Meson Mass Splitting in $U(3)$ Quasilocal Quark Model

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Abstract

By using the SPVA, U(3)-extension of effective Quasilocal Quark Model and the large-N_c and log-approximations, the mass spectrum of both ground meson states and their first radial excitations is obtained. The model-independent relations for the meson mass splitting are derived. Fits and predictions for SPVA meson masses and their first excitations are presented, and comparisons are made with Particle Date Group results.

1 Introduction

It is well known that the standard perturbation theory applied to Quantum Chromodynamics (QCD) fails in describing low-energy phenomena of the strong interactions and one has no universal non-perturbative method for QCD so far. In principle, one should try to resolve the problem in terms of evolution of the renormalization group from short distances to long distances. Unfortunately, it is difficult to carry out this evolution explicitly, because at intermediate energies, typically of a few GeV, non-perturbative dynamics like the dynamical chiral symmetry breaking and confinement sets in.

Nevertheless, in order to investigate the hadron spectrum of resonances at low and intermediate energies one can employ effective Quasilocal Quark Model (QQM) with four quark interaction including derivatives [1-6]. Such models describe not only ground states of scalar (S), pseudoscalar (P), vector (V), and axial-vector (A) mesons and also their first radial excitations in contrast to local four-fermion NJL-type models [7-16]. There is a number of different, non-local [17, 18], quark model approaches (see, for example, [19]) to the study of higher meson resonances. The effective QQM in SP and VA, $SU(2)$ cases was developed in [6, 20].

In this report we present our investigation of the mass spectrum and meson mass splitting for the low-lying meson states and their first excitations in $SPVA$, $U(3)$-extension of QQM. The model fits and predictions for the meson spectrum and their comparison with the PDG [21] data are displayed.

2 Structure of $SPVA$, $U(3)$ Quasilocal Quark Model

The minimal structure of SP, $SU(2)$ QQM in n-channel case was discussed in [1, 4, 6]. The same arguments can be applied to $SPVA$, $U(3)$ QQM and we will focus on 2-channel case, i.e. we describe the ground states and their first radial excitation only. In the Euclidean space the Lagrangian under consideration has the form:

$$L = i\bar{q}\left(\hat{\not\partial} + \hat{m}\right)q + L^{I}_{SPVA}, \tag{1}$$

where S, P, V, A denote the scalar, pseudoscalar, vector, and axial-vector case and

$$L^I_{SPVA} = \frac{1}{4N_f N_c \Lambda^2} \sum_{k,l=1}^{2} Tr \left\{ a^a_{kl} \sum_{j=1}^{2} \bar{q} f_k(\tau) \Gamma^a_j q \bar{q} f_l(\tau) \Gamma^a_j q + b^a_{kl} \sum_{j=3}^{4} \bar{q} f_k(\tau) \Gamma^a_j q \bar{q} f_l(\tau) \Gamma^a_j q \right\} . \tag{2}$$

a^a_{kl} and b^a_{kl} represent here symmetric matrices of real coupling constants in SP and VA case respectively. Symbols Γ^a_j mean:

$$\Gamma^a_1 \equiv \lambda^a, \quad \Gamma^a_2 \equiv i\gamma_5 \lambda^a, \quad \Gamma^a_3 \equiv i\gamma_\mu \lambda^a, \quad \Gamma^a_4 \equiv i\gamma_5 \gamma_\mu \lambda^a; \qquad a = 0, ..., 8 \,, \tag{3}$$

where

$$\lambda^a = \frac{1}{\sqrt{2}} \lambda^a_{G-M} \,, \quad a = 1, ..., 7 \,, \tag{4}$$

$$\lambda^0 = \frac{\lambda^0_{G-M} + \lambda^8_{G-M}}{\sqrt{6}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{5}$$

$$\lambda^8 = \frac{-\lambda^0_{G-M} + \sqrt{2}\, \lambda^8_{G-M}}{\sqrt{6}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix} , \tag{6}$$

with λ^a_{G-M} being the standard set of Gell-Mann matrices. The current quark mass matrix is

$$\hat{m} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} . \tag{7}$$

In the following we adopt the exact isospin symmetry $m_u = m_d$. The symbol $\tilde{u}$ will stand everywhere for the u, d, $\bar{u}$, $\bar{d}$ quarks. The symbol $\tilde{s}$ will denote s or $\bar{s}$ quarks. We choose the polynomial form-factors to be orthogonal on the unit interval

$$\int_0^1 f_k(\tau) f_l(\tau) d\tau = \delta_{kl} \,, \tag{8}$$

and select here:

$$f_1(\tau) = 2 - 3\tau; \qquad f_2(\tau) = -\sqrt{3}\, \tau; \qquad \tau \equiv -\frac{\partial^2}{\Lambda^2} \,. \tag{9}$$

The parameter Λ is the four-momentum cutoff for virtual quark momenta in quark loops. N_c denotes number of colors and $N_f = 3$ is the number of quark flavors.

We will work with bosonized action. Thus, one introduces auxiliary $SPVA$-fields following the standard procedure:

$$L_{aux} = i\bar{q} \left(\hat{\partial} + \hat{m} + \sum_{k=1}^{2} \varphi^a_{k,j} \Gamma^a_j f_k(\tau) \right) q + N_c N_f \Lambda^2 \sum_{k,l=1}^{2} Tr\, \varphi^a_{k,j} \left(c^a_{kl} \right)^{-1} \varphi^a_{l,j} \,, \tag{10}$$

where $\varphi^a \equiv \sigma^a, \pi^a, \rho^a, A^a$ represents auxiliary S, P, V, A fields and c^a_{kl} denotes a^a_{kl} for SP-case and b^a_{kl} for VA-case.

For the cancellation of quadratic divergences of order Λ^2 the following parameterization of the coupling constants is accepted in SP-case:

$$8\pi^2 \left(a^a_{kl}\right)^{-1} = \delta_{kl} - \frac{\Delta^a_{kl}}{\Lambda^2}; \qquad \Delta_{kl} \ll \Lambda^2 \,, \tag{11}$$

where the physical mass parameters Δ^a_{kl} satisfy the relations:

$$\Delta^m_{kl} = \Delta^0_{kl} \,, \quad \Delta^n_{kl} = \frac{1}{2}\left(\Delta^0_{kl} + \Delta^8_{kl}\right) ; \qquad m = 1,2,3 \,, \quad n = 4,5,6,7 \,. \tag{12}$$

Conditions (11), (12) can be rewritten in the equivalent form:

$$8\pi^2 \varphi^a_j \left(a^a_{kl}\right)^{-1} \varphi^a_j = \varphi^a_j \lambda^a \left(\delta_{kl}\hat{I} - \frac{\hat{\Delta}_{kl}}{\Lambda^2}\right) \varphi^a_j \lambda^a \tag{13}$$

with

$$\hat{\Delta}_{kl} = \begin{pmatrix} \Delta^0_{kl} & 0 & 0 \\ 0 & \Delta^0_{kl} & 0 \\ 0 & 0 & \Delta^8_{kl} \end{pmatrix} \tag{14}$$

and $\hat{I}$ being the unit matrix. The same conditions are valid for VA-case with the replacement

$$\left(a^a_{kl}\right)^{-1} \to 2\left(b^a_{kl}\right)^{-1} , \qquad \Delta^a_{kl} \to \frac{4}{3}\bar{\Delta}^a_{kl}. \tag{15}$$

For convenience we redefine the scalar fields as follows:

$$\hat{m} + \sum_{k=1}^{2} \sigma^a_k f_k(\tau)\lambda^a \longrightarrow \sum_{k=1}^{2} \sigma^a_k f_k(\tau)\lambda^a. \tag{16}$$

After integrating over the quark fields one comes to the bosonic effective action:

$$S_{eff} = N_c N_f \int d^4x \left\{ \Lambda^2 \sum_{k,l=1}^{2} Tr\, \varphi^a_{k,j} \left(c^a_{kl}\right)^{-1} \varphi^a_{l,j} - Tr \ln\left[1 + \frac{i\sum_{k=1}^{2} \varphi^a_{k,j}\Gamma^a_j f_k(\tau)}{i\hat{\partial} + i\hat{M}(\tau)}\right]\right\} , \tag{17}$$

where the dynamical mass function was introduced:

$$\hat{M}(\tau) \equiv \sum_{k=1}^{2} \sigma^i_k f_k(\tau)\lambda^i. \tag{18}$$

Throughout the paper $i = 0, ..., 8$.

When differentiating the effective action (17) with respect to the scalar fields one obtains the mass-gap equation (large-N_c approximation is used in QQM) which determines the minimum to the effective potential V_{eff} (by definition $S_{eff} = \int V_{eff} d^4x$):

$$\Delta^i_{kl}\sigma^i_k + \left(\Lambda^2\delta_{kl} - \Delta^i_{kl}\sigma^i_k\right) n_k m^i_0 \simeq (M^i_0)^3 f_l(0) \ln\frac{\Lambda^2}{(M^i_0)^2} + \int dp^2 \frac{(M^i(p^2))^3 f_l(p^2) - (M^i_0)^3 f_l(0)}{p^2 + (M^i(p^2))^2} \,, \tag{19}$$

where

$$m_0^0 = -m_{\tilde{u}}\,, \quad m_0^8 = -m_{\tilde{s}}\,; \qquad n_1 = \frac{1}{2}\,, \quad n_2 = -\frac{\sqrt{3}}{2}\,; \qquad M_0^i \equiv M^i(0)\,. \tag{20}$$

There exist non-zero solutions of equations (19). They indicate to the dynamical breaking of chiral symmetry.

If one retains only quadratic parts in both boson field fluctuations and in soft external momenta in (17), one comes to the bosonic effective action (17):

$$S^0_{eff} \simeq \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \sum_{k,l=1}^{2} Tr\,\hat{\varphi}_{k,j}\hat{F}^{\varphi_j}_{kl}\,\hat{\varphi}_{l,j}\,. \tag{21}$$

Here $\hat{\varphi} \equiv \lambda^a\varphi^a$ and

$$\hat{F}^{\varphi_j}_{kl} = \hat{A}^{\varphi_j}_{kl}p^2 + \hat{B}^{\varphi_j}_{kl}\,, \tag{22}$$

where two symmetric matrices represent the kinetic term ($\hat{A}$) and the momentum independent part ($\hat{B}$). In (21) we did not take into account both the $\pi - a_1$ mixing and different mixing terms which are proportional to the current quark masses, since these contributions do not influence the meson mass spectrum considered in the next section.

3 Mass spectrum and mass splitting of $SPVA$, $U(3)$ QQM

The physical mass spectrum of $SPVA$, $U(3)$ QQM is defined by the solutions of the corresponding secular equation

$$det\hat{F}^{\varphi_j}_{kl} = 0\,; \qquad m^2_{phys} = -p^2. \tag{23}$$

We use the large-log approximation $\ln\frac{\Lambda^2}{(M^i_0)^2} \gg 1$, which is compatible with confinement [20]. The self-consistency of the mass spectrum turns out to impose the following scale conditions:

$$\Delta^i_{kl} \sim \bar{\Delta}^i_{kl} \sim (M^i_0)^2\ln\frac{\Lambda^2}{(M^i_0)^2}\,; \qquad m^i_0\Lambda^2 \sim 1\,. \tag{24}$$

In the following we drop the index i.

The expressions for the mass spectrum are rather cumbersome and displayed in the Appendix. Here we present the mass relations, which are independent of model parameters in the large-log approximation. Some comments concerning our notations and approach are in order. We do not suppose any concrete physical particle in indices for S and A case, since the choice of a particle may depend on a model. The gluon anomaly is not considered in the present report. Thus, the η'-meson is excluded. However, we do not see any reason for appearance of $U_A(1)$ problem for excited states. Below the prime denotes excited states and the symbol $(\eta)'$ means the first radial excitation of η-meson.

First of all, in the model holds the Gell-Mann-Okubo relation and the singlet state has no admixture of s-quark in $U(3)$ case:

$$m^2_{\alpha,\tilde{u}\tilde{u}} + m^2_{\alpha,\tilde{s}\tilde{s}} = 2m^2_{\alpha,\tilde{s}\tilde{u}}\,; \qquad m_{\alpha,\tilde{u}\tilde{u}} = m_{\alpha,singlet}\,. \tag{25}$$

Here $\alpha \equiv S, V, A; S', V', A', P'$. For P-case one has the $SU(3)$ relation:

$$m^2_{P,\tilde{u}\tilde{u}} + 3m^2_{P,\tilde{s}\tilde{s}} = 4m^2_{P,\tilde{s}\tilde{u}}\,. \tag{26}$$

All other relations presented below are derived within the framework of $SPVA$, $U(3)$ QQM.

For the ground SP-meson states one has:

$$m^2_{\sigma,\tilde{u}\tilde{u}} - 3m^2_\pi = m^2_{\sigma,\tilde{s}\tilde{u}} - 3m^2_K = m^2_{\sigma,\tilde{s}\tilde{s}} - 3\left(2m^2_K - m^2_\pi\right) = 4\mathcal{M}^2_0 + \mathcal{O}\left(\frac{1}{\ln\frac{\Lambda^2}{M^2_0}}\right)\,. \tag{27}$$

Here and below $\mathcal{M}_0 \equiv M_0\,|_{m_0=0}$. Note that (27) differs from the Nambu relation due to extraction from M_0 of the current mass contribution. In particular, in the case $M = const$ the contribution $\mathcal{O}\left(\ln^{-1}\frac{\Lambda^2}{M^2_0}\right)$ in (27) disappears, and the following relations for the excited SP-meson states can be derived:

$$m^2_{\sigma',\tilde{u}\tilde{u}} - m^2_{\pi'} = m^2_{\sigma,\tilde{u}\tilde{u}} - m^2_\pi\,, \tag{28}$$

$$m^2_{\sigma',\tilde{s}\tilde{u}} - m^2_{K'} = m^2_{\sigma,\tilde{s}\tilde{u}} - m^2_K\,, \tag{29}$$

$$m^2_{\sigma',\tilde{s}\tilde{s}} - m^2_{(\eta)'} = m^2_{\sigma,\tilde{s}\tilde{s}} - \frac{1}{2}\left(3m^2_\eta - m^2_\pi\right)\,, \tag{30}$$

$$m^2_{(\eta)'} - m^2_{K'} \simeq m^2_{K'} - m^2_{\pi'} \simeq m^2_K - m^2_\pi. \tag{31}$$

In the VA-meson sector the relations for ground states look as follows:

$$m^2_{a_1,\tilde{u}\tilde{u}} - m^2_\rho \simeq \frac{3}{2}\left(m^2_{\sigma,\tilde{u}\tilde{u}} - m^2_\pi\right)\,, \tag{32}$$

$$m^2_{a_1,\tilde{s}\tilde{u}} - m^2_{K^*} \simeq \frac{3}{2}\left(m^2_{\sigma,\tilde{s}\tilde{u}} - m^2_K\right)\,, \tag{33}$$

$$m^2_{a_1,\tilde{s}\tilde{s}} - m^2_\varphi \simeq \frac{3}{2}\left[m^2_{\sigma,\tilde{s}\tilde{s}} - \frac{1}{2}\left(3m^2_\eta - m^2_\pi\right)\right]\,. \tag{34}$$

We suppose equal couplings for all VA-mesons. It amounts to the equality $\bar{\Delta}^0_{kl} = \bar{\Delta}^8_{kl}$. Then the following relation takes place:

$$m^2_\varphi = m^2_\rho\left[1 + \frac{m^2_K - m^2_\pi}{\mathcal{M}^2_0\left(2 + \frac{8\pi^2(f^2_\pi - f^2_{\pi,0})}{N_c m^2_\pi}\right)}\right]\,. \tag{35}$$

In (35) $f_{\pi,0}$ denotes the π-meson weak decay constant in the chiral limit, $f_{\pi,0} = 87$ MeV. For the excited VA-meson states one has:

$$m^2_{a'_1,\tilde{u}\tilde{u}} - m^2_{\rho'} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{u}\tilde{u}} - m^2_{\pi'}\right)\,, \tag{36}$$

$$m^2_{a'_1,\tilde{s}\tilde{u}} - m^2_{K^{*\prime}} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{s}\tilde{u}} - m^2_{K'}\right)\,, \tag{37}$$

$$m^2_{a'_1,\tilde{s}\tilde{s}} - m^2_{\varphi'} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{s}\tilde{s}} - m^2_{(\eta)'}\right)\,. \tag{38}$$

4 Fits and predictions of $SPVA$ meson masses

Our choice of correspondence of the model $SPVA$ meson states with the physical particles is presented in the table. Using the mass relations above one can predict different masses of $SPVA$ mesons having given some masses as input parameters. We put also $\mathcal{M}_0 =$ 320 MeV. The square brackets denote the standard approximation in NJL-like models, $M_0 = const$. As it is seen from the table the predictive power of this approximation is stronger. All the fits are compared with the corresponding experimental values [21]. Several comments are in order in connection to the SPA cases.

The $SPVA$, $U(3)$ QQM masses of mesons and their first excitations (in MeV)

Particle, th.	Particle, exp.	Input	Prediction	Experiment	Difference, %	Case
$Singlet$						
$\tilde{u}\tilde{u}$	π	140		135-140		P
$\tilde{s}\tilde{u}$	K	500		494-498		
$\tilde{s}\tilde{s}$	η		570	547.30±0.12	4	
$(Singlet)'$	$\eta(1295)$		1300	1297.0±2.8	<1	
$\tilde{u}'\tilde{u}'$	$\pi(1300)$	1300		1300±100		P'
$\tilde{s}'\tilde{u}'$	$K(1460)$	1400	[1390]	1400-1460	[<1]	
$\tilde{s}'\tilde{s}'$	$\eta(1440)$		1490 [1470]	1400-1470	1 [<1]	
$Singlet$	$f_0(400-1200)$		680	400-1200	(?)	
$\tilde{u}\tilde{u}$	$a_0(980)$		680	984.8±1.4	30	S
$\tilde{s}\tilde{u}$	$K_0^*(960)$ (?)		1080	905±50 (?)	19 (?)	
$\tilde{s}\tilde{s}$	$f_0(1500)$ (?)		1360	1500±10	9	
$(Singlet)'$	$f_0(1370)$		1470 [1460]	1200-1500	(?)	
$\tilde{u}'\tilde{u}'$	$a_0(1450)$	1470	[1460]	1474±19	[1]	S'
$\tilde{s}'\tilde{u}'$	$K_0^*(1430)$		1590 [1690]	1412±6	13 [20]	
$\tilde{s}'\tilde{s}'$	$f_0(1710)$	1710	[1880]	1715±7	[9]	
$Singlet$	$\omega(782)$		770	782.57±0.12	2	
$\tilde{u}\tilde{u}$	$\rho(770)$	770		769.3±0.08		V
$\tilde{s}\tilde{u}$	$K^*(892)$		890	892-896	<1	
$\tilde{s}\tilde{s}$	$\varphi(1020)$		990	1019.417±0.014	3	
$(Singlet)'$	$\omega(1420)$		1460	1419±31	3	
$\tilde{u}'\tilde{u}'$	$\rho(1450)$	1460		1465±25		V'
$\tilde{s}'\tilde{u}'$	$K^*(1410)$		1570	1414±15	11	
$\tilde{s}'\tilde{s}'$	$\varphi(1680)$	1680		1680±20		
$Singlet$	$f_1(1285)$		1120	1281.9±0.6	12	
$\tilde{u}\tilde{u}$	$a_1(1260)$		1120	1230±40	9	A
$\tilde{s}\tilde{u}$	$K_1(1400)$		1470	1402±7	5	
$\tilde{s}\tilde{s}$	$f_1(1510)$		1740	1512±4	15	
$(Singlet)'$	(?)		1670	(?)	(?)	
$\tilde{u}'\tilde{u}'$	$a_1(1640)$		1670	1640±40	2	A'
$\tilde{s}'\tilde{u}'$	$K_1(1650)$		1960	1650±50	19	
$\tilde{s}'\tilde{s}'$	(?)		2210	(?)	(?)	

P-mesons. As it was discussed above, we do not incorporate the η'-meson in our model. However, we assume restoration of $U(3)$ symmetry for excited states. One of the arguments in favor of this hypothesis is appearance of singlet state with almost the same mass as that of triplet particles in accordance with (25).
S-mesons. The meson $f_0(1370)$ is a natural candidate for the singlet excited state. A large uncertainty in width and mass indicates the meson $f_0(400-1200)$ to be its ground state rather than $f_0(980)$ (the last one can be a $\bar{K}K$ bound state [21]). The state $K_0^*(1430)$ does not seem to be a ground state, because of its large mass. As a possible candidate for the ground su-state we accept $K_0^*(960)$-meson [22], which is not included in the Particle Date Group [21]. The resonance $f_0(1500)$ could be, in principle, replaced be the broad state $f_0(1420^{+150}_{-70})$ [19]. A rather big discrepancy in prediction of $a_0(980)$-meson mass seems to be an artifact of large-N_c approximation: the next-to-leading correction to the scalar mass degenerates masses for the triplet and singlet states, resulting in an anomalous large enhancement of the triplet mass [23].
A-mesons. Our fits show that the candidate for the ground su-meson state is rather $K_1(1400)$-meson than $K_1(1270)$. The last particle may be regarded as a $K\rho$ bound state. We have no any information concerning possible candidates for excited singlet and $s\bar{s}$ states.

5 Summary

In present work we derived the mass splitting relations for ground and excited $SPVA$ meson states in the framework of the $U(3)$ Quasilocal Quark Model. These relations are independent of model parameters and generalize the corresponding $SU(2)$ results [6, 20]. The agreement with experiment is within 4% for pseudoscalar mesons, 30% for scalar mesons, 11% for vector mesons, and 19% for axial-vector mesons.

All the calculations are carried out in the large-N_c and large-log approximations. The second one represents a model of quark confinement within this approach, since quark-antiquark threshold contributions are suppressed in two-point functions in the leading-log approximation. Improvement of this approximation may mean that details of confinement are of importance in the case under consideration.

Finally we would like to mention future possibilities. The $SPVA$, $U(3)$ QQM can be matched with QCD with the help of Operator Product Expansion [24], as it was done for the SP, VA $SU(2)$ QQM [6, 20]. As we expect, the QCD matching based on the Chiral Symmetry Restoration sum rules helps to select out the relevant pattern of DCSB as well as to enhance predictability of $SPVA$ QQM.

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Appendix

Here we write down the solutions of equations for the meson mass spectrum (23). The isospin index i is dropped in all expressions. Let us introduce notations:

$$d \equiv 3\bar{\Delta}_{11}+2\sqrt{3}\bar{\Delta}_{12}-15\bar{\Delta}_{22}\,, \quad \bar{\Delta} \equiv 3\bar{\Delta}_{11}+2\sqrt{3}\,\bar{\Delta}_{12}+\bar{\Delta}_{22}\,, \quad \Delta \equiv 3\Delta_{11}+2\sqrt{3}\,\Delta_{12}+\Delta_{22}\,, \tag{A.1}$$

$$\bar{\sigma}_{11} \equiv -\frac{159}{4}\sigma_1^2-\frac{5\sqrt{3}}{2}\sigma_1\sigma_2+\frac{3}{4}\sigma_2^2,\ \bar{\sigma}_{12} \equiv -\frac{5\sqrt{3}}{4}\sigma_1^2+\frac{3}{2}\sigma_1\sigma_2+\frac{\sqrt{3}}{4}\sigma_2^2,\ \bar{\sigma}_{22} \equiv \frac{3}{4}\sigma_1^2+\frac{\sqrt{3}}{2}\sigma_1\sigma_2+\frac{9}{4}\sigma_2^2, \tag{A.2}$$

$$< \bar{q}q > \equiv -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1-\sqrt{3}\sigma_2)\,, \tag{A.3}$$

$$f_\pi^2 \equiv \frac{N_c M_0^2}{4\pi^2}\ln\frac{\Lambda^2}{M_0^2}\,. \tag{A.4}$$

Notations (A.3) and (A.4) represent the analytic expressions for the quark condensate and π-meson weak decay constant respectively (in the framework of QQM model). The symbol $\hat{\bar{\Delta}}$ will signify the matrix $\bar{\Delta}_{kl}$.

Spectrum for vector and axial-vector mesons

Ground states:

$$m_1^2 = \left(2vM_0^2-\frac{\det\hat{\bar{\Delta}}}{2\bar{\Delta}_{22}\ln\frac{\Lambda^2}{M_0^2}}\right)\left(1-\frac{d}{8\bar{\Delta}_{22}\ln\frac{\Lambda^2}{M_0^2}}\right)-\frac{3\det{}^2\hat{\bar{\Delta}}}{16\bar{\Delta}_{22}^3\ln^2\frac{\Lambda^2}{M_0^2}}+$$

$$\frac{v}{2\bar{\Delta}_{22}\ln\frac{\Lambda^2}{M_0^2}}\left[\frac{\det\hat{\bar{\Delta}}}{4\bar{\Delta}_{22}}\left(3M_0^2-4\bar{\sigma}_{22}\right)+\bar{\Delta}_{11}\bar{\sigma}_{22}-2\bar{\Delta}_{12}\bar{\sigma}_{12}+\bar{\Delta}_{22}\bar{\sigma}_{11}\right]+\mathcal{O}\left(\frac{1}{\ln^2\frac{\Lambda^2}{M_0^2}}\right). \tag{A.5}$$

Excited states:

$$m_2^2 = -\frac{4}{3}\bar{\Delta}_{22}+\frac{4}{3}v\bar{\sigma}_{22}+\left(2vM_0^2-\frac{\bar{\Delta}}{6\ln\frac{\Lambda^2}{M_0^2}}\right)\left(1+\frac{2}{\ln\frac{\Lambda^2}{M_0^2}}\right)+$$

$$\frac{v\left(3\bar{\sigma}_{11}+2\sqrt{3}\,\bar{\sigma}_{12}+\bar{\sigma}_{22}\right)}{6\ln\frac{\Lambda^2}{M_0^2}}-m_1^2+\mathcal{O}\left(\frac{1}{\ln^2\frac{\Lambda^2}{M_0^2}}\right). \tag{A.6}$$

In the expressions (A.5), (A.6) $v=0$ corresponds to the V-case and $v=3$ does to the A-one.

Spectrum for scalar and pseudoscalar mesons

Taking into account the mass-gap equations (19) one has for the ground states:

$$m_1^2 = (v-1)\left\{2M_0^2+\frac{\sigma_2^3(9\sigma_2+4\sqrt{3}\,\sigma_1)-\sigma_1^2(39\sigma_1^2+6\sigma_2^2+4\sqrt{3}\,\sigma_1\sigma_2)}{8M_0^2\ln\frac{\Lambda^2}{M_0^2}}\right\}-$$

$$\frac{2<\bar{q}q>m_0}{f_\pi^2}+\mathcal{O}\left(\frac{1}{\ln^2\frac{\Lambda^2}{M_0^2}}\right). \qquad (A.7)$$

Excited states:

$$m_2^2=-\frac{4}{3}\Delta_{22}+\frac{4}{3}v\bar{\sigma}_{22}+\left(2vM_0^2-\frac{\Delta}{6\ln\frac{\Lambda^2}{M_0^2}}\right)\left(1+\frac{2}{\ln\frac{\Lambda^2}{M_0^2}}\right)+$$

$$\frac{v\left(3\bar{\sigma}_{11}+2\sqrt{3}\,\bar{\sigma}_{12}+\bar{\sigma}_{22}\right)}{6\ln\frac{\Lambda^2}{M_0^2}}-m_1^2+\mathcal{O}\left(\frac{1}{\ln^2\frac{\Lambda^2}{M_0^2}}\right). \qquad (A.8)$$

In (A.7), (A.8) $v=1$ corresponds to the P-case and $v=3$ does to the S-one. Moreover, dependence on the current quark mass m_0 can be extracted from M_0:

$$(M_0^i)^2=(\mathcal{M}_0^i)^2\left(1+\frac{(m_{1,P}^i)^2}{2(\mathcal{M}_0^i)^2}\right)+\mathcal{O}\left(\frac{1}{\ln^2\frac{\Lambda^2}{(M_0^i)^2}}\right). \qquad (A.9)$$

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