Q_2 = 0$. The quasirotational states corresponding to each root of this equation $\omega = \omega_n$, $n = 1,2,\ldots$ are

$$X_i^\mu(\tau_i,\sigma) = \check{X}^\mu_{i(rot)} + |A_i^z|\sqrt{1+Q_1^2/\omega_n^2}\, e_3^\mu \sin\omega_n\sigma\cdot\cos(\omega_n\tau+\psi_n). \tag{19}$$

Here $\tau_i = \tau$ and for each n we have two independent modes of the oscillations (stationary waves) with the amplitudes A_i^z connected by the relation $A_1^z + A_2^z + A_3^z = 0$.

For the second equation (18), the stationary waves are described by the same relation (19), but with $\cos\omega_n\sigma$ substituted by $\sin\omega_n\sigma$, and by the equality $A_1^z = A_2^z = A_3^z$. If the oscillations of three string segments are synchronous and equal, the junction oscillates also. For all modes, the roots of Eqs. (18) are real numbers.

The planar quasirotational motions of the three-string are more complicated. The system for the amplitudes A_i^0, A_i, δ_i similar to Eqs. (17) consists of 8 independent equations

$$\begin{gathered}
i\tilde{\omega}\Gamma\left[Q_1(C_{\tilde{\omega}}+i)\,\delta_i + C_{\tilde{\omega}}(A_1 - A_i)\right] + \left[Q_1(1+v_1^{-2})C_{\tilde{\omega}} - \tilde{\omega}\right](A_1^0 - A_i^0) = 0,\\
\epsilon_i v\Gamma^2(\tilde{\omega}^2 - \omega^2)\,A_i + 2K_1A_1^0 + (K_1 - i\epsilon_i K_2)\,A_i^0 = 0,\\
2K_1A_1 + (K_1 + i\epsilon_i K_2)\,A_i + \epsilon_i K_3 A_i^0 = 0,\\
i\tilde{\omega}\Gamma(A_1 + A_2 + A_3) + \left[Q_1(1+v_1^{-2}) + \tilde{\omega}C_{\tilde{\omega}}\right](A_1^0 + A_2^0 + A_3^0) = 0,\\
K_4\left[2\tilde{\omega}A_1^0 - (\tilde{\omega} - i\sqrt{3}\,\omega_1)\,A_2^0 - (\tilde{\omega} + i\sqrt{3}\,\omega_1)\,A_3^0\right] = 0,\\
\left[K_4(\tilde{\omega}^2 - \omega_1^2) + 2iK_1(\tilde{\omega}^2 + \omega_1^2)\right](A_2^0 - A_3^0) = 0.
\end{gathered}$$

Here in the first 3 equations $i = 2, 3$, $C_{\tilde\omega} = \cot \pi\tilde\omega$, $\epsilon_i = (-1)^i\sqrt{3}$, $q_1 = Q_1^2(1 + v_1^{-2})$, $K_1 = q_1 - \tilde\omega^2 + 2Q_1\tilde\omega C_{\tilde\omega}$, $K_2 = \Gamma v_1\left[(\tilde\omega^2 + \omega_1^2)\, C_{\tilde\omega} + 2Q_1\tilde\omega\right]$, $K_3 v_1 \Gamma^2(\tilde\omega^2 - \omega_1^2) = K_2^2 - K_1^2$, $K_4 = (q_1 - \tilde\omega^2)\, C_{\tilde\omega} - 2Q_1\tilde\omega$.

This system has 3 branches of nontrivial solutions. The first two branches are described by the dimensionless frequencies $\tilde\omega = \tilde\omega_n$ satisfying the equations

$$\frac{\tilde\omega^2 - q_1}{2Q_1\tilde\omega} = \cot \pi\tilde\omega\,, \qquad \frac{\tilde\omega^2 - q_1}{2Q_1\tilde\omega} = -\tan \pi\tilde\omega\,. \tag{20}$$

The first Eq. (20) looks like Eq. (15) if $Q_2 = 0$ or $m_2 \to \infty$. These oscillations are planar-symmetric stationary waves with the amplitudes $A_1 = A_2 = A_3$, $A_1^0 = A_2^0 = A_3^0$ and with real frequencies.

The frequencies of the third branch are roots of the equation

$$2\frac{Q_1\tilde\omega(\omega_1^2 - \tilde\omega^2) - i(\tilde\omega^2 - q_1)(\tilde\omega^2 + \omega_1^2)}{(\tilde\omega^2 - q_1)(\tilde\omega^2 - \omega_1^2) - 4iQ_1\tilde\omega(\tilde\omega^2 + \omega_1^2)} = \cot \pi\tilde\omega, \tag{21}$$

and are always complex numbers (except for the roots $\tilde\omega = \pm\omega_1$). For example, the first roots of Eq. (21) for the motion (16) with $m_i = 1$, $\gamma = 1$, $R_i = v_i/\Omega = 0.3$ are $\tilde\omega_1 \simeq 0.276 + 0.0297i$, $\tilde\omega_2 \simeq 1.05 + 0.157i$, $\tilde\omega_3 \simeq 2.027 + 0.17i$, $\tilde\omega_4 \simeq 3.0183 + 0.173i$. Imaginary parts of the roots of Eq. (21) are always positive, so the disturbances of this class (branch) exponentially grow in time with the factor

$$\exp(-i\tilde\omega_n\tau) = \exp(-i\Re\tilde\omega_n\tau)\exp(\Im\tilde\omega_n\tau)\,.$$

Arbitrary quasirotational motion (a disturbance of the rotational motion (16)) may be expanded in the Fourier series with harmonics of all classes described above. So only for the disturbances with the special symmetry (when all amplitudes of the modes (21) are equal to zero), these disturbances do not grow exponentially. So we proved that the rotational motions (16) for the three-string configuration are unstable even in the linear approximation.

The evolution of this instability was calculated numerically [13, 19] using the developed approach of solving the initial-boundary value problem for this model. The numerical experiments [13, 19] show that the picture of motion is qualitatively identical for any small asymmetric disturbance. Starting from some point in time, the junction begins to move. During this complicated motion, the distance between the junction and the rotational center increases, and the lengths of the string segments vary quasiperiodically unless one of the material points inevitably merges with the junction.

Note that for the massless ($m_i = 0$) three-string model [6, 7] the rotational motions are also unstable. In this case, the equation (21) has the form

$$2i(\tilde\omega^2 + \omega_1^2)/(\omega_1^2 - \tilde\omega^2) = \cot \pi\tilde\omega, \qquad m_i = 0$$

and its roots are complex numbers.

The stability problem for the string baryon models q-q-q and Δ was studied in [5, 13] by solving the initial-boundary value problem for these systems. The results were different: the classical rotational motions (2) for the triangle model appeared to be stable and for the linear q-q-q configuration they are unstable. That means that any small asymmetric perturbation results in complicated motion with quasi-periodical varying of the distance between the nearest two quarks. However, the minimal value of this distance is not zero, in other words, the system q-q-q is not transformed in the quark-diquark (q-qq) one, as was formerly supposed in Ref. [4].

Conclusion

For all string hadron models considered above (see Fig. 1), the rotational motions may be used under some assumptions for describing orbital excitations on the parent meson and baryon Regge trajectories. However, the choice of the most adequate string baryon model among the four existing ones depends on stability of these rotational states.

The stability problem for classical rotations is solved for all string meson and baryon models. Rotational motions (1) for the relativistic string with massive ends are classically *stable* in the linear approximation. Any small disturbances of these motions may be represented in the form of Fourier series (13). Rotational motions (1) and (16) for the string baryon models q-q-q and Y are *unstable* on the classical

level (any their asymmetric disturbances grow exponentially), but similar motions (2) of the triangle configuration are *stable*. For the three-string baryon model, instability of the state (16) is connected with presence in the spectrum of quasirotational excitations of the modes with growing amplitudes or complex frequencies (21).

Quasirotational motions of the string with massive ends in the form (13) may be used as the basis of quantization in the linear vicinity of the stable solution (1). Expression (13) satisfies the constraint (5), so, in this case, we have no analog of the Virasoro conditions. Progress in quantization of the nonlinear meson and baryon string models considered by us is necessary for describing not only orbital, but also radial and other excitations of hadrons, known from potential models [14]. Quantization in the linear vicinity is possible only for stable solutions, so we have some problems here for the linear q-q-q and three-string models. However, this doesn not mean that these models are finally closed for further applications.

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