# Hadronic Corrections to the Muon Decay*

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**Abstract**

We consider the $\mathcal{O}(\alpha^2)$ hadronic corrections to the energy spectrum of the electron produced in muon decay. We find that the correction can be described, within good approximation, by a linear function of the electron energy. Explicit expressions for the form- factors needed in an approach based on dispersion integrals are given.

At present, the best value of the muon life time is $\tau_\mu = (2.19703 \pm 0.00004)\,\mu s$ [1]. In fact, experimental data have reached such a precision that quantum corrections have to be taken into account when comparing this experimental result with theoretical predictions. To match this accuracy from the theory side, two-loop radiative corrections to the muon decay in the full electroweak Standard Model are needed. To perform the required calculations is a formidable, but not an impossible, task. A step in this direction is the calculation of the purely electromagnetic corrections to the total decay rate at order $\alpha^2$ in the Fermi theory, which has been performed by van Ritbergen and Stuart [2, 3] (see also [4]). Besides that, also the $\mathcal{O}(N_f\alpha^2)$ corrections in the Standard Model have been considered in Refs. [5, 6]. The decay spectrum is known to order $\alpha$ since long; however, the corresponding corrections for polarized muon decay were calculated only recently in [7]. The leading logarithmic QED corrections of order $\mathcal{O}\left(\alpha^2 \ln^2(m_\mu/m_e)\right)$ to the muon decay spectrum were considered in Ref. [8].

The spectrum calculation is different from the one for the life time since the Kinoshita-Lee-Nauenberg theorem [9] is not in effect. Consequently, powers of the large logarithm $\ln(m_\mu/m_e)$ do not cancel in the calculation of the electromagnetic corrections to the spectrum. This becomes obvious when fitting the spectrum corrected to order $\mathcal{O}(\alpha)$ to the Michel spectrum: the resulting effective Michel parameter differs by about 6 % from its lowest-order value [10], a correction which is more than 10 times larger than the corresponding correction to the muon life time. At order $\mathcal{O}(\alpha^2)$ the radiative corrections may be expected to be of order of several per-mille and will become important for future high-precision experiments.

Given this perspective we review the calculation of the hadronic contribution to the energy spectrum of the final-state electron in muon decay. This contribution is not expected to be logarithmically enhanced, but nonetheless is required for a complete second-order calculation. Further details of the calculation are given in Ref. [11].

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We consider the decay of a muon in its rest system,

$$\mu^-(p) \to e^-(p') + \nu_\mu(q_1) + \bar{\nu}_e(q_2)\,, \tag{1}$$

and define momenta as shown in Fig.(1). It is convenient to introduce the dimensionless variable $x = 2E_e/m$ to denote the ratio of the energy of the produced electron $E_e$ with respect to the muon mass $m_\mu \equiv m$. Neglecting the electron mass, $p'^2 = 0$, the kinematically allowed range is $0 \le x \le 1$, and the momentum transferred from the charged particles to the neutrino pair, $q = q_1 + q_2 = p - p'$ is determined by $q^2 = (1-x)m^2$.

The matrix element $\mathcal{M}$ for (1) in the Fermi theory can be calculated most conveniently after a Fierz rearrangement factorizing the amplitude into a current $J_\mu$ which describes the $\mu e$ transition and a current for the $\nu_\mu\nu_e$ interaction. After squaring and summing (averaging) over spins, one can write $|\mathcal{M}|^2$ as a product of two corresponding tensors. The one pertaining to the neutrino interaction can be integrated over the unobserved momenta of the neutrinos independently, leading to

$$N_{\mu\nu} = q_\mu q_\nu - g_{\mu\nu} q^2\,. \tag{2}$$

This tensor will be contracted with $C_{\mu\nu} = J^*_\mu J_\nu$, with $J_\mu = \bar{u}_e(p')\Lambda_\mu(q)u_\mu(p)$, where $\Lambda_\mu(q)$ is the effective vertex of the four-fermion interaction. At the lowest order, $\Lambda_\mu(q)$ is identified with

$$\Lambda^0_\mu = \frac{G_F}{\sqrt{2}}\gamma_\mu\,(1-\gamma_5)\,, \tag{3}$$

where $G_F = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant.

Hadronic contributions to the radiative corrections to the current $J_\mu$ at order $\mathcal{O}(\alpha^2)$ are described by the Feynman diagrams shown in Fig. 1. The hadronic vacuum polarization

$$\Pi^{\rm had}_{\mu\nu}(k^2) = \frac{-{\rm i}\Pi^{\rm had}(k^2)}{k^2 + {\rm i}0}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \tag{4}$$

is inserted in a one-loop vertex correction. The vacuum polarization can be related to the measured cross section for $e^+e^- \to hadrons$ with the help of a dispersion relation

$$\Pi^{\rm had}(k^2) = \frac{\alpha}{3\pi}\int_{s_{\rm thr}}^\infty \frac{{\rm d}s}{s} R(s)\frac{k^2}{k^2 - s + {\rm i}0}\,, \qquad R(s) = \frac{\sigma(s; e^+e^- \to hadrons)}{\sigma(s; e^+e^- \to \mu^+\mu^-)}, \tag{5}$$

where the integration starts at the two-pion threshold, $s_{\rm thr} = 4m_\pi^2$. Therefore, the calculation corresponds to a one-loop vertex with a photon of mass $\sqrt{s}$, i.e. using a propagator

$$\frac{-{\rm i}}{k^2 - s + {\rm i}0}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\,. \tag{6}$$

The result can be written in the form

$$\Lambda_\mu(q) = \frac{\alpha}{3\pi}\int_{s_{\rm thr}}^\infty \frac{{\rm d}s}{s} R(s)\tilde{\Lambda}_\mu(s; q, m^2) \tag{7}$$

where the vector function $\tilde{\Lambda}_\mu$ can be decomposed into Lorentz-covariants as

$$\tilde{\Lambda}_\mu(s; q, m^2) = \gamma_\mu\omega_L\left[1 + \tilde{f}(s; q^2, m^2)\right] + \frac{p_\mu + p'_\mu}{m}\omega_R\tilde{g}_+(s; q^2, m^2) + \frac{q_\mu}{m}\omega_R\tilde{g}_-(s; q^2, m^2)\,, \tag{8}$$

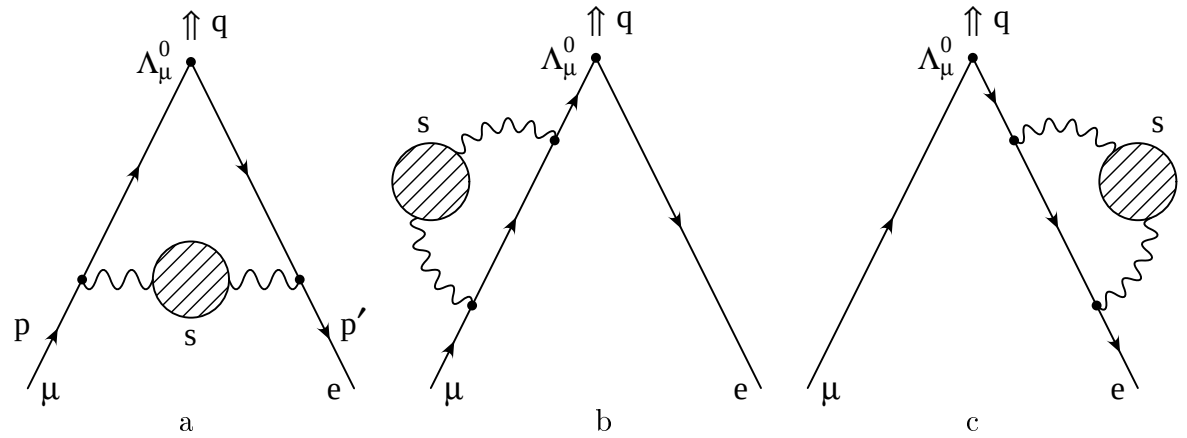


Figure 1: *Feynman diagrams describing a self energy insertion in the photon one-loop corrections to the μe vertex.*

with $\omega_{R,L} = (1 \pm \gamma_5)/2$. The calculation is straightforward and corresponds to that of the one-loop vertex correction in QED, with the difference that (*i*) the exchanged "photon" is massive with mass $\sqrt{s}$, (*ii*) the coupling is purely left-handed, and (*iii*) two fermion lines have different masses. In fact, except for small $q^2$, the electron mass can safely be neglected. After contraction with the neutrino tensor $N_{\mu\nu}$, Eq. (2), only the form-factors $\tilde{f}(s;q^2,m^2)$ and $\tilde{g}_+(s;q^2,m^2)$ will remain in the final result.

Explicit results for the relevant form factors $\tilde{f}$ and $\tilde{g}_+$ are [11]

$$\begin{aligned}
\tilde{f}(s;q^2,m^2) &= -\frac{\alpha}{4\pi}\Bigg\{2(m^2-q^2-s)\left[1+\frac{q^2 s}{(m^2-q^2)^2}\right]C_0 \\
&\quad + \left[\frac{2q^2 s}{(m^2-q^2)^2} - \frac{2q^2}{(m^2-q^2)} + 1\right] B_0(q^2;m^2,0) \\
&\quad + \left[\frac{2q^2}{m^2-q^2} - \frac{s(m^2+q^2)}{(m^2-q^2)^2}\right] B_0(m^2;m^2,s) + \left(\frac{1}{s} - \frac{1}{m^2-q^2}\right) A(s) + 2\Bigg\} \\
&\quad + \tilde{f}_{\rm SE}(s;q^2,m^2)\,, \qquad (9)
\end{aligned}$$

$$\begin{aligned}
\tilde{g}_+(s;q^2,m^2) &= -\frac{\alpha}{4\pi}\frac{m^2}{q^2}\Bigg\{\frac{2q^2 s}{m^2-q^2}\left[\frac{3q^2 s}{(m^2-q^2)^2}+2\right]C_0 \\
&\quad - \frac{q^2}{m^2-q^2}\left[\frac{6q^2 s}{(m^2-q^2)^2}+1\right] B_0(q^2;m^2,0) - \frac{q^2}{m^2-q^2} \\
&\quad + \frac{q^2}{m^2-q^2}\left[\frac{6m^2 s}{(m^2-q^2)^2} - \frac{s(4m^2-q^2)}{m^2(m^2-q^2)} + 2\right] B_0(m^2;m^2,s) \\
&\quad + \frac{q^2}{m^2-q^2}\left[\frac{3}{m^2-q^2} - \frac{1}{m^2}\right] A(s) + \left[\frac{1}{m^2-q^2} - \frac{1}{m^2}\right] A(m^2)\Bigg\}\,, \qquad (10)
\end{aligned}$$

where $C_0 = C_0(m^2,0,q^2;m^2,s,0)$ is the three-point integral (cf. Fig. 1a), whereas $B_0$ and $A$ denote the tadpole and two-point integrals [17, 18], respectively (see Eqs. (12)–(14) below). The complete set of results, including that for $\tilde{g}_-$, is presented in [11].

Self-energy diagrams (Fig. 1b,c) contribute to the coefficient of $\gamma_\mu$ only. The result is

$$\tilde{f}_{\rm SE}(s;q^2,m^2) = \frac{\alpha}{8\pi}\Bigg\{2(s+2m^2)\left.\frac{\partial B_0(p^2;m^2,s)}{\partial p^2}\right|_{p^2=m^2} - \frac{s}{m^2}B_0(m^2;m^2,s)$$

$$-\frac{s+m^2}{sm^2}A(s)+\frac{1}{m^2}A(m^2)+1+\frac{3}{2}\Bigg\}, \tag{11}$$

where the last term, $\frac{3}{2}$, comes from the self energy on the massless (electron) leg. Using recurrence relations [19] (see also Appendix A of [20]), the derivative in (11) can be represented as

$$\left.\frac{\partial B_0(p^2;m^2,s)}{\partial p^2}\right|_{p^2=m^2}=\frac{1}{m^2(s-4m^2)}\left[m^2-(s-3m^2)B_0(m^2;m^2,s)+A(m^2)-\left(1-\frac{2m^2}{s}\right)A(s)\right].$$

The required tadpole and two-point integrals are [17, 18]

$$A(m^2) = m^2\left[-\Delta-1+\ln\frac{m^2}{\mu^2_{\rm DR}}\right], \tag{12}$$

$$B_0(m^2;m^2,s) = \Delta+2-\ln\frac{m^2}{\mu^2_{\rm DR}}-\frac{s}{2m^2}\ln\frac{s}{m^2}+\frac{s}{2m^2}\beta\ln\left(\frac{1+\beta}{1-\beta}\right), \tag{13}$$

$$B_0(q^2;m^2,0) = \Delta+2-\ln\frac{m^2}{\mu^2_{\rm DR}}+\frac{m^2-q^2}{q^2}\ln\left(\frac{m^2-q^2}{m^2}\right), \tag{14}$$

where $\beta\equiv\sqrt{1-4m^2/s}$, and $\mu_{\rm DR}$ is the scale parameter of dimensional regularization. In Eqs. (12)–(14), terms containing $\Delta=1/\varepsilon-\ln\pi-\gamma_{\rm E}$ represent the ultraviolet singularities which cancel in the final results (9)–(10).

Finally, we need the three-point scalar function $C_0$ [17] for positive values of $q^2$. For arbitrary values of $m_\mu$ and $m_e$, it can be presented in the following symmetric form:

$$\begin{aligned} C_0 = {} & \frac{1}{\sqrt{\lambda(m_e^2,{m_\mu}^2,q^2)}}\Bigg\{-{\rm Li}_2\left(1-\frac{cm_e}{m_\mu}\right)-{\rm Li}_2\left(1-\frac{cm_\mu}{m_e}\right) \\ & +{\rm Li}_2\left(1-\frac{cb_\mu}{b_e}\right)+{\rm Li}_2\left(1-\frac{cb_e}{b_\mu}\right)+{\rm Li}_2\left(1-\frac{c}{b_eb_\mu}\right)+{\rm Li}_2\left(1-cb_eb_\mu\right) \\ & +\ln^2 b_e+\ln^2 b_\mu+\frac{1}{2}\ln^2 c-\frac{1}{2}\ln^2\frac{m_e}{m_\mu}-\ln c\ln\frac{m_em_\mu}{s}\Bigg\}, \end{aligned} \tag{15}$$

where

$$b_e\equiv\sqrt{\frac{1-\beta_e}{1+\beta_e}},\quad b_\mu\equiv\sqrt{\frac{1-\beta_\mu}{1+\beta_\mu}},\quad \beta_e\equiv\sqrt{1-\frac{4m_e^2}{s}},\quad \beta_\mu\equiv\sqrt{1-\frac{4{m_\mu}^2}{s}}, \tag{16}$$

$$c\equiv\sqrt{\frac{m_e^2+{m_\mu}^2-q^2-\sqrt{\lambda(m_e^2,{m_\mu}^2,q^2)}}{m_e^2+{m_\mu}^2-q^2+\sqrt{\lambda(m_e^2,{m_\mu}^2,q^2)}}} \tag{17}$$

and $\lambda(x,y,z)\equiv x^2+y^2+z^2-2xy-2yz-2zx$ is the standard notation for the Källen function. This expression is regular in the limit $m_e\to 0$. A compact result for this case is given in Eq. (34) of [11].

Performing the dispersion integral, one obtains the form-factors

$$f(q^2,m^2)=\frac{\alpha}{3\pi}\int_{s_{\rm thr}}^{\infty}\frac{{\rm d}s}{s}R(s)\tilde{f}(s;q^2,m^2)\,,\quad g_+(q^2,m^2)=\frac{\alpha}{3\pi}\int_{s_{\rm thr}}^{\infty}\frac{{\rm d}s}{s}R(s)\tilde{g}_+(s;q^2,m^2)\,. \tag{18}$$

Since we are interested in the $\mathcal{O}(\alpha^2)$ correction, it is sufficient to keep only terms of first order in $f$ and $g_+$ in the decay spectrum which then can be written in the form

$$\frac{1}{\Gamma_0}\frac{{\rm d}\Gamma}{{\rm d}x}=2x^2\left[(3-2x)\left(1+2f(x)\right)+xg_+(x)\right]=\left.\frac{1}{\Gamma_0}\frac{{\rm d}\Gamma}{{\rm d}x}\right|_{\rm Born}\left(1+r(x)\right), \tag{19}$$

with

$$\Gamma_0=\frac{G_F^2m^5}{192\pi^3}\,,\qquad \left.\frac{{\rm d}\Gamma}{{\rm d}x}\right|_{\rm Born}=2x^2(3-2x)\Gamma_0\,,\qquad r(x)=2f(x)+\frac{x}{3-2x}g_+(x)\,, \tag{20}$$

where $f(x)\equiv f\left((1-x)m^2,m^2\right)$ and $g_+(x)\equiv g_+\left((1-x)m^2,m^2\right)$.

For our purpose, the function $R(s)$ describing the hadronic cross section of $e^+e^-$ annihilation can be modeled by a combination of experimental data and analytical results from perturbative QCD. Since we are going to calculate a small correction, it is not necessary to invoke the most sophisticated treatment as needed, for example, when calculating the hadronic contribution to the fine structure constant $\alpha(m_Z)$. At low $s<2.5$ GeV$^2$ we use experimental data from ALEPH parametrized in [12] or provided directly by ALEPH [13] from a measurement of the isovector $\tau$ spectral function. These data are complemented by the resonance contributions from the isospin-0 light mesons $\omega$ and $\phi$. Above $s=2.5$ GeV$^2$ we use the QCD prediction for $R(s)$ due to light quarks at order $\mathcal{O}(\alpha_{\rm s})$.

Since the data in the $c\bar{c}$-channel published by various groups are in a large part of the energy range inconsistent, we apply in this case the QCD-based approach of analytic continuation by duality [14]. The data region can be chosen to extend only over the sub-threshold resonances, i.e. one can calculate the contribution coming from the $c\bar{c}$-channel by a combination of data describing the $J/\Psi(1S)$ and $J/\Psi(2S)$ resonances and the prediction of perturbative QCD. We checked that the results obtained this way are consistent with those of the standard approach using the new BES data [15].

The correction to the total decay rate, $\Delta\Gamma=\int_0^1{\rm d}x\,({\rm d}\Gamma/{\rm d}x)$, was calculated before in [2]. Our result,

$$\Delta\Gamma_{\rm had}\simeq-0.0421\,(\alpha/\pi)^2\,\Gamma_0\,, \tag{21}$$

agrees perfectly with the corresponding number $-0.042$ given in [2]. The resulting corrections to the spectrum (19) are shown in Fig. 2. At small $x$, the corrections are positive, but the correction to the total decay width is dominated by the negative values at $x\gtrsim0.18$. The dependence of the form factors on $x$ is to a very good approximation linear:

$$\begin{aligned} f(x) &\simeq (0.0071-0.0378x)\,(\alpha/\pi)^2\,\Gamma_0\,,\\ g_+(x) &\simeq -0.0067\,(\alpha/\pi)^2\,\Gamma_0\,,\\ r(x) &\simeq (0.0148-0.0813x)\,(\alpha/\pi)^2\,\Gamma_0\,. \end{aligned} \tag{22}$$

For $g_+$, the coefficient of the term linear in $x$ is very small and is therefore omitted. Note that this behavior cannot be described by a simple redefinition of the Michel parameter.

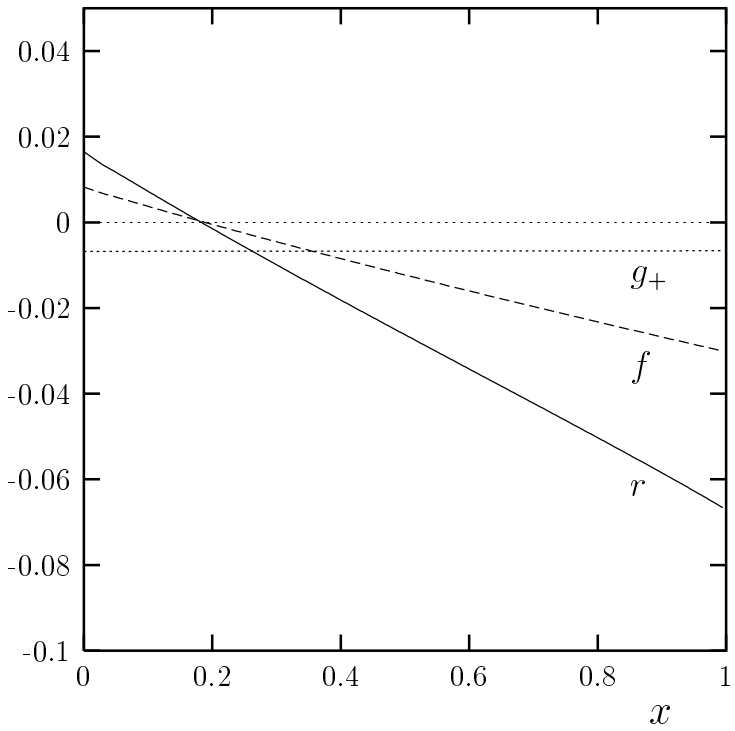


Figure 2: *Results for the form factors $f$, $g_+$ and $r$ defined in (18), (20).*

Since $G_F$ is a free parameter in the Fermi theory, the correction to the total decay width is not observable; it can be absorbed by a suitable redefinition of the Fermi constant. However, the modification of the spectrum is, in principle, observable.

The correction to the total decay rate can be split up into the various contributions to the hadronic vacuum polarization, as shown in Table 1. The form factor $f$ of the $\gamma_\mu$ term contributes $-0.0387$, whereas the correction due to $g_+$, $-0.0033$, is smaller by one order of magnitude. The total correction (the contributions due to $f$ and $g_+$) is saturated to 81 % (80.8 % and 89 %, respectively) by the contributions from small $s$ below 2.5 GeV$^2$. Only 5 % of $\Delta\Gamma_{\rm had}$ is due to charmed states, and the bottom sector is completely negligible.

The numerical results given here take into account the $\mathcal{O}(\alpha_{\rm s})$ QCD corrections in $R(s)$. Using the leading-order expression for $R(s)$, i.e. without the correction factor $(1+\alpha_{\rm s}/\pi)$ in the light-quark contribution at large $s$ and the charm-quark contribution, the final result would be $-0.04149$, i.e. changed by 1.4 %. We conclude that a more refined treatment which would include higher orders of perturbative QCD and mass-dependent corrections in the heavy-quark sector is not required for our purpose.

The evaluation of the dispersion integrals has been performed using standard numerical integration routines up to a value $s_{\rm max}$ of several hundred GeV$^2$. The contribution above this value was obtained with the help of `Maple` using the asymptotic expansion of the form factors for large $s$ (see in the Appendix of Ref. [11]). For intermediate values of $s$, good consistency of both procedures has been verified.

The same set of formulas can be used to calculate the contributions from a $\mu^+\mu^-$ or a $\tau^+\tau^-$ loop insertion. We find that the tau loop gives a very small contribution, about

| Contributions to $\Delta\Gamma_{\rm had}$ | | | |
|---|---|---|---|
| 1 | $0 < s < 0.2$ GeV$^2$ | $-0.00129$ | 3.1 % |
| 2 | $\omega$ | $-0.00223$ | 5.3 % |
| 3 | $\phi$ | $-0.00264$ | 6.3 % |
| 4 | $0.2 < s < 2.5$ GeV$^2$ | $-0.02804$ | 66.6 % |
| 5 | $s > 2.5$ GeV$^2$ | $-0.00564$ | 13.4 % |
| 6 | $J/\Psi(1S)$ | $-0.00066$ | 1.6 % |
| 7 | $J/\Psi(2S)$ | $-0.00017$ | 0.4 % |
| 8 | charm, $s > 4m_c^2$ | $-0.00138$ | 3.3 % |
| 9 | bottom, $s > 4m_b^2$ | $-0.00003$ | 0.1 % |
| | Sum | $-0.04207$ | 100 % |

Table 1: Contributions to the corrections of the total decay rate

1.5 % of the one from the muon loop, in agreement with Ref. [2]. Therefore we give only results for the muon loop, where one has to insert in Eq. (18)

$$R(s) \to \left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}} \quad \text{and} \quad s_{\rm thr} \to 4m^2\,. \tag{23}$$

With this input we obtain

$$\Delta\Gamma_{\rm muon} \simeq -0.0364\,(\alpha/\pi)^2\,\Gamma_0\,, \tag{24}$$

which perfectly agrees with the exact result given in Ref. [2]. The results of a linear fit of the form factors for the muon-loop insertion are:

$$\begin{aligned} f_{\rm muon}(x) &\simeq (0.0130 - 0.0414x)\,(\alpha/\pi)^2\,\Gamma_0\,, \\ g_{+,\rm muon}(x) &\simeq (-0.0090 + 0.0005x)\,(\alpha/\pi)^2\,\Gamma_0\,, \\ r_{\rm muon}(x) &\simeq (0.0267 - 0.0898x)\,(\alpha/\pi)^2\,\Gamma_0\,. \end{aligned} \tag{25}$$

When analyzing the similar contribution with an $e^+e^-$ loop insertion (which represents one of the pure QED contributions), one should carefully treat logarithmic singularities occurring in the limit $m_e \to 0$. For $m_e \neq 0$, the general result (15) can be used.

One can see that the energy spectrum in the decay of an unpolarized muon is corrected by a smooth function of $x$ due to hadronic contributions of order $\mathcal{O}(\alpha^2)$. At both ends of the spectrum no particularly large enhancement or suppression is observed.

The calculations described in this paper constitute only the most straightforward part of a full calculation which would be necessary before the expected future high-precision data can be confronted with theoretical predictions. This will not only be necessary for a meaningful test of the electroweak Standard Model, but also when searching for physics beyond the Standard Model [16].

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