Semileptonic Decays of the Bottom-Charm Hadrons

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Abstract

We present the results of a study of the semileptonic decays of the B_c-meson and the lowest lying doubly heavy baryons using the relativistic quark model. We do not employ a heavy quark mass expansion, but keep the masses of the heavy quarks and hadrons finite. We calculate all relevant form factors and decay rates.

The semileptonic decays of heavy mesons and baryons are ideally suited to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The heaviest flavored bottom-charm B_c-meson was observed by the CDF Collaboration [1] in the analysis of the decay mode $B_c \to J/\psi \bar{l}\nu$. The discovery of the B_c-meson raises hopes that doubly heavy flavored baryons will also be discovered in the near future. The theoretical treatment of the systems with two heavy quarks is complicated by the fact that one cannot make use of an expansion in terms of the inverse heavy quark masses. Previously, non-relativistic potential models, diquark approximation, QCD sum rules and non-relativistic QCD have been used to describe the spectroscopy of doubly heavy baryons and to estimate the inclusive and some exclusive decay modes of such systems (for review, see [2]-[5] and references therein).

We present here our recent results [6, 7] of exploration of the semileptonic decays of the doubly heavy B_c-meson and the lowest lying doubly heavy Ξ-baryons within a relativistic constituent quark model. This model [8] can be viewed as an effective quantum field theory approach based on an interaction Lagrangian of hadrons interacting with their constituent quarks. Universal and reliable predictions for exclusive processes involving both mesons composed from a quark and antiquark and baryons composed from three quarks result from this approach. The coupling strength of hadrons H to their constituent quarks is determined by the compositeness condition $Z_H = 0$ [9, 10] where Z_H is the wave function renormalization constant of the hadron. The quantity $Z_H^{1/2}$ is the matrix element between a physical particle state and the corresponding bare state. The compositeness condition $Z_H = 0$ enables us to represent a bound state by introducing a hadronic field interacting with its constituents so that the renormalization factor is equal to zero. This does not mean that we can solve the QCD bound state equations but we are able to show that the condition $Z_H = 0$ provides an effective and self-consistent way to describe the coupling of the particle to its constituents. One starts with an effective interaction Lagrangian written down in terms of quark and hadron variables. Then, by using Feynman rules, the S-matrix elements describing hadron-hadron interactions are given in terms of a set of quark diagrams. In particular, the compositeness condition enables one to avoid the double counting of quark and hadron degrees of freedom. This approach is self-consistent and all calculations of physical observables are straightforward. There is a small set of model parameters: the values of the constituent quark masses and the scale

parameters that define the size of the distribution of the constituent quarks inside a given hadron. The shapes of the vertex functions and the quark propagators can in principle be determined from an analysis of the Bethe-Salpeter (Fadde'ev) and Dyson-Schwinger equations, respectively, as done e.g. in [11, 12]. In the present paper we, however, choose a more phenomenological approach where the vertex functions are modeled by Gaussian forms and the quark propagators are given by local representations. We have demonstrated in our papers [13] that the relativistic constituent model is consistent with the heavy quark symmetry in the limit of infinite quark masses.

We start with the effective interaction Lagrangian which describes the coupling between hadrons and their constituent quarks. For example, the couplings of the bottom-charm hadrons into their constituents are given by

$$\begin{aligned}
\mathcal{L}_{\rm int}(x) &= g_{B_c}\, B_c(x)\, J_{B_c}(x) + \Big(g_{\Xi_{bc}}\, \bar{\Xi}_{bc}(x)\, J_{\Xi_{bc}}(x) + {\rm h.c.}\Big)\ , \qquad (1)\\
J_{B_c}(x) &= \int dx_1 \int dx_2 \Phi_{B_c}(x;x_1,x_2)\ \Big(\bar{b}(x_1) i\gamma^5 c(x_2)\Big)\ ,\\
J_{\Xi_{bc}}(x) &= \int dx_1 \int dx_2 \int dx_3\ \Phi_{\Xi_{bc}}(x;x_1,x_2,x_3)\, \gamma^\mu\gamma^5 b_{a_1}(x_1)\, (c_{a_2}(x_2)\, C\gamma^\mu\, q_{a_3}(x_3))\, \varepsilon^{a_1a_2a_3}\ ,
\end{aligned}$$

where $q = u, d$. The vertex function Φ_H is taken to be invariant under the translation $x \to x + a$ which guarantees Lorentz invariance for the interaction Lagrangian Eq. (1).

The matrix elements of the weak current $\bar{b}O^\mu c$ are written in our approach as

$$\begin{aligned}
< \eta_c(p')|\bar{b}O^\mu c|B_c(p) > &= 3 g_{B_c} g_{\eta_c} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_{B_c}(-k^2) \tilde{\Phi}_{\eta_c}(-k^2) \qquad (2)\\
&\quad \cdot {\rm tr}\Big[S_c(\not{k} + \not{p}') O^\mu S_b(\not{k} + \not{p}) \gamma^5 S_c(\not{k}) \gamma^5 \Big]\\
&= f_+(q^2)\ (p + p')^\mu\ +\ f_-(q^2)\ (p - p')^\mu\ ,\\
< J/\psi(p'), \epsilon^*|\bar{b}O^\mu c|B_c(p) > &= 3 g_{B_c} g_{J/\psi} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_{B_c}(-k^2) \tilde{\Phi}_{J/\psi}(-k^2) \qquad (3)\\
&\quad \cdot {\rm tr}\Big[S_c(\not{k} + \not{p}') O^\mu S_b(\not{k} + \not{p}) \gamma^5 S_c(\not{k})\ \not{\epsilon}^* \Big]\\
&= -\epsilon^{*\mu}(m_{B_c} + m_{J/\psi})\, A_1(q^2) + (p + p')^\mu\, p\cdot\epsilon^*\, \frac{A_2(q^2)}{m_{B_c} + m_{J/\psi}} +\\
&\quad + (p - p')^\mu\, p\cdot\epsilon^*\, \frac{A_3(q^2)}{m_P + m_V} - i\varepsilon^{\mu\nu\alpha\beta}\epsilon^{*\nu} p^\alpha p'^{\,\beta}\, \frac{2\ V(q^2)}{m_P + m_V}\ ,\\
< \Xi_{cc}(p')|\bar{b}O^\mu c|\Xi_{bc}(p) > &= 12 g_{\Xi_{cc}} g_{\Xi_{bc}} \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i}\, \tilde{\Phi}_{\Xi_{cc}}(-K^2)\, \tilde{\Phi}_{\Xi_{bc}}(-K^2)\\
&\quad \cdot \gamma^\alpha\gamma^5\, S_q(k_2)\, \gamma^\beta\, S_c(k_1 + k_2)\, \gamma^\alpha\, S_c(k_1 + p')\, O^\mu\, S_b(k_1 + p)\, \gamma^\beta\gamma^5\ ,\\
&= \gamma^\mu\, (F_1^V - F_1^A\, \gamma^5)\ +\ i\sigma^{\mu\nu} q^\nu\, (F_2^V - F_2^A\, \gamma^5)\ +\ q^\mu\, (F_3^V - F_3^A\, \gamma^5)
\end{aligned}$$

where $K^2 \equiv k_1^2 + (k_1+k_2)^2 + k_2^2$. The quark propagator is chosen to have a local form

$$S_i(k) = \frac{1}{m_i - \not{k}} \qquad (i = u, d, s, c, b) \tag{4}$$

with m_i being a constituent quark mass. We assume that $m_H < \sum_{i=1}^{n} m_{q_i}$ (n=2 for mesons and n=3 for baryons) in order to avoid the appearance of imaginary parts in the physical amplitudes. This is a reliable approximation for the heavy pseudoscalar mesons. The above condition is not always met for heavy vector mesons. We shall therefore take equal masses for the heavy pseudoscalar and vector mesons in our form factor calculations, but use physical masses for the phase space.

Generally, $\tilde{\Phi}_H$ is a function of external momenta, however, in the impulse approximation employed in our approach, we assume that it only depends on the sum of relative momentum squared.

The coupling constants g_H are determined by the so called *compositeness condition* proposed in [9] and extensively used in [10]. The compositeness condition means that the renormalization constant of the meson field is equal to zero $Z_H = 1 - \tilde{\Pi}'_H = 0$, where $\tilde{\Pi}'_H$ is the derivative of the mass function. The meson-mass functions are defined as

$$\Pi_P(p^2) = \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P^2(-k^2) \mathrm{tr}\Big[\gamma^5 S_3(\not{k}) \gamma^5 S_1(\not{k} + \not{p})\Big] , \tag{5}$$

$$\Pi_V(p^2) = \frac{1}{3}\Big[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\Big] \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_V^2(-k^2) \mathrm{tr}\Big[\gamma^\mu S_3(\not{k}) \gamma^\nu S_1(\not{k} + \not{p})\Big] . \tag{6}$$

In the baryon case the compositeness condition may be rewritten in a form suitable for determination of the coupling constants:

$$-12\, g_{q_1q_2q_3}^2 \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i}\, \tilde{\Phi}^2_{q_1q_2q_3}(-k^2)\, D^\mu_{q_1q_2q_3} \,|_{\not{p}=m_H} \;=\; \gamma^\mu \,, \tag{7}$$

$$D^\mu_{q_1q_2q_3} = \gamma^\alpha\gamma^5\, S_{q_1}(k_1+p)\, \gamma^\mu\, S_{q_1}(k_1+p)\gamma^\beta\gamma^5\, \mathrm{tr}\left(S_{q_2}(k_1+k_2)\, \gamma^\alpha\, S_{q_3}(k_2)\, \gamma^\beta\right) ,$$

$$(q_1q_2q_3) = (bcq)\,, (bsq)\,, (csq)\,,$$

$$D^\mu_{ccq} = \gamma^\alpha\gamma^5\, S_q(k_2)\, \gamma^\beta\gamma^5\, \mathrm{tr}\left(\gamma^\alpha\, S_c(k_1+p)\, \gamma^\mu\, S_c(k_1+p)\, \gamma^\beta\, S_c(k_1+k_2)\right) .$$

The calculational techniques are outlined in Refs. [6, 7].

Before presenting our numerical results we need to specify our values for the constituent quark masses and shapes of the vertex functions. As concerns the vertex functions, we found a good description of various physical quantities [6, 8, 13] adopting a Gaussian form. Here we apply the same procedure using $\tilde{\Phi}_H(k_E^2) = \exp\{-k_E^2/\Lambda_H^2\}$ in the Euclidean region. The magnitude of Λ_H characterizes the size of the vertex function and is an adjustable parameter in our model. Parameters Λ_H in the meson sector were determined [6] by a minimal-squares fit to experimental data and lattice results. The quality of the fit may be seen from Table 1 for the leptonic decay constants. The nucleon Λ_N parameter was determined from the best description of the electromagnetic properties of the nucleon [8]. Parameters Λ_H for baryons with one heavy quark (bottom or charm) are determined by

analyzing available experimental data on bottom and charm baryon decays [13]. Since there is no experimental information on the properties of doubly heavy baryons yet, we use the simple observation that the magnitude of Λ_H is increasing with the mass of the hadron whose shape it determines. Keeping in mind that $\Lambda_N = 1.25$ GeV, $\Lambda_{Qqq} = 1.8$ GeV and $\Lambda_{B_c} = 2.43$ GeV, we simply choose the value of $\Lambda_{QQq} = 2.5$ GeV for the time being. We found that variations of this value by 10 % does not significantly affect the values of form factors. We use the same values for the quark masses (see, Eq.(8)) as have been used previously for the description of light and heavy mesons (baryons) [8, 13]. Note that the values of the light quark masses in the meson case are different from those in the baryon case as a result of the lack of confinement in our approach.

We thus use (in GeV)

m_u	m_s	m_c	m_b
(0.235) 0.420	(0.333) 0.570	1.67	5.06

(8)

Λ_π	Λ_K	Λ_D	Λ_{D_s}	$\Lambda_{J/\psi}$	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_Υ
1.16	1.82	1.87	1.95	2.12	2.16	2.27	2.43	4.425

(9)

The resulting form factors are *approximated* by the interpolating form

$$f(q^2) = \frac{f(0)}{1 - a_1 q^2 + a_2 q^4} \tag{10}$$

It is interesting that for most of the form factors the numerical fit for values of a_1 and a_2 obtained from the interpolating form (10) are such that the form factors can be represented by monopole function in the case of B_c-meson and dipole formula in the case of doubly heavy baryons:

$$f(q^2) \approx \frac{f(0)}{(1 - q^2/m_V^2)^n} \qquad (n = 0, 1). \tag{11}$$

The values of m_V in this representation are very close to the values of the appropriate lower-lying $(\bar{q}q')$ vector mesons ($m_{D_s^*}$=2.11 GeV for (c-s)-transitions and $m_{B_c^*} \approx m_{B_c}$=6.4 GeV for (b-c)-transitions). In Fig.1 we show two representative form factors and their dipole approximations. It is gratifying to see that our relativistic quark model with the Gaussian vertex function and free quark propagators reproduces the monopole in the meson case and the dipole in the baryon case for most of the form factors.

Finally, in Table 2 we present our predictions for the branching ratios of the semileptonic B_c-decay rates and compare them with other approaches. In Table 3 the predictions for the decay widths of the doubly heavy Ξ-baryons are given. We compare them with the free quark decay widths. One notes that the rates for the exclusive modes $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$ are rather small when compared with the total semileptonic inclusive rates. The remaining part of the inclusive rate would have to be filled in by decays into excited or multi-body baryonic states. Note that the smallness of the exclusive/inclusive ratio of the above exclusive modes markedly differs from that of the mesonic semileptonic $b \to c$ transitions, where the exclusive transitions to the ground state S-wave mesons $B \to D, D^*$ make up approximately 66% of the total semileptonic $B \to X_c$ rate [21].

For $\Lambda_b \to \Lambda_c$ transitions one expects even higher semileptonic exclusive-inclusive ratio of amount 80% [14]. Note that the rate for $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ is of the same order of magnitude as the rates calculated for the corresponding double heavy mesonic decays $B_c \to \eta_c + l\bar{\nu}$ and $B_c \to J/\Psi + l\bar{\nu}$ [6]. The QCD sum rule and potential model predictions for the rates of $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{cs} + l\bar{\nu}$ given in [3] exceed our rate predictions by factors of 10 and 3 respectively. In fact, the exclusive semileptonic rates given in [3] tend to saturate the inclusive semileptonic rates.

In Table 4 we present values for the invariant form factors at $q^2_{\rm min} = 0$ and $q^2_{\rm max} = (m_i - m_f)^2$. Note that the values of the axial vector form factor F_1^A are rather small for the two decays $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$. This provides for a partial explanation of why the rates of these two modes are small compared to the inclusive semileptonic rate. Also the zero recoil values of the vector form factors are significantly below the value of one which one would expect from a naive application of the heavy quark limit. The smallness of the vector form factors provide for the remaining explanation of the smallness of the predicted respective rates. We mention that the QCD sum rule and potential model estimates of the zero recoil values of both the vector and axial vector form factors F_1^V and F_1^A given in [3] are close to one. In the model of [3] the form factors F_2^V and F_2^A are set to zero. In our approach we find that the numerical values of F_2^V and F_2^A are quite small compared to those of F_1^V and F_1^A in all cases when expressed in terms of the mass scale $(m_i + m_f)$.

References

[1] F. Abe *et al.* CDF Collaboration, Phys. Rev. Lett. 81 (1998) 2432;
Phys. Rev. D58 (1998) 112004

[2] S.S. Gershtein *et al.*, "Theoretical status of the B_c meson", hep-ph/9803433;
in Proc. "Progress in Heavy Quark Physics", eds.: M. Beyer, T. Mannel, H. Schroder, (University of Rostock, Germany), 1998, p. 272.

[3] V.V. Kiselev and A.K. Likhoded, "Baryons with two heavy quarks", hep-ph/0103169.

[4] J.G. Körner, M. Krämer, D. Pirjol, Prog. Part. Nucl. Phys. 33 (1994) 787

[5] B. Guberina, B. Melic, H. Stefancic, Eur. Phys. J. C9 (1999) 213

[6] M.A. Ivanov, J.G. Körner, P. Santorelli, Phys. Rev. D63 (2001) 074010

[7] A. Faessler, Th. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij,
Phys. Lett. B518 (2001) 55

[8] M.A. Ivanov, M.P. Locher, V.E. Lyubovitskij, Few-Body Syst. 21 (1996) 131; M.A. Ivanov and V.E. Lyubovitskij, Phys. Lett. B408 (1997) 435;
M.A. Ivanov and P. Santorelli, Phys. Lett. B456 (1999) 248

[9] A. Salam, Nuovo Cim. 25 (1962) 224; S. Weinberg, Phys. Rev. 130 (1963) 776;
K. Hayashi et al., Fort. Phys. 15(1967) 625

[10] G.V. Efimov and M.A. Ivanov, Int. J. Mod. Phys. A4 (1989) 2031;
"The Quark Confinement Model of Hadrons", IOP, London, 1993.

[11] C.D. Roberts and A.G. Williams, Prog. Part. Nucl. Phys. 33 (1994) 477

[12] M.A. Ivanov, Yu.L. Kalinovsky, C.D. Roberts, Phys. Rev. D60 (1999) 034018

[13] M.A. Ivanov, V.E. Lyubovitskij, J.G. Körner, P. Kroll, Phys. Rev. D56 (1997) 348;
M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, A.G. Rusetsky,
Phys. Rev. D57 (1998) 5632; D60 (1999) 094002; Phys. Lett. B476 (2000) 58

[14] J.G. Körner and B. Melic, Phys. Rev. D62 (2000) 074008

[15] V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko, Nucl. Phys. B259 (2000) 473; V.V.
Kiselev, A.E. Kovalsky, A.K. Likhoded, Nucl. Phys. B585 (2000) 353

[16] A. Abd El-Hady, J. H. Muñoz and J.P. Vary, Phys. Rev. D62 (2000) 014019

[17] A.Yu. Anisimov, P.Yu. Kulikov, I.M. Narodetskii, K.A. Ter-Martirosyan,
Phys. Atom. Nucl. 62 (1999) 1739

[18] M.A. Nobes and R.M. Woloshin, J. Phys. C26 (2000) 1079

[19] C.-H. Chang and Y.-Q. Chen, Phys. Rev. D49 (1994) 3399

[20] P. Colangelo and F. De Fazio, Phys. Rev. D61 (2000) 034012

[21] Particle Data Group, D.E. Groom *et al.* Eur. Phys. J. C15 (2000) 1

Table 1: Leptonic decay constants f_H (MeV) used in the least-squares fit.

Meson	This model	Expt./Lattice
π^+	131	$130.7 \pm 0.1 \pm 0.36$
K^+	160	$159.8 \pm 1.4 \pm 0.44$
D^+	191	191^{+19}_{-28}
D_s^+	206	206^{+18}_{-28}
B^+	172	172^{+27}_{-31}
B_s^+	196	$171 \pm 10^{+34+27}_{-9-2}$
B_c	360	360
J/ψ	404	405± 17
Υ	711	710±37

Table 2: Branching ratios BR(%) for the semileptonic decays $B_c^+ \to Hl^+\nu$, calculated with the CDF central value $\tau(B_c) = 0.46$ ps [1].

H	This model	[15]	[16]	[19]	[20]	[17]	[18]
$\eta_c\, e\, \nu$	0.98	0.8±0.1	0.78	1.0	0.15(0.5)	0.6	0.52
$\eta_c\, \tau\, \nu$	0.27						
$J/\psi\, e\, \nu$	2.30	2.1±0.4	2.11	2.4	1.5(3.3)	1.2	1.47
$J/\psi\, \tau\, \nu$	0.59						
$D^0\, e\, \nu$	0.018		0.003	0.006	0.0003(0.002)		
$D^0\, \tau\, \nu$	0.0094						
$D^{*0}\, e\, \nu$	0.034		0.013	0.019	0.008(0.03)		
$D^{*0}\, \tau\, \nu$	0.019						
$B^0\, e\, \nu$	0.15		0.08	0.16	0.06(0.07)		
$B^{*0}\, e\, \nu$	0.16		0.25	0.23	0.19(0.22)		
$B_s^0\, e\, \nu$	2.00	4.0	1.0	1.86	0.8(0.9)	1.0	0.94
$B_s^{*0}\, e\, \nu$	2.6	5.0	3.52	3.07	2.3(2.5)		1.44

Table 3: Calculated decay widths of lowest lying $J^P = 1/2^+$ doubly heavy Ξ-baryons. Inclusive widths are calculated using the current quark pole masses.

Mode	Decay widths, ps^{-1}	
	RTQM	Inclusive width
$\Xi_{\mathrm{bc}} \to \Xi_{\mathrm{cc}} + l\,\bar{\nu}$	0.012	$2 \cdot \Gamma_0(b \to c) = 0.162$
$\Xi_{\mathrm{bc}} \to \Xi_{\mathrm{bs}} + l\,\bar{\nu}$	0.043	$\Gamma_0(c \to s) = 0.122$
$\Xi_{\mathrm{cc}} \to \Xi_{\mathrm{cs}} + l\,\bar{\nu}$	0.224	$2 \cdot \Gamma_0(c \to s) = 0.244$

Table 4: Values of F_1^V and F_1^A form factors at maximum and zero recoil.

	$\Xi_{bc} \to \Xi_{cc}$		$\Xi_{bc} \to \Xi_{bs}$		$\Xi_{cc} \to \Xi_{cs}$	
	F_1^V	F_1^A	F_1^V	F_1^A	F_1^V	F_1^A
$q^2 = 0$	0.46	-0.091	0.39	0.061	0.47	0.61
$q^2 = q^2_{\mathrm{max}}$	0.83	-0.086	0.58	0.065	0.59	0.77

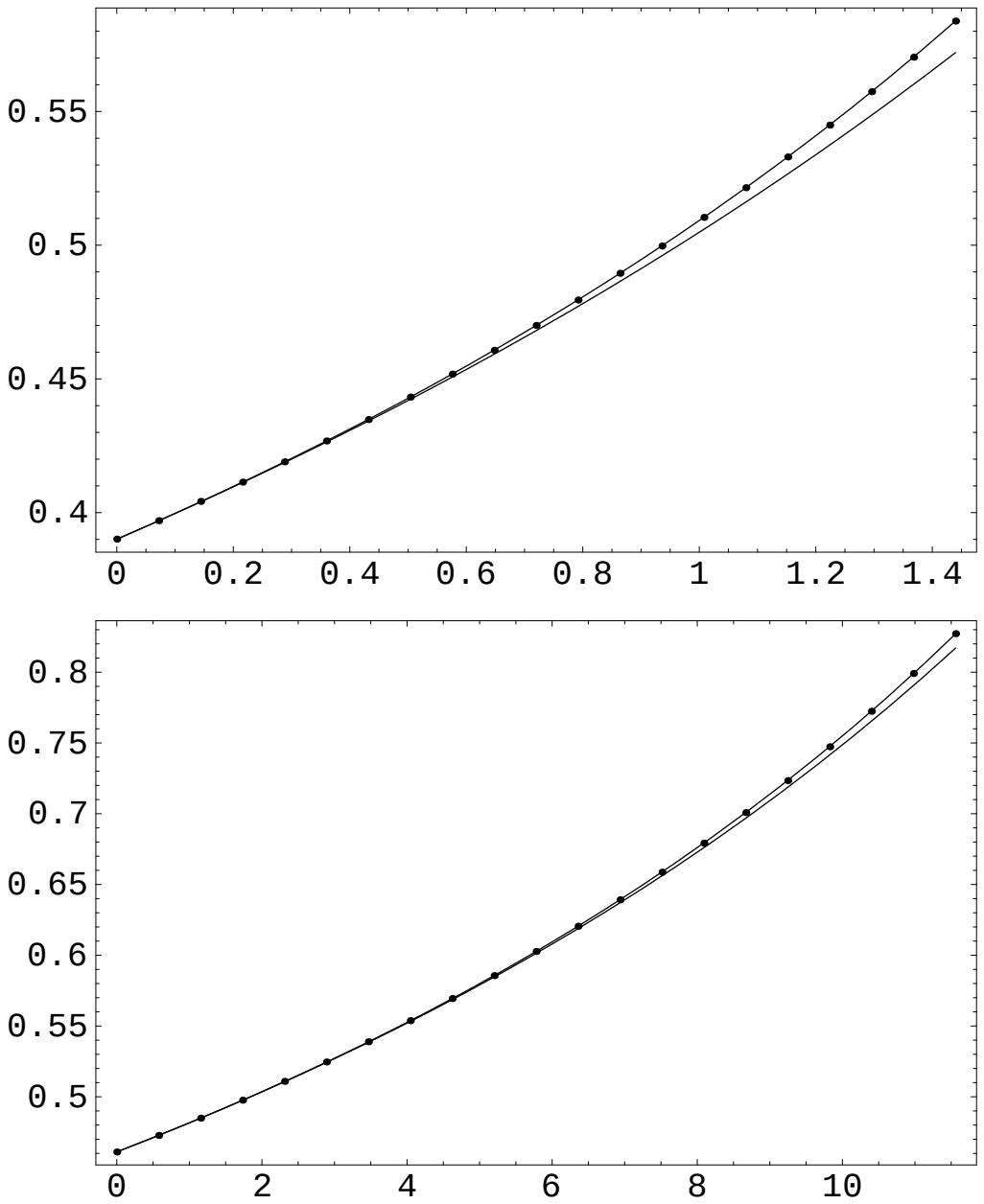

Figure 1: Upper panel: Form factor $F_1^V(q^2)$ (solid-dotted line) for $bc \to bs$ transitions and its dipole approximation (solid line) with $m_{cs} = 2.88$ GeV; Lower panel: Form factor $F_1^V(q^2)$ (solid-dotted line) for $bc \to cc$ transitions and its dipole approximation (solid line) with $m_{bc} = 6.81$ GeV.