Intercept Parameter λ of the Two-Pion (-Kaon) Correlation Functions in q-Bose Gas Model

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Abstract

Within recently proposed approach aimed to effectively describe the observed non-Bose type behavior of the intercept λ of two-particle correlation function $C(p, K)$ of identical pions or kaons detected in heavy-ion collisions, the q-deformed oscillators and q-Bose gas picture are employed. For the intercept λ, connected with deformation parameter q, the model predicts (at fixed q) a fully specified dependence of λ on pair mean momentum $\mathbf{K}$. The intercepts λ_π and λ_K for pions and kaons, differing noticeably at small mean momenta $\mathbf{K}$, should merge at $\mathbf{K}$ large enough, i.e., in the range $|\mathbf{K}| \geq 800$ MeV/c, where the effect of resonance decays is negligible. In this paper we compare, fixing q appropriately, the predicted dependence $\lambda_\pi = \lambda_\pi(\mathbf{K})$ with recent results from STAR/RHIC for $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs. Using the same q, we also predict behavior of λ for kaons.

Two-particle correlations in momentum space can be used to extract information about the space-time structure of the emitting sources created in heavy ion collisions. The method exploits in an essential way the quantum mechanical uncertainty relation between coordinates and momenta, and thus any formal treatment of two-particle correlations must be based on a quantum mechanical description. For the so-called "chaotic" sources where two particles are emitted independently the description can be based on the *single-particle* Wigner density $S(x, K)$ of the source.

In the standard quantum mechanical treatment the Bose-Einstein correlations are due to symmetrization of the two-particle (many-particle) wave function (suppose particles are emitted independently) $\psi_{\gamma_a\gamma_b}(\mathbf{x}_a, \mathbf{x}_b, t) = \frac{1}{\sqrt{2}}[\psi_{\gamma_a}(\mathbf{x}_a, t)\,\psi_{\gamma_b}(\mathbf{x}_b, t) + e^{i\alpha}\psi_{\gamma_a}(\mathbf{x}_b, t)\,\psi_{\gamma_b}(\mathbf{x}_a, t)]$ with $\alpha = 0$ ($\alpha = \pi$) for identical bosons (fermions). The indices γ_a, γ_b of the 1-particle wave functions label complete sets of 1-particle quantum numbers. In the following, we consider two-particle correlations of noninteracting spin-zero identical bosons. The correlation function, with $P_1(\mathbf{k})$ and $P_2(\mathbf{k}_a, \mathbf{k}_b)$ being single- and two-particle probabilities to detect particles with given momenta, is defined as

$$C(\mathbf{k}_a, \mathbf{k}_b) = \frac{P_2(\mathbf{k}_a, \mathbf{k}_b)}{P_1(\mathbf{k}_a)\; P_1(\mathbf{k}_b)}. \tag{1}$$

In the absence of final state interactions (FSI, see [1]), for chaotic source, the correlation function can be expressed as follows [2]:

$$C(\mathbf{k}_a, \mathbf{k}_b) = 1 + \cos\alpha\, \frac{\left|\int d^4x\, e^{ip\cdot x} S(x, K)\right|^2}{\int d^4x\, S(x, k_a) \int d^4y\, S(y, k_b)} \tag{2}$$

with the 4-momenta $K = \frac{1}{2}(k_a + k_b)$ as pair mean momentum and $p = k_a - k_b$ as relative momentum. The source function $S(x, K)$ (single-particle Wigner density) is defined by

the single-particle states $\psi_\gamma(x)$ at freeze-out time and the source density matrix $\rho_{\gamma\gamma'}$ as, e.g., in [2].

Obviously, from (2) at zero relative momentum $\mathbf{k}_a = \mathbf{k}_b$ one gets $C(\mathbf{k}_a, \mathbf{k}_a) \equiv 1 + \lambda = 1 + \cos\alpha$ and since for bosons $\alpha = 0$ it follows that $C(\mathbf{k}_a, \mathbf{k}_a) = 2$, i.e., $\lambda = 1$.

From the very first experiments it was deduced that the factor $\cos\alpha$ in (2) is less than one. The correlation function of *identical bosons* is usually presented as $C(p, K) = 1 + \lambda f(p, K)$ with λ obtained from a fit to a data, with typical experimental values $\lambda = 0.4$ - 0.9 and $f(p, K)$ commonly taken as Gaussian so that $f(\mathbf{p} = 0, K) = 1$. Obviously, the second term in (2) should be viewed as manifestation of quantum-mechanical interference, and deviation of λ from unity manifests weakening of the interference effects which can occur due to different reasons - influence of long-living resonances, coherent emission, etc.

Let us explain the key idea of the model developed in [2, 3] (dubbed as AGI-model in what follows) and further exploited in this letter. In two-boson correlations the deviation of intercept λ from unity (in addition to the contribution due to effects from long-living resonances) can also be caused by the averaged softening of quantum-statistical effects in the short-living many-particle systems formed in relativistic heavy ion collisions.

What are the physical reasons for modifying the conventional approach? First, in a short-living small system the symmetrization angle α of $\psi_{\gamma_a\gamma_b}(\mathbf{x}_a, \mathbf{x}_b, t)$ can be distorted by an additional phase due to a inhomogeneity of the system at freeze-out times (strong radial and azimuthal flows). These peculiarities can determine the effect analogous to the Aharonov-Bohm one. As a result, a finite value of averaged symmetrization angle can appear: $\overline{\alpha} > 0$ for bosons and $\overline{\alpha} < \pi$ for fermions.

Trying to explain experimental data with formula (2) in mind, it is natural to relate parameter λ with the averaged angle $\overline{\alpha}$ to get *the reduction factor* λ by means of $\cos\overline{\alpha}$. In other words, deviation of the intercept λ from unity is due to fluctuations of symmetrization angle α, i.e.

$$\lambda = \cos\overline{\alpha}\,. \tag{3}$$

To implement this idea, we exploit quantum field theory with q-deformed commutation relations (qDCR) and the techniques of q-boson statistics (see [4] and refs. therein) which reflects a partial suppression of the quantum statistical effects. In [5, 6] it was argued that the algebra of qDCR is connected, *for real q only*, to the so-called non-extensive statistics introduced by Tsallis [7]. This type of generalized statistics has already found numerous applications in diverse branches of modern physics (see [8] for refs.). In particular, recently non-extensive statistics was successfully applied to the problems of high-energy nuclear collisions ([9] and refs. therein). Meanwhile, the techniques of q-boson statistics based on qDCR allows the use of complex values as well as the real values for the deformation parameter q, depending on a choice of algebraic realization of qDCR. The physical reasons for the use of qDCR, and subsequent interpretation of q, essentially differ depending on the case of q real or complex. Introducing deformed statistics with q real, enables one to effectively account for interaction effects by means of non-interacting ideal gas of "modified" particles. On the other hand, the approach based on qDCR provides the ability to model the effects involving the Aharonov-Bohm-like phase, intimately connected with symmetrization properties of wave functions.

For the system of pions or kaons produced in heavy ion collisions, we employ the ideal q-Bose gas picture. Physical meaning or explanation of the origin of q-deformation in the

considered phenomenon sharply differs in the case of real deformation parameter q from the case when q is a pure phase factor, as will be seen in what follows.

The AGI-model exploits two different sets of qDCR. The first is the multi-mode Biedenharn-Macfarlane (BM-type) q-oscillator defined as [10]: $[N_j, b_j] = -b_j$, $[N_j, b_j^\dagger] = b_j^\dagger$, $b_j b_j^\dagger - q^{-1} b_j^\dagger b_j = q^{N_j}$, where different modes ($i \neq j$) commute. Then, $b_i^\dagger b_i = [N_i]_q$ (here the "q-bracket" means $[r]_q = (q^r - q^{-r})/(q - q^{-1})$) so that $b_i^\dagger b_i = N_i$ is recovered in the "classical" $q = 1$ (or "no deformation") limit. Below, by BM-type q-oscillators it is meant that

$$q = \exp(i\theta) \ , \qquad 0 \leq \theta < \pi/2 \ . \tag{4}$$

The second multi-mode q-oscillator used in AGI-model is the set of Arik-Coon (AC-type) q-oscillators, defined by the relations [11] $[\mathcal{N}, a] = -a$, $[\mathcal{N}, a^\dagger] = a^\dagger$, and $aa^\dagger - qa^\dagger a = 1$ (subscript suppressed). Again, at $q \neq 1$, the bilinear $a_i^\dagger a_i$ does not equal the number operator $\mathcal{N}_i$ (as is true for usual bosonic oscillators, i.e., at $q = 1$). Instead, $a_i^\dagger a_i = [[\mathcal{N}_i]]$ where now the notation $[[r]] \equiv (1 - q^r)(1 - q)$, is used. The q-bracket $[[A]]$ for an operator A is understood as a formal expression (formal series). At $q \to 1$, from $[[r]]$ resp. $[[A]]$ one recovers r resp. A. In what follows we set $-1 \leq q \leq 1$. For each such value of the deformation parameter q, the operators $a_i^\dagger$ and a_i are mutually conjugate. Note that the inverse of the relation $a_i^\dagger a_i = [[\mathcal{N}_i]]$ is given by a formula expressing the operator $\mathcal{N}_i$ as a formal series of creation/annihilation operators.

For a multi-pion (-kaon) system, viewed as ideal gas of q-bosons the Hamiltonian is taken as

$$H = \sum_i \omega_i \, N_i \tag{5}$$

with i labeling energy eigenvalues, $\omega_i = \sqrt{m^2 + \mathbf{k}_i^2}$, and N_i defined as above. This is unique truly noninteracting Hamiltonian with additive spectrum [4]. We assume discrete 3-momenta of particles (the system is in a box of volume $\sim L^3$). For the set of AC-type q-oscillators, one takes $\mathcal{N}_i$ instead of N_i in (5).

Statistical properties are obtained by evaluating thermal averages $\langle A \rangle = \mathrm{Sp}(A\rho)/\mathrm{Sp}(\rho)$, $\rho = \mathrm{e}^{-\beta H}$, with the Hamiltonian (5). Here $\beta = 1/T$.

With $b_i^\dagger b_i = [N_i]_q$ and $q + q^{-1} = [2]_q = 2\cos\theta$, the q-deformed distribution function is obtained as [4, 12, 2]

$$\langle b_i^\dagger b_i \rangle = \frac{1}{\mathrm{e}^{\beta\omega_i} - (1 - \delta_i)} \, , \qquad \delta_i = 2 \, \frac{1 - \cos\theta}{1 - \mathrm{e}^{-\beta\omega_i}} \ . \tag{6}$$

If $\theta \to 0$, it yields Bose-Einstein (BE) distribution. Note that the q-distribution function (6) is real.

The q-distribution (6) deviates from the quantum Bose-Einstein just in the "right direction" toward the classical Boltzmann distribution, that *reflects a decrease of quantum statistical effects.* For kaons, whose mass m_K is bigger than m_π, analogous curve should lie closer, than pion's one, to that of BE distribution.

In the case of AC-type q-bosons with real q from (4), one arrives at the distribution function (cf. [4, 12, 2]):

$$\langle a_i^\dagger a_i \rangle = \frac{1}{\mathrm{e}^{\beta\omega_i} - q} \ . \tag{7}$$

In the no-deformation limit $q \to 1$, this also reduces to the Bose-Einstein distribution, since at $q = 1$ we return to the standard system of bosonic commutation relations.

Deviation from the standard BE statistics is a usual thing if one considers the system of interacting particles versus that of non-interacting particles (ideal gas). For instance, natural type of interaction is the hard-core repulsion of particles that occupy finite self-volume. This type of interaction, as was shown in [13], results in the same kind (7) of modified statistics. At microscopic level, a finite self-volume arising due to composite structure of particles naturally leads to q-deformed commutation relations [14] and subsequently results in certain q-deformed statistics of the gas of these particles.

The two-particle distribution corresponding to the BM-type q-oscillators reads

$$\langle b_i^\dagger b_i^\dagger b_i b_i \rangle = \frac{2\cos\theta}{\mathrm{e}^{2\beta\omega_i} - 2\cos(2\theta)\mathrm{e}^{\beta\omega_i} + 1} \ . \tag{8}$$

From this and (5) one obtains the intercept $\tilde{\lambda}_i \equiv \lambda_i + 1 = \langle b_i^\dagger b_i^\dagger b_i b_i \rangle / (\langle b_i^\dagger b_i \rangle)^2$ of two-particle correlations (subscript omitted) as

$$\lambda = -1 + \frac{2\cos\theta \ (\cosh(\beta\omega) - \cos\theta)^2}{(\cosh(\beta\omega) - 2\cos^2\theta + 1)(\cosh(\beta\omega) - 1)} \, . \tag{9}$$

At $\beta\omega \to \infty$ (i.e., at low temperature and fixed momenta or large momenta and fixed temperature) the asymptotics of intercept is given merely by the deformation angle θ (recall that $q = \exp(\mathrm{i}\theta)$):

$$\lambda^{\mathrm{asympt}} = 2\cos\theta - 1 \quad (T \to 0 \ \ \text{or} \ \ |\mathbf{K}| \to \infty) \, . \tag{10}$$

From this and (3) we have the (asymptotic) relation $\cos\theta = \cos^2(\overline{\alpha}/2)$. Let us also note that, if the the intercept is less than one due to the decays of resonances (commonly accepted idea), all the curves would tend to the value $\lambda = 1$ in the large $|\mathbf{K}|$ asymptotics. It is not the case when one considers a medium created in relativistic heavy ion collisions as the q-Bose gas.

In the case of AC-type q-oscillators, the formula for two-particle distribution

$$\langle a_i^\dagger a_i^\dagger a_i a_i \rangle = (1+q)(\mathrm{e}^{\beta\omega_i} - q)^{-1}(\mathrm{e}^{\beta\omega_i} - q^2)^{-1} \tag{11}$$

combined with (6) results in the following

$$\lambda = -1 + \langle a_i^\dagger a_i^\dagger a_i a_i \rangle / \langle a_i^\dagger a_i \rangle^2 = q - \frac{q \ (1-q^2)}{\mathrm{e}^{\beta\omega} - q^2} \, . \tag{12}$$

In the limit $T \to 0$ or $|\mathbf{K}| \to \infty$ we have an asymptotic value $\lambda^{\mathrm{asympt}} = q$.

Below, the two versions (9) and (12) corresponding to BM- and AC-types of q-deformation are compared to experimental data. The experimental values for intercept parameter λ in Figs. 1 and 2 are taken from Ref. [15]. The theoretical values are obtained by averaging over given rapidity y and transverse momentum K_t intervals:

$$\lambda_i = \frac{1}{\Delta_y} \int\limits_{-\Delta_y/2}^{\Delta_y/2} \mathrm{d}y \, \frac{1}{\Delta_i} \int\limits_{K_t^{i,\min}}^{K_t^{i,\max}} \mathrm{d}K_t \ \lambda(q, m, T, y, K_t), \tag{13}$$

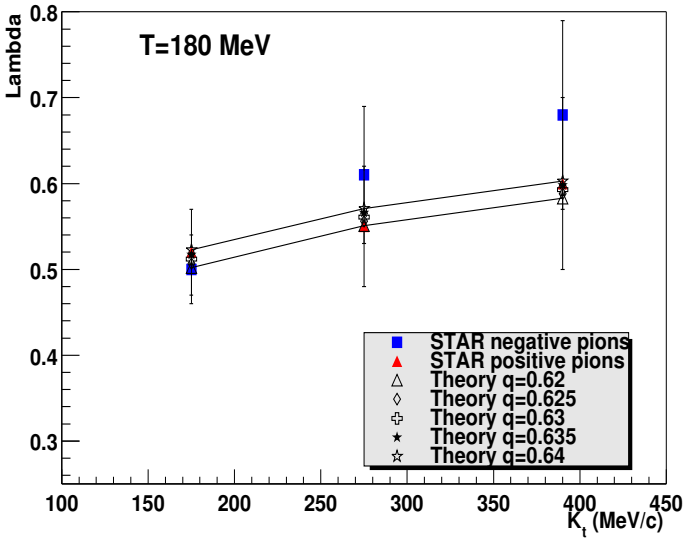

Figure 1: The transverse momentum $|\mathbf{K}_t|$ dependence of the intercept λ of two-pion correlation. Deformation parameter q is a real quantity, $0 \leq q \leq 1$.

where m is particle mass.

Expressions (12) and (9) for $\lambda(q, m, T, y, K_t)$ were used for calculations of theoretical points shown in Figs. 1 and 2 respectively. It can be seen from Figures 1 and 2 that the agreement between experimentally measured and theoretically calculated values of intercept parameters λ is very good.

The AGI-model predicts that parameter λ will asymptotically reach a constant value $\lambda^{\rm asympt} < 1$, determined by q only, at sufficiently large (500$-$600 MeV/c) pion pair mean momentum $|\mathbf{K}|$. Hence, in order to check this prediction, measurements at higher K_t are necessary. Such measurements would be available in near future at RHIC. Detailed comparison with experiment gives that the values λ_i obtained from (13),(12), i.e. with real q, fit better the three experimental values for the intercept of $\pi^+\pi^+$ correlations. On the other hand, the values calculated with (13),(10), i.e. for the case of q pure phase factor, agree better with the three experimental values for the intercept of $\pi^-\pi^-$ correlations. One of the possible explanations of the observed difference between experimental values of intercept for $\pi^-\pi^-$-pairs and $\pi^+\pi^+$-pairs could be the influence of the Coulomb FSI of these charged pions with the positive charge of fireball protons.

Below, for the prediction of intercept of kaons, we use the values of q which provide the best fit of experimental data for pions (see Figs. 1 and 2): $q = 0.63$ or $\theta = 28.5°$. We assume a universality of the deformation parameter for description of excited hot hadronic matter. The result of averaging in rapidity $-0.5 \leq y \leq 0.5$, as in the first integral on the right hand side in (13), is shown in Fig. 3 as solid curve in each of the triplet of curves. The other two curves in each triplet correspond to fixed value of rapidity: $y = 0$ (dotted curve) and $y = 0.5$ (dashed curve). As it is clearly seen, the cases of real q and q a phase factor yield significantly different values for the kaon intercept λ_K. It is tempting to use just this feature for making preference to particular version of the deformation

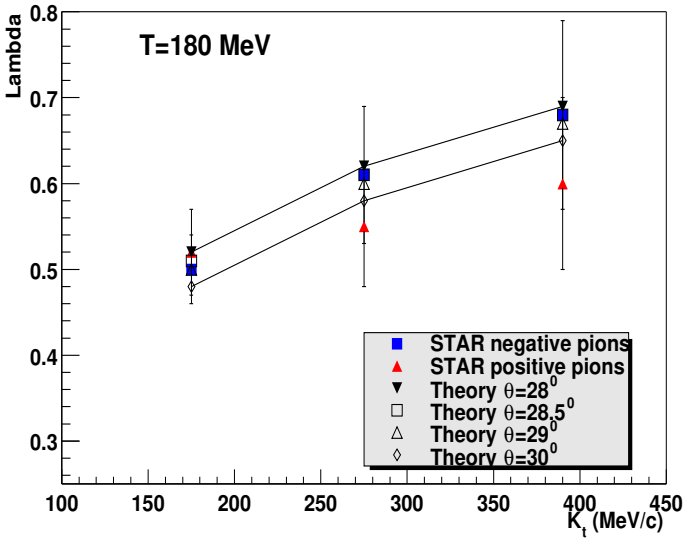

Figure 2: The transverse momentum $|\mathbf{K}_t|$ dependence of the intercept λ of two-pion correlation. The deformation parameter q is taken in the form $q = e^{i\theta}$.

parameter q - real or pure phase. The choice is important because of different physics behind these two versions: real q may reflect, for instance, particle finite size effects [13] or particle composite structure [14], and complex-valued q may refer to deformed symmetrization properties of wave functions (like in Aharonov-Bohm effects), relevant for short-lived systems occurring in heavy-ion collisions. It is also possible that a phase-factor q encodes [16] the effects from mixing at the composite (quark) level. Recent data from NA44 [17], i.e. $\lambda = 0.84 \pm 0.13$ and $\lambda = 0.61 \pm 0.36$ resp. for $\langle K_t \rangle \approx 0.25$ GeV/c and $\langle K_t \rangle \approx 0.91$ GeV/c do not help in making choice of optimal version for q.

In summary, we presented comparison between AGI model and experimental data on two-particle correlations at RHIC. We used parameters extracted from comparison to pions to predict behavior of intercept of kaon correlation functions. We stress the importance of correlation measurements at high transverse momentum in order to check predicted asymptotic "saturation" of intercept parameters.

Measurements of $|\mathbf{K}|$ in the range up to $500-600$ MeV/c for pions (up to $700-800$ MeV/c for kaons) should be possible by RHIC detectors such as STAR and PHENIX. The asymptotic behavior of intercept parameter λ within the proposed model, see (10), for the case of complex q, should determine the actual value of parameter q which is considered as a universal quantity.

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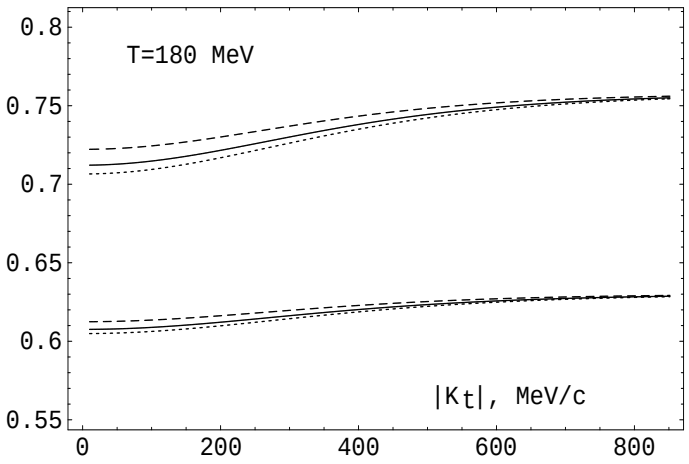

Figure 3: The transverse momentum $|\mathbf{K}_t|$ dependence of the intercept λ of two-kaon correlation. Both the case of real $q = 0.63$ (lower triple of curves) and the case of $q = e^{i\theta}$ (upper triple of curves) with $\theta = 28.5°$ are shown.

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