Bundles Over the Moduli Space of Skyrme Model

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Abstract

The problem of the construction of a fiber bundle over the moduli space of the Skyrme model is considered. The extension of the original Skyrme model including the interaction with fermions is constructed to obtain a fiber bundle structure over the moduli space. The possibility of the non-trivial holonomy appearance is considered. It is shown that the effect of the fermion interaction turns the n-skyrmion moduli space into a real vector bundle with natural $SO(2n+1)$ connection.

It was already noted [1] that the solutions of the Skyrme model [2], especially at the low energy, look like monopoles with the baryon number being identified with the monopole topological number. This correspondence looks more clear by using the moduli space approach [3].

In the recent paper by N. Manton and B Schroers [4] the effect of fermion interaction with BPS monopoles have been studied. The result is that the n-monopoles moduli space turns into a vector bundle with $O(n)$ natural connection constructed from the fermion zero modes.

In this note we would like to investigate if one could expand the analogy between the moduli spaces of the BPS monopoles and Skyrme model to the case when the interaction with the fermions is included. The fundamental field $U(\mathbf{x},t)$ of Skyrme's model is a map from coordinate space M_4 to the configuration space of the mesons, in the simplest case given by the group $SU(2)$. The rescaled Lagrangian of the model[1] is [2]

$$\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\,\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32}\,\mathrm{tr}\,\left[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger\right]^2. \tag{1}$$

The first term here corresponds to the non-linear σ-model and the second one stabilizes the soliton solutions of the model. In general it is useful to parametrise U via a quartet of scalar fields (ϕ_0, phi^a) as $U(\mathbf{r}) = (\phi_0(\mathbf{r}) + i\phi^a(\mathbf{r})\cdot\sigma^a)$ with the constraint $\phi_0^2 + (\phi^a)^2 = 1$. In this note we consider the hedgehog configuration [2] in static case described by the ansatz

$$U(\mathbf{r})_{n=1} = e^{i(\sigma^a\cdot\hat{r}^a)F(r)} = \cos F(r) + i(\sigma^a\cdot\hat{r}^a)\sin F(r) \tag{2}$$

that defines the chiral angle $F(r)$. Here σ^a are standard Pauli matrices. The boundary conditions $F(0)=\pi, F(\infty)=0$ correspond to the sector with baryon (topological) number $n=1$.

Extending the original Skyrme model to include the interaction with fermions, the minimal chiral coupling of the fermion $SU(2)$ doublet with Skyrme field is given by the Lagrangian

$$\mathcal{L}_{int} = \bar\psi\left(i\gamma^\mu\partial_\mu + gU^{\gamma_5}\right)\psi = \bar\psi\left[i\gamma^\mu\partial_\mu + g\left(\phi_0 + i\gamma_5(\phi^a\cdot\sigma^a)\right)\right]\psi \tag{3}$$

where $U^{\gamma_5} = \exp\{i\gamma_5\,(\sigma^a\cdot\hat{r}^a)\,F(r)\} = \dfrac{1+\gamma_5}{2}U + \dfrac{1-\gamma_5}{2}U^\dagger$. The expression (3) can be considered as an analogue of the Lagrangian of the monopole-fermion interaction. The obvious difference is the effect of the $SU(2)$ magnetic monopole gauge potential A_i to define the covariant derivative as $D_i = \partial_i + A_i$. Note that the topology of configurations is fixed by the hedgehog ansatz in both cases and, on the other hand, after a chiral rotation of the spinor field

$$\psi \to S\psi = \exp\left(-\frac{i}{2}\gamma_5\,(\sigma^a\cdot\hat{r}^a)F(r)\right)\psi$$

the Lagrangian (3) transforms to the

$$\begin{aligned}\mathcal{L}'_{int} &= \bar\psi\left[i\gamma^\mu\mathcal{D}_\mu + g\right]\psi = \bar\psi\left[i\gamma^\mu\left(\partial_\mu + gS^{-1}\partial_\mu S\right) + g\right]\psi \\ &= \bar\psi\left[i\gamma^\mu\left(\partial_\mu + V_\mu + A_\mu\gamma_5\right) + g\right]\psi,\end{aligned} \tag{4}$$

[1] assuming zero pion mass

where

$$V_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right), \quad A_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right); \quad \xi = \exp\left\{\frac{i}{2}(\sigma^a \cdot \hat{r}^a)F(r)\right\} = U^{1/2}.$$

The effective chiral Lagrangian of this form was suggested in [5]. Thus, the connection $S^\dagger\partial_\mu S$ is generating an effective vector potential which relates sectors with different baryon numbers.

Another difference between the cases of monopole-fermion and skyrmion-fermion interaction is the mass term. Indeed, using the Dirac representation of the γ matrices, the Hamiltonian of the Dirac field chirally coupled to skyrmion can be written as [6], [7], [8]:

$$H\psi = \left(\alpha^a \cdot p^a + g\beta\cos F(r) + ig\gamma_5\ (\tau^a \cdot \hat{r}^a)\sin F(r)\right)\psi \equiv \tag{5}$$
$$\begin{pmatrix} M(r) & \mathcal{D} \\ \mathcal{D}^\dagger & -M(r) \end{pmatrix}\psi \equiv \begin{pmatrix} g\cos F(r) & \sigma^a \cdot p^a + ig(\tau^a \cdot \hat{r}^a)\sin F(r) \\ \sigma^a \cdot p^a - ig(\tau^a \cdot \hat{r}^a)\sin F(r) & -g\cos F(r) \end{pmatrix}\psi = E\psi$$

where $\mathbf{p} = -i\nabla$ and the Dirac operator is $\mathcal{D} = \sigma^a \cdot p^a + ig(\tau^a \cdot \hat{r}^a)\sin F(r)$. Thus the equation (5) describe the spinor field which has a space dependent dynamical complex mass. Its counterpart is the Hamiltonian of the monopole-fermion interaction

$$H_{BPS}\psi = \left(\alpha^a \cdot \left(p^a + \vec{A}\right) + iq\gamma_5\ \beta(\tau^a \cdot \hat{r}^a)H(r)\right)\psi \equiv \tag{6}$$
$$\begin{pmatrix} 0 & \mathcal{D}' \\ \mathcal{D}'^\dagger & 0 \end{pmatrix}\psi \equiv \begin{pmatrix} 0 & \sigma^a \cdot (p^a + A^a) + iq(\tau^a \cdot \hat{r}^a)H(r) \\ \sigma^a \cdot (p^a + A^a) - iq(\tau^a \cdot \hat{r}^a)H(r) & 0 \end{pmatrix}\psi = E\psi$$

where we suppose that the fermion mass appears as a result of pseudoscalar coupling between the Higgs field and fermions. Here the Dirac operator is $\mathcal{D}' = \sigma^a \cdot (p^a + A^a) + iq(\tau^a \cdot \hat{r}^a)H(r)$ and

$$H(r) = \left(\frac{1}{r} - \cosh r\right), \quad \text{and} \quad A_i^a = \varepsilon_{iab}\frac{r^b}{r}\left(\frac{1}{r} - \frac{1}{\sinh r}\right)$$

are well known BPS monopole solutions.

Obviously, there are common features of the monopole-fermion and skyrmion-fermion interaction. In both cases the Hamiltonians are commute with the generalized angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{T} + \mathbf{S}$ which composed operators of standard orbital momentum $\mathbf{L}$, isospin $\mathbf{T}$ and spin $\mathbf{S}$. However, the Hamiltonian H (5) also commutes with the Dirac parity operator defined by

$$\psi(\vec{r}) \to \beta\psi(-\vec{r})$$

Therefore, the Hamiltonian of the fermion-monopole interaction is invariant under a joint parity transformation and magnetic charge conjugation.

The most important from point of view of our consideration are the low-energy modes of the Hamiltonian H (5). One could expect that this the low energy state has spherical symmetry in the sense that it is invariant under a combined spatial and isospin rotation. Then the Dirac $j = 0$ positive parity spinor can be written as

$$\psi(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} i\rho(r) \\ (\tau^a \cdot \hat{r}^a)\lambda(r) \end{pmatrix}|\chi> \tag{7}$$

where $|\chi>$ is the spin-isospin spinor, i.e. $(\tau^a + \sigma^a)\,|\chi>= 0$ that means $|\chi>= -i\sigma_2 \otimes I_2$.

The equations for spinor components of the ψ are straightforward:

$$\begin{aligned} \left(\frac{d}{dr} + g\cos F(r) - \frac{1}{r}\right)\rho = &\quad (E - g\sin F(r))\,\lambda; \\ \left(\frac{d}{dr} - g\cos F(r) + \frac{1}{r}\right)\lambda = &\ -(E + g\sin F(r))\,\rho \end{aligned} \tag{8}$$

Note, that the effect of the mass term is that even in the case $E = 0$ these equations do not decouple into two independent equations for the spinors ρ and λ as it was in the case of monopole-fermion interaction.

It means, that a chirally coupled skyrmion may or may not give rise to a zero energy fermion mode depending on the strength of the coupling (effective mass) [10]. The investigation of the constraints under which a zero energy normalisable solution of the system (7) exists have been done in [6]-[9].

In contrast with the BPS monopole-fermion system there is no analytical solution of this equation and in order to simplify the numerical calculations some ansatzs for shape function (chiral angle $F(r)$) were used. The general result of such calculations is that there is a spectral flow of the Hamiltonian (5): for some small value of parameter g a single eigenvalue of H emerges from the positive energy continuum, crosses zero and goes to the negative continuum if g increases [11]. In contrast, in the case of a BPS monopole, the Hamiltonian (6) has a single zero energy eigenvalue that does not depend from the coupling constant [4]. Nevertheless the index theorem also can be applied to the Hamiltonian of the form (5) [11]: Writing it in the form

$$H = \begin{pmatrix} M & \mathcal{D} \\ \mathcal{D}^\dagger & -M \end{pmatrix} = H_0 + M\sigma_3, \qquad H^2 \geq M^2, \tag{9}$$

we see that $\mathrm{Ind}[H_0] = \mathrm{Dim\ Ker}[D^\dagger] - \mathrm{Dim\ Ker}[D] = n$. The difference from the BPS monopole - fermion system is that zero mode of the Dirac operator does not lies in the the kernel of D.

In order to describe the moduli space of the Skyrme model note that its symmetry group is $G = E_3 \times SO(3)$, where Euclidean group $E_3 = SO(3) \times R_3$. This group is 9-dimensional but when it acts on a field of the hedgehog form (2) spatial and isospin rotations are equivalent. Hence the orbit of the standard skyrmion under symmetry group is a six-dimensional manifold $\mathcal{M}_1$. This is the manifold which is used as the moduli space in the $n = 1$ topological sector. It is diffeomorphic to $R_3 \times SO(3)$ and its elements are fully specified by their position and orientation. On the moduli space $\mathcal{M}_1$ the potential energy is constant and equal to the skyrmion's rest mass M. The induced dynamics is determined by the kinetic energy which can be found [12] by inserting the adiabatically rotating and moving ansatz into (1):

$$U(\mathbf{r},t) = S(t)U_0(\mathbf{X(t)} - \mathbf{r})S(t)^\dagger, \quad \psi(\mathbf{r},t) \to S(t)\psi(\mathbf{r},t), \tag{10}$$

where $S(t)$ is an $SU(2)$ cranking matrix and $\mathbf{X}(t)$ are collective coordinates on R_3. Exploiting the identity

$$S^\dagger \dot{S} = \frac{\tau^a}{2}\ \mathrm{tr}\left(\tau^a S^\dagger \dot{S}\right)$$

after a lengthy calculation one obtains the induced Lagrangian

$$L = -M + \frac{1}{2}M\dot{\mathbf{X}}^2 + \Lambda\ \mathrm{tr}\ \left(\dot{S}^\dagger \dot{S}\right) + \frac{i}{2}\ \mathrm{tr}\left(\tau^a S^\dagger \dot{S}\right)\Sigma^a + O(\dot{A}^3), \tag{11}$$

where $\Sigma^a = \int d^3x\ \psi^\dagger \tau^a \psi$ is the quarks contribution to the total angular momentum of the configuration, Λ is the moment of inertia associated with the collective rotations, and the fermion kinetic energy term is ignored.

Let us remind that spatial rotations act on $S(t)$ by right multiplication on an $SU(2)$ matrix, while isospin rotations act by left multiplication. Thus according to the Noether's theorem the invariance of the effective Lagrangian under space rotations $S(t) \to S(t)\exp\{i\varepsilon^a\sigma^a/2\}$ leads to the conservation of the total momentum

$$\mathbf{J} = i\Lambda\ \mathrm{tr}\ \left(\sigma^a S^\dagger \dot{S}\right) - \Sigma^a, \tag{12}$$

and the effective Lagrangian (11) can be written as

$$L = -M + \frac{1}{2}M\dot{\mathbf{X}}^2 + \frac{1}{2\Lambda}\left[\mathbf{J}^2 - {\Sigma^a}^2\right] \tag{13}$$

The common parameterization of the $SU(2)$ cranking matrix $S(t)$ is

$$S(t) = \exp\{\frac{i}{2}\sigma_k\omega_k(t)\} = a_0(t) + i\sigma_k a_k(t) = \begin{pmatrix} a_0 + ia_3 & a_2 + ia_1 \\ -a_2 + ia_1 & a_0 - ia_3 \end{pmatrix} \tag{14}$$

where $a_\mu^2 = 1$ i.e. the a_μ are the coordinates of a point on an S_3. As usually we can introduce the Euler angles α, β, γ on the three-sphere according to:

$$S = \begin{pmatrix} \cos\frac{\beta}{2} e^{\frac{i}{2}(\gamma+\alpha)} & \sin\frac{\beta}{2} e^{\frac{i}{2}(\gamma-\alpha)} \\ -\sin\frac{\beta}{2} e^{-\frac{i}{2}(\gamma-\alpha)} & \cos\frac{\beta}{2} e^{-\frac{i}{2}(\gamma+\alpha)} \end{pmatrix} \tag{15}$$

Recall that left and right rotations on $SO(3)$ in terms of the Euler angles are generated by one-forms familiar in the analysis of rigid body rotations:

$$L = dS \cdot S^{-1} = \frac{i}{2}\sigma_k L_k; \qquad R = S^{-1} \cdot dS = \frac{i}{2}\sigma_k R_k$$

with the property $S^{-1} \cdot dS = -dS^{-1} \cdot S$. Here components of the velocities are

$$\begin{aligned} L_1 &= \dot\beta \sin\alpha - \dot\gamma \sin\beta \cos\alpha; & R_1 &= -\dot\beta \sin\gamma + \dot\alpha \cos\gamma \sin\beta; \\ L_2 &= \dot\beta \cos\alpha + \dot\gamma \sin\beta \sin\alpha; & R_2 &= \dot\beta \cos\gamma + \dot\alpha \sin\gamma \sin\beta; \\ L_3 &= \dot\alpha + \dot\gamma \cos\beta; & R_3 &= \dot\gamma + \dot\alpha \cos\beta \end{aligned} \tag{16}$$

Thus the straightforward calculation yields two equivalent forms of the Lagrangian of collective motion on one-skyrmion moduli space written via the operators of left and right rotation respectively:

$$\begin{aligned} L &= -M + \frac{1}{2}M\dot{\mathbf{X}}^2 + \frac{\Lambda}{2}L_k^2 + \frac{1}{2}\Sigma^k L_k; \\ L &= -M + \frac{1}{2}M\dot{\mathbf{X}}^2 + \frac{\Lambda}{2}R_k^2 - \frac{1}{2}\Sigma^k R_k \end{aligned} \tag{17}$$

Note that in this parameterization the metric on the group manifold is non-diagonal

$$dS^2 = \frac{1}{4}d\alpha^2 + \frac{1}{4}d\beta^2 + \frac{1}{4}d\gamma^2 + \frac{1}{2}\cos\beta \; d\alpha d\gamma \tag{18}$$

Thus it is more convenient to introduce orthogonal coordinates

$$\psi = \gamma + \alpha; \qquad \chi = \gamma - \alpha \tag{19}$$

in terms of which the metric becomes

$$dS^2 = \frac{1}{4}d\beta^2 + \frac{1}{4}d\psi^2 \cos^2\frac{\beta}{2} + \frac{1}{4}d\chi^2 \sin^2\frac{\beta}{2} \tag{20}$$

Consider now the complete path around the group manifold. Obviously it is parameterized by the values of the Euler angles ranging within intervals $\beta \in [0,\pi]$; $\alpha \in [0, 2\pi]$; $\gamma \in [0, 2\pi]$. However, since these rotations act independently on the chiral components of spin-isospinor wave function $\psi_R \to U_R\psi_R$; $\psi_L \to U_L\psi_R$, the lower energy state transforms as

$$U_R \mid \chi >= -U_L|\chi >$$

Therefore, there is a non-trivial holonomy on the sphere and we obtain $SO(3)$ bundle over the moduli space of the Skyrme model with a unit topological charge. Indeed, the corresponding $SO(3)$ connection can be easily calculated if we consider the canonical momenta which correspond to the effective Lagrangian (17):

$$\begin{aligned} \mathbf{P} &= M\dot{\mathbf{X}}; \qquad \Pi_\alpha = \Lambda(\dot\alpha + \dot\gamma\cos\beta) + \Sigma_3; \\ \Pi_\beta &= \Lambda\dot\beta + \Sigma_1 \sin\alpha - \Sigma_2 \cos\alpha; \\ \Pi_\gamma &= \Lambda(\dot\gamma + \dot\alpha) - \Sigma_1 \sin\beta\cos\alpha - \Sigma_2 \sin\beta \sin\alpha + \Sigma_3 \cos\beta \end{aligned}$$

and one can see that the fermion interaction term gives rise to the effective non Abelian gauge potential $A_i = A_i^a \Sigma^a$

$$A_\alpha = \frac{1}{2}A_\alpha^a\sigma^a = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \qquad A_\beta = \frac{1}{2}A_\beta^a\sigma^a = \frac{i}{2}\begin{pmatrix} 0 & e^{-i\alpha} \\ -e^{i\alpha} & 0 \end{pmatrix};$$

$$A_\gamma = \frac{1}{2}A_\gamma^a\sigma^a = -\frac{1}{2}\begin{pmatrix} -\cos\beta & \sin\beta e^{-i\alpha} \\ \sin\beta e^{i\alpha} & \cos\beta \end{pmatrix} \tag{21}$$

The mechanism of the effective potential generation is well known [13], [16]. It is connected with the nontrivial holonomy on sphere S_3. Indeed, the well-known Hopf fibration arises if one considers the S_3 as a principal fibre bundle with base S_2 and a structure group $U(1)$. Then the additional term in the formula (11) is simple the Balachandran-Aitchison monopole effective Lagrangian [13, 14]:

$$L_{eff} = \frac{i}{2} s \ \mathrm{tr}\left(\sigma_3 S^\dagger \dot{S}\right). \tag{22}$$

This term is in fact a Wess-Zumino-type one. Indeed, as it was suggested by Diakonov and Petrov [15], one can introduce a unit three-vector $n^a = \frac{1}{2}\ \mathrm{tr}\left(S\sigma^a S^\dagger \sigma_3\right)$ and rewrite the effective Lagrangian (22) as

$$L_{eff} = i\frac{s}{4} \int d\sigma \ \varepsilon_{abc}\varepsilon_{ij} \ n^a \partial_i n^b \partial_j n^c, \tag{23}$$

where the surface integral actually is a full derivative and define the topological charge on the group space. Then the charge quantization condition takes place.

Indeed, it is clear that the model is still invariant under a $U(1)$ time-dependent gauge transformation $U \to U \exp\{i\hat{Q}\alpha(t)\} = U\exp\{i\sigma_3\alpha(t)\}$ where $\hat{Q} = 2s$ is $U(1)$ generator. The Lagrangian (22) transforms as $L_{eff} \to L_{eff} - 2s\dot{\alpha}$ that is full time derivative. Thus the symmetry group is $SU(2)/U(1)$ rather then $SU(2)$ and in quantum theory the physical states Φ that are eigenfunctions of the Hamiltonian on moduli space have to be restricted to be also eigenfunctions of operator $\hat{Q}$:

$$\hat{Q}\Phi = 2s\Phi. \tag{24}$$

As a result the parameter s have to be quantized as $s = n/2, n \in Z$ where n is a winding number associated with above mentioned $U(1)$ gauge degrees of freedom [13]. That means that coupling of the skyrmion with fermions could affect the statistics. Indeed, under 2π rotations $\Phi \to \Phi\exp\{i\pi\sigma_3\} = \Phi\exp\{i\pi\hat{Q}\}$. The condition (24) means that $\Phi \to \Phi\exp\{2i\pi s\}$, i.e skyrmion coupled with odd number of quarks transforms as a fermion and as a boson if it is coupled with even number of quarks.

In order to generalize this construction to the case of the skyrmion with topological charge n let us consider quantum mechanics on the moduli space. The canonical quantization prescription gives the quantum Hamiltonian

$$H = -\frac{1}{2M}\frac{\partial^2}{\partial \mathbf{X}^2} - \frac{1}{2\Lambda}\left(\mathbf{J}^2 - (\Sigma^a)^2\right) \equiv \nabla_\alpha \nabla^\alpha \tag{25}$$

where $\mathbf{J}$ is the angular momentum operator defined by eq.(12) and we introduce the shorthand ∇_α, $\alpha = 1, 2 \ldots 6$, for the covariant derivative associated with the connection on the moduli space of Skyrme model $\mathcal{M}$ parameterized by the set of coordinates $\xi_\alpha = (X_k, \omega_k)$.

Now one can apply the adiabatic approximation for the eigenfunctions of the Hamiltonian (25) taking into account the fermionic degrees of freedom:

$$\Phi(\xi_\alpha, \mathbf{r}) = \Psi(\xi_\alpha)\psi(\xi_\alpha, \mathbf{r}) \tag{26}$$

where $\psi(\xi_\alpha, \mathbf{r})$ is the single fermion quasi-zero mode (7).

Using the Born-Oppenheimer adiabatic approximation [4],[16] we can consider these fermionic degrees of freedom as the "fast" variables and the "slow" variables, that are coordinates ξ_α on 6-dimensional moduli space $\mathcal{M}_1$, describe the effective quantum dynamic of the skyrmion. Multiply the Schrödinger equation with the Hamiltonian (25)

$$H\Phi = \nabla_\alpha \nabla^\alpha \Phi = E\Phi$$

on the left by $\psi^\dagger(\xi_\alpha, \mathbf{r})$ and integrate over fermionic coordinates $\mathbf{r}$ one can obtain exploiting the orthogonality of $\psi(\xi_\alpha, \mathbf{r})$:

$$\nabla_\alpha\nabla^\alpha\Psi + 2 < \psi, \nabla_\alpha\psi > \nabla^\alpha\Psi + \Psi < \psi, \nabla_\alpha\nabla^\alpha\psi >= E\Psi. \tag{27}$$

Introduce a local gauge potential $\mathcal{A}^{eff}_\alpha = i\langle\psi, \nabla_\alpha\psi\rangle$ and neglecting the transitions between the fermions levels, that is the Born-Oppenheimer approximation, we can see that the matrix-valued Hamiltonian sandwiched between the "fast" degrees of freedom is

$$H_{eff} = \left(\nabla_\alpha - i\mathcal{A}^{eff}_\alpha\right)^2 \tag{28}$$

This result can be obtained also if we note that adiabatically rotating fermionic field of the form (10) can be expand in the complete set of unrotating zero modes $\psi_n(\mathbf{x})$. According the undex theorem the number of these modes is equal to the topological charge n and therefore the Hamiltonian H for each band n may be regarded as an element of the algebra $SO(2n+1)$.

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