Perturbative Fragmentation of Doubly Heavy Diquark into Baryon

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Abstract

In the framework of perturbative QCD and diquark model of baryons we have obtained the fragmentation functions for doubly heavy scalar $(QQ')_0$ and vector $(QQ')_1$ diquarks that split into spin-3/2 and spin-1/2 baryons. Using nonrelativistic approximation we predict exactly the total fragmentation probabilities for the following processes: $(bb)_1 \to \Omega_{bbc}, (bb)_1 \to \Omega^*_{bbc}, (bc)_1 \to \Omega_{bcc}, (bc)_1 \to \Omega^*_{bcc}, (bc)_0 \to \Omega'_{bcc}$.

Doubly heavy baryons take the special place among baryons which contain heavy quarks. The existence of two heavy quarks causes the brightly expressed quark - diquark structure of the (QQq)-baryon, in a wave function which one's the configuration with compact heavy (QQ)-diquark dominates. Regularity in a spectrum of mass of doubly heavy baryons appear in many respects to a similar case of mesons containing one heavy quark [1, 2, 3, 4]. Production mechanisms for (QQq)-baryons and $(Q\bar{q})$-mesons also have common features. At the first stage compact heavy (QQ)-diquark is formed, than it fragments in a final (QQq)-baryon, picking up a light quark. Such a picture is considered as a fragmentation approach and applied following relation between relevant time scales $\tau_Q << \tau_D << \tau_H$, where τ_Q is the time of heavy quark production in a hard interaction, τ_D is the time of a doubly heavy diquark creation, τ_H is the time of a final baryon creation. Usually, to describe the doubly diquark fragmentation into baryon, the phenomenological fragmentation function is used [5]. However, some years ago it was suggested the approach [6, 7, 8], based on perturbative QCD and diquark model of baryons, in which one can obtain fragmentation functions for doubly heavy diquarks that split into spin-3/2 and spin-1/2 baryons.

We consider doubly heavy diquarks as point-like objects in the colour anti-triplet states. Diquark interaction with a gluon field is described by following vertex functions

$$\begin{aligned} T^{VVg}_{\alpha\mu\nu}(p,p') &= -ig_sT^aV^{VVg}_{\alpha\mu\nu}(p,p')F_V((p-p')^2) \\ V^{VVg}_{\alpha\mu\nu}(p,p') &= g_{\mu\nu}(p+p')_\alpha - g_{\mu\alpha}((1+\kappa)p'-\kappa p)_\nu - g_{\nu\alpha}((1+\kappa)p-\kappa p')) \end{aligned} \quad (1)$$

for the vector diquark and

$$T^{SSg}_{\alpha}(p,p') = -ig_sT^a(p+p')_\alpha F_S((p-p')^2) \quad (2)$$

for the scalar diquark. The $\kappa = +1$ being the anomalous chromomagnetic moment of the vector doubly heavy diquark [4, 9]. The diquark form factors may be parameterized using dipole formula as follows

$$F_S(k^2) = \frac{k^2_{0S}}{k^2_{0S}-k^2} \text{ for the scalar diquark,} \quad (3)$$

$$F_V(k^2) = \left(\frac{k^2_{0V}}{k^2_{0V}-k^2}\right)^2 \text{ for the vector diquark.} \quad (4)$$

Here the parameters k_{0S}^2 and k_{0V}^2 are determined through the diquark radiuses $< r_{QQ}^2 >$

$$k_{0S}^2 = \frac{6}{< r_{QQ'}^2 >}, \qquad k_{0V}^2 = \frac{12}{< r_{QQ'}^2 >}. \tag{5}$$

The values of the diquark radiuses may be obtained by solving nonrelativistic Schredinger equation for doubly heavy diquark wave function. In our calculation we use results which were obtained in Ref. [3]. The diquark radiuses and relevant values of k_0^2 are shown in the Table 1.

In our approach we use perturbative QCD for calculation the hard amplitude of diquark fragmentation which is obtained after factorization on soft and hard parts of the total amplitude. Unconditionally, such approach is precise enough if hardness of amplitude comes from large mass of the quark which formed the baryon together with the initial doubly heavy diquark. For example, it takes place in the fragmentation processes $(bb)_1 \to \Omega_{bbc}, (bb)_1 \to \Omega_{bbc}^*$. However the obtained result may be used as a QCD motivated parameterization and in the case of doubly heavy baryons containing one light quark.

The fragmentation function is given by the following expression [10]

$$D_{(QQ')\to H_{QQ'}}(z,\mu_0) = \frac{1}{16\pi^2}\int_{s_{min}}^{\infty} ds \lim_{q_0\to\infty}\frac{|\mathcal{M}|^2}{|\mathcal{M}_o|^2}, \tag{6}$$

where $\mathcal{M}$ is the amplitude for the produced $H_{QQ'}$ baryon plus antiquark $\bar{q}$ with the total four-momentum $q = p + q'$ and invariant mass $s = q^2$, $\mathcal{M}_o$ is the amplitude for the production of a on-shell diquark with the same three-momentum $\vec{q}$ from the same source. The lower limit in the integral is:

$$s_{min} = \frac{M^2}{z} + \frac{m_q^2}{1-z}, \quad \text{where} \quad z = \frac{p_0 + p_3}{q_0 + q_3},$$

$p = (p_0, 0, 0, p_3)$ is the $H_{QQ'}$ baryon four-momentum. In the axial gauge for the gluon propagator associated with four-vector $n = (1, 0, 0, -1)$

$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{(kn)},$$

the fragmentation contribution comes only from the Feynman diagram shown in Fig.1.

After some obvious simplification we have obtained the matrix elements for vector and scalar diquark fragmentation into baryons. For example:

$$\mathcal{M}((bb)_1 \to \Omega_{bbc} + \overline{c}) = \frac{\Psi_{\Omega_{bbc}}(0)}{2\sqrt{3m_c}}\frac{4\delta^{ij}}{3\sqrt{3}}(4\pi\alpha_s)\frac{F_D^V(k^2)}{k^2(s-m_{bb}^2)}\bar{U}(p)\gamma_5(\gamma^\alpha + \frac{p^\alpha}{M}) \times$$
$$\times\gamma^\alpha V(q')(-g_{\alpha\beta} + \frac{k^\alpha n^\beta + k^\beta n^\alpha}{(kn)})V_{\mu\nu\beta}^{VVg}(-g_{\lambda\nu} + \frac{q^\lambda q^\nu}{s})G_\lambda \tag{7}$$

$$\mathcal{M}((bb)_1 \to \Omega_{bbc}^* + \overline{c}) = \frac{\Psi_{\Omega_{bbc}^*}(0)}{2\sqrt{m_c}}\frac{4\delta^{ij}}{3\sqrt{3}}(4\pi\alpha_s)\frac{F_D^V(k^2)}{k^2(s-m_{bb}^2)}\bar{\Psi}_\alpha(p)\gamma^\alpha V(q') \times$$
$$\times(-g_{\alpha\beta} + \frac{k^\alpha n^\beta + k^\beta n^\alpha}{(kn)})V_{\mu\nu\beta}^{VVg}(-g_{\lambda\nu} + \frac{q^\lambda q^\nu}{s})G_\lambda \tag{8}$$

$$\mathcal{M}((bc)_0 \to \Omega^{'}_{bcc} + \bar{c}) = \frac{\Psi_{\Omega_{bcc}}(0)}{\sqrt{2m_c}} \frac{4\delta^{ij}}{3\sqrt{3}} (4\pi\alpha_s) \frac{F^S_D(k^2)}{k^2(s - m^2_{bc})} \bar{U}(p_q)\gamma^\alpha V(q^{'}) \times$$
$$\times(-g_{\alpha\beta} + \frac{k^\alpha n^\beta + k^\beta n^\alpha}{(kn)})(q + (1 - r)p)_\beta G, \tag{9}$$

where G is the diquark production vertex, $\bar{U}(p)$ is the spin-$\frac{1}{2}$ baryon spinor, $\bar{\Psi}_\alpha(p)$ is the Rarita-Schwinger spinor for the spin-$\frac{3}{2}$ baryon, k is the four-vector of virtual gluon.

Omitting the next details, we write here the obtained analytical result for the fragmentation function of the process $(bc)_0 \to \Omega'_{bcc}$:

$$D_{(bc)_0\to\Omega'_{bcc}}(z, \mu_0) = \frac{32|\Psi_{\Omega_{bcc}}|^2\alpha_s^2}{81M^3r^2\bar{r}k_0^6} \frac{1}{(1 - \bar{r}z)^6(rM^2(1 - \bar{r}z)^2 - z(1 - z)k_0^2)} \times$$
$$\Bigg(6rM^2 \ln[\frac{rM^2(1 - \bar{r}z)^2}{zk_0^2(z - 1) + rM^2(1 - \bar{r}z)^2}]((z - 1)zk_0^2 + rM^2(1 - \bar{r})^2) \times$$
$$(2(z - 1)zk_0^2 + 3M^2\bar{r}r(1 - \bar{r}z)(r - ((3 + \bar{r}) + 1)z + 1)k_0^2 + 16M^4r^3\bar{r}^2 \times \tag{10}$$
$$(1 - z\bar{r})^2)(1 - \bar{r}z)^4 - k_0^2(z - 1)z((z - 1)^2z^2((r(1 + 3\bar{r}) + 3)z^2)(z - 1)^2 -$$
$$-2(r + 3)z\bar{r}^2 + 3(r^2 + 1))k_0^6 - rM^2(z - 1)z(1 - \bar{r}z)^2((r - 3)\bar{r}^2(9r + 7)z^2 +$$
$$-2\bar{r}((r - 10)r - 21)z + 3(3r^2 - 7))k_0^4 + 6\bar{r}r^2M^4(1 - \bar{r}z)(\bar{r}(3 - r(3 -$$
$$-r(3r - 20))z^2 + 2r(7r - 10)z - 6z + 3r + 3)k_0^2 + 96\bar{r}^2r^4M^6(1 - \bar{r}z)^6),$$

where $\mu_0 = m_{(bc)} + 2m_c$, $\Psi_{\Omega_{bcc}}(0)$ is the Ω_{bcc} wave function at the origin in the quark-diquark approximation, $\bar{r} = 1 - r$, $r = m_c/M$, $M = m_{(bc)} + m_c$. If the picked quark is light ($m_q << m_{c,b}, r = m_q/M$), the diquark form factor approximately equals unity and fragmentation function may be written as follows (see, also, [11]):

$$D_{(bc)_0\to\Omega'_{bcq}}(z, \mu_0) = \frac{32|\Psi_{\Omega_{bcq}}(0)|^2\alpha_s^2}{81M^3r^2\bar{r}} \frac{z^2(1 - z)^2}{(1 - \bar{r}z)^6} \cdot \Bigg(3 + 3r^2 -$$
$$-(6 - 10r + 2r^2 + 2r^3)z + (3 - 10r + 14r^2 - 10r^3 + 3r^4)z^2\Bigg). \tag{11}$$

However, there is no evidence in this case to use correctly the value of wave function at the origin $\Psi_{\Omega_{bcq}}(0)$, which is obtained by solving nonrelativistic Schredinger equation. Such a way our prediction in the case of doubly heavy baryons has arbitrary normalization.

The fragmentation functions for the vector diquark are calculated in the same way. We don't write analytical formulae here because they are unwieldy. The fragmentation probabilities

$$P_{(QQ')\to H} = \int_0^1 D_{(QQ')\to H}(z, \mu)dz$$

does not evolve with scale μ at leading order in α_s. The obtained values of probabilities for different baryons are presented in the Table 2.

The fragmentation function $D_{(QQ')\to H}(z, \mu)$ satisfies the DGLAP evolution equation [11]:

$$\mu\frac{\partial}{\partial\mu}D_{(QQ')\to H}(z, \mu) = \int_z^1 \frac{dy}{y} P_{(QQ')\to(QQ')}(\frac{z}{y}, \mu)D_{(QQ')\to H}(y, \mu), \tag{12}$$

where

$$P_{(QQ')\to(QQ')}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{2x}{1-x}\right]_+ ,$$

$$f(x)_+ = f(x) - \delta(1-x)\int_0^1 f(x')dx'.$$

The QCD evolution reduces the average fraction of the baryon momentum $< z >$. We have obtained for Ω_{bbc}-baryon that $< z >= 0.84$ at the $\mu = \mu_0$ and $< z >= 0.73$ at the $\mu = M_Z/2$. This result is independent of the baryon spin.

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References

[1] S. Fleck and J. Richard, Prog.Theor.Phys. 82 (1989) 760

[2] E. Bagan et al., Z.Phys. C64 (1994) 57

[3] V.V. Kiselev et al., Phys.Lett. B332 (1994)411 ; S.S. Gershtein et al. Phys.Rev. D63 (2000) 054021

[4] D. Ebert et al., Z.Phys. C76 (1997) 111

[5] C. Peterson, Phys.Rev. D27 (1983) 105

[6] A.P. Martynenko and V.A. Saleev, Phys.Lett. B385 (1996) 296

[7] V.A. Saleev, Phys.Lett. B426 (1998) 384

[8] A.D. Adamov, G.R. Goldstein, Phys.Rev. D56 (1997) 7381

[9] V.A. Saleev, Mod.Phys.Lett. A14 (1999) 2615

[10] E. Braaten, K. Cheung, T.C. Yan, Phys.Rev. 48 (1993) 4230

[11] V.V. Kiselev, Phys.Rev. D58 (1998) 054008 ; V.V.Kiselev and A.E. Kovalsky, Phys.Atom.Nucl. 63 (2000) 1640

[12] V.N. Gribov and L.N. Lipatov, Sov.J.Nucl.Phys. 15 (1972) 438 ; Yu.A. Dokshitser, Sov.Phys.JETP, 46 (1977) 641 ; G. Altarelli and G. Parisi, Nucl.Phys., B126 (1977) 298

Table 1.

Diquark	$\sqrt{< r^2_{QQ} >}$, fm	k^2_o, GeV
$(bb)_1$	0.33	4.39
$(bc)_1$	0.48	2.09
$(bc)_0$	0.48	1.04
$(cc)_1$	0.58	1.40

Table 2.

Diquark	Baryon	$\|\Psi_H(0)\|^2$ GeV^3	$P_{(QQ)\to H}$
$(bb)_1$	Ω_{bbc}	0.16	$1.5 \cdot 10^{-6}$
$(bb)_1$	Ω^*_{bbc}	0.16	$3.2 \cdot 10^{-6}$
$(bc)_1$	Ξ_{bcc}	0.15	$3.1 \cdot 10^{-8}$
$(bc)_1$	Ξ^*_{bcc}	0.15	$6.9 \cdot 10^{-8}$
$(bc)_0$	$\Xi^{'}_{bcc}$	0.15	$9.2 \cdot 10^{-7}$

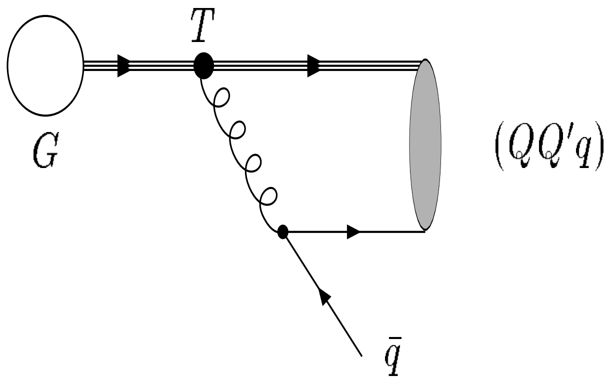

Figure 1: The diagram used for description of the doubly heavy diquark to split into baryon.

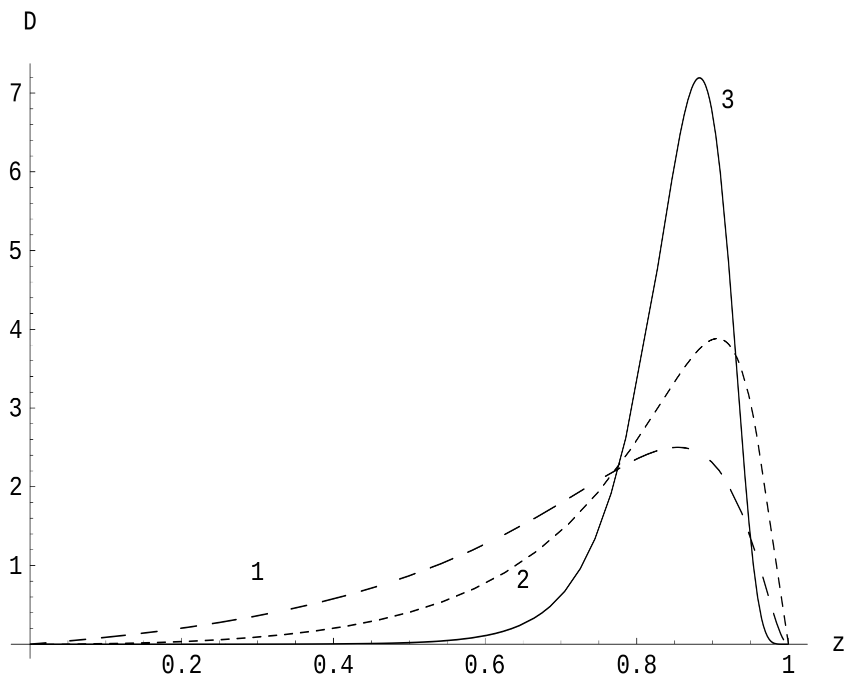

Figure 2: The fragmentation function $D_{(bc)_0}(z, \mu)$ normalized to unity at $\mu = \mu_0$. The curve 1 is Peterson's parameterization [5], the curve 2 is our result with $F_s = 1$ and curve 3 is our result with diquark form factor (3).

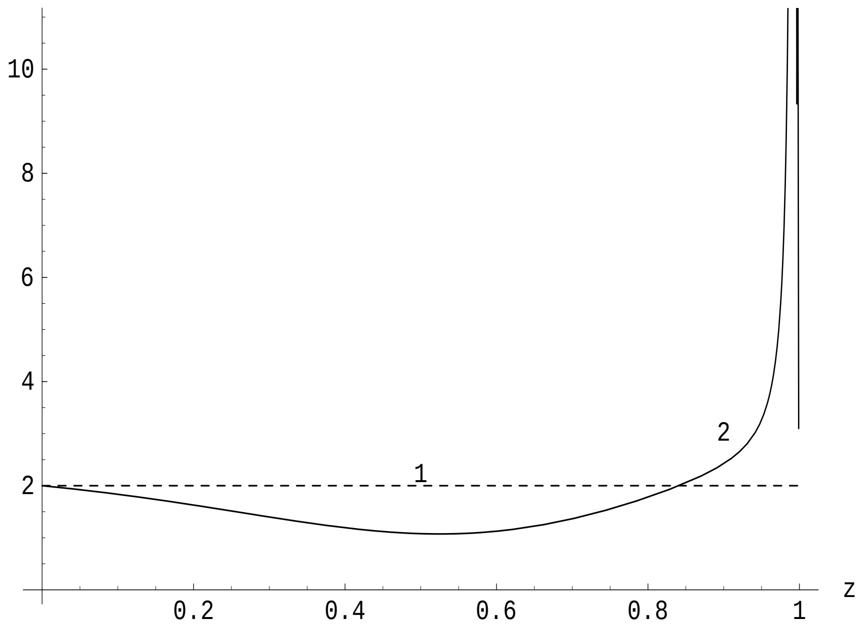

Figure 3: The ratio of the fragmentation functions $D_{(bb)\to\Omega^*_{bbc}}(z, \mu)/D_{(bb)\to\Omega_{bbc}}(z, \mu)$ at $\mu = \mu_0$.