Application of the Formalism of Two-Point Green's Functions for the Study of Bremsstrahlung in Matter

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Abstract

On a basis of the formalism of two-particle Green's functions in matter bremsstrahlung by high energy particles in a medium is studied. In the case of multiple elastic scattering of the particles in the static matter the probability of bremsstrahlung photons is calculated. It is shown the new mode of the formation of bremsstrahlung spectrum takes place as compared with one has been considered by A.B.Migdal in the study of the Landau-Pomeranchuk. In this mode when the scattering in the matter is rather strong the emission spectrum depends sufficiently on the time spent by the particle before the photon emission. It results in the rearrangement of the spectrum in the short wave region and leads to the nonlinear dependence of spectral distribution of the particle emission on the time spent by the particle in the matter.

1 Introduction

The influence of scattering in the matter on bremsstrahlung has been studied by L.D.Landau and I.Y.Pomeranchuk [1,2] for the first time. It was pointed to the suppression of the bremsstrahlung intensity due to the multiple elastic scattering of ultrarelativistic particles in the matter (the Landau-Pomeranchuk effect) in those papers. The quantitative theory of this effect has been derived by A.B.Migdal (the LPM effect) in papers [3,4]. The theory of the LPM effect has been developed further during research of the dispersion properties of the scattering matter [5], its boundaries [6], the Columb scattering of particles in the matter [7] etc. [1] But the results obtained in the papers above mentioned are restricted to the neglect of the influence of the multiple scattering of particles in the matter on photon production during the period preceding the photon emission. Such simplification shows itself as the linear dependence of the bremsstrahlung intensity on the observation time [1-8]. Besides it still has been no quantitative estimations of the main approximation used in the papers [3,4] which is the factorization of the two-time distribution function.

On a basis of the formalism of two-particle Green's function in non-equilibrium matter [9] the production probability of photons by high energy particles scattered multiply elastically in the matter is obtained. When the scattering in the medium is rather weak the spectrum coincides with one obtained in [3,4]. In the opposite case of the strong scattering in the matter the another mode of the bremsstrahlung formation appear what leads to the rearrangement of the emission spectrum. It manifests itself in the dependence

[1] The detailed review of both the pointed out papers and other methods of the calculation of the LPM effect is given in the paper [8].

of the probability of the photon production on the problem parameters such as the photon energy, the observation time, the energy of the particle. Particular, the probability becomes the significantly nonlinear function of the observation time and the photon rate appears to be strongly suppressed in the short wave region of the spectrum as compared with the spectral distribution obtained in Ref.[3,4].

2 Bremsstrahlung by high energy particles in the matter

Let us consider fast ($E \gg \omega$; ω is the photon energy) ultrarelativistic particles ($m \ll E$; m is the particle mass, E is its energy) which undergo multiple elastic collisions in the static matter.

The probability of photon production is given by the expression in such case [9] :

$$dW = \frac{\alpha}{4\omega\pi^2 E^2} \int_{-T}^{T} dt_1 \int_{-T}^{T} dt_2 \left(\vec{k} \times \vec{p}\,'\right) \cdot \left(\vec{k} \times \vec{p}\right) \exp\left(-i\omega t_1 + i\omega t_2\right)$$
$$F_{\vec{k}}(\vec{p}; \vec{p}\,'; t_1; t_2) d^3\vec{k} \tag{1}$$

where $\vec{p}$ and $\vec{p}\,'$ are the particle momenta at moments t_1 and t_2, respectively, ω and $\vec{k}$ are the energy and momentum of the photon; T is the observation time (the time spent by the particle in the matter); α is the fine structure constant; $F_{\vec{k}}(\vec{p}; \vec{p}\,'; t_1; t_2)$ is the two-particle Green's functions. It satisfies the following equations [9] :

$$\left(\frac{\partial}{\partial t_1} - i\vec{k}\vec{v}\right) F_{\vec{k}}(\vec{p}\,'; \vec{p}\,'; t_2; \vec{p}; \vec{p}; t_1) = \frac{\pi n m}{E(\vec{p})} \cdot \int d^3\vec{q} |V(\vec{q})|^2 \cdot$$
$$\Big\{ \Big(F_{\vec{k}}(\vec{p}\,'; \vec{p}\,'; t_2; \vec{p} + \vec{q}; \vec{p} + \vec{q}; t_1) - F_{\vec{k}}(\vec{p}\,'; \vec{p}\,'; t_2; \vec{p}; \vec{p}; t_1) \Big)$$
$$\cdot \delta(E(\vec{p} + \vec{q}) - E(\vec{p})) + \Big[F_{\vec{k}}(\vec{p}\,'; \vec{p}\,' + \vec{q}; \vec{p} + \vec{q}; t_2; \vec{p}; t_1) \cdot$$
$$\delta(E(\vec{p}\,') - E(\vec{p}\,' + \vec{q})) - F_{\vec{k}}(\vec{p}\,'; \vec{p}\,' - \vec{q}; t_2; \vec{p}; \vec{p} + \vec{q}; t_1)$$
$$\cdot \delta(E(\vec{p}\,') - E(\vec{p}\,' - \vec{q})) \Big] \cdot \cos\Big(t_1 \Big(E(\vec{p} + \vec{q}) - E(\vec{p})\Big)\Big) \Big\} \tag{2}$$

$$\Big(\frac{\partial}{\partial t_2} + i\vec{k}\vec{v}\,'\Big) F_{\vec{k}}(\vec{p}\,'; \vec{p}\,'; t_2 \vec{p}; \vec{p}; t_1) = \frac{\pi n m}{E(\vec{p})} \cdot \int d^3\vec{q} |V(\vec{q})|^2 \cdot$$
$$\Big\{ \Big(F_{\vec{k}}(\vec{p}\,' + \vec{q}; \vec{p}\,' + \vec{q}; t_2; \vec{p}; \vec{p}; t_1) - F_{\vec{k}}(\vec{p}\,'; \vec{p}\,'; t_2; \vec{p} +; \vec{p}; t_1) \Big)$$
$$\cdot \delta(E(\vec{p}\,' + \vec{q}) - E(\vec{p}\,'))$$
$$\Big[F_{\vec{k}}(\vec{p}\,'; \vec{p}\,' + \vec{q}; t_2; \vec{p} + \vec{q}; \vec{p}; t_1) \cdot \delta(E(\vec{p}) - E(\vec{p} + \vec{q})) -$$

$$F_{\vec{k}}(\vec{p}\,';\vec{p}\,'+\vec{q};t_2;\vec{p};\vec{p}-\vec{q};t_1)\cdot\delta(E(\vec{p}-E(\vec{p}-\vec{q}))\Big]$$

$$\cdot\cos\Big(t_2\Big(E(\vec{p}\,'+\vec{q})-E(\vec{p}\,')\Big)\Big)\Big\}, \tag{3}$$

where $E(\vec{p})$ is the energy of the particle having momentum $\vec{p}$ (the dispersion law is supposed to be known); $\vec{v}=\vec{p}/E(\vec{p});\vec{v}\,'=\vec{p}\,'/E(\vec{p}\,')$ are the particle velocities; $V(\vec{q})$ is the Fourier image of the potential of the interaction of the particle with the individual scattering center; n is the number of the centers per unit volume.

Since the motion of ultrarelativistic particles in matter can be described in terms of the small angle approximation [1-4] the momentum transferred in the result of a single collision of two particles is small as compared with the initial momentum of the particle.

First we consider the situation when the following inequality takes place :

$$t_{1,2}|E(\vec{p}+\vec{q})-E(\vec{p}\,'+\vec{q})|\ll 1. \tag{4}$$

where $\vec{q}$ is the momentum transferred in the result of a single collision of the particles in the matter which is small as compared with the initial momentum of the high energy particle .

Note that $\vec{p}$ and $\vec{p}\,'$ in this formula are the particle momentum before and after the photon emission.

Then we can write

$$t_{1,2}|E(\vec{p}+\vec{q})-E(\vec{p}\,'+\vec{q})|\sim t\frac{(\vec{p}\vec{p}\,')\vec{q}}{p}\sim t(<{\Theta_s}^2>\tau)^{1/2}q_\perp\ll t\tau<{\Theta_s}^2>k\ll 1, \tag{5}$$

where $q_\perp$ is the momentum which is perpendicular to the direction of the particle movement; $<{\Theta_s}^2>$ is the mean square of the multiple scattering of particle per unit path length.

On the other hand the last inequality means that the cosines in the right-hand sides of Eqs.(2), (3) are approximately unit.

Then we expand the collision integrals in the formulae (2), (3) over the small parameter $q/p\ll 1$. As a result, we have the following equations for the function $F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)$:

$$\frac{\partial F_{\vec{k}}(\vec{p},\vec{v}\,',t_1,t_2)}{\partial t_1}-i\vec{k}\cdot\vec{v}\cdot F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)=$$
$$\frac{<{\Theta_s}^2>}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\eta}^2}+\frac{\partial^2}{\partial\vec{\eta}\partial\vec{\zeta}}\right\}F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)+ \tag{6}$$

$$\frac{\partial F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)}{\partial t_1}+i\vec{k}\cdot\vec{v}\,'\cdot F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)=$$
$$\frac{<{\Theta_s}^2>}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\zeta}^2}+\frac{\partial^2}{\partial\vec{\eta}\partial\vec{\zeta}}\right\}F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2) \tag{7}$$

In Eqs. (7), (8) we introduce notations :

$$\vec{p} = p_0 \vec{e}_z (1 - \frac{\vec{\eta}^2}{2}) + p_0 \vec{\eta}; \qquad \vec{\eta} \perp \vec{e}_z; \qquad | \vec{\eta} | \ll 1$$
$$\vec{p}\,' = p_0 \vec{e}_z (1 - \frac{\vec{\zeta}^2}{2}) + p_0 \vec{\zeta}; \qquad \vec{\zeta} \perp \vec{e}_z; \qquad | \vec{\zeta} | \ll 1$$
$$< {\Theta_s}^2 > = 2mn {p_0}^{-2} E^{-2} \int d^3\vec{q} q |V(\vec{q})| \delta(E(\vec{p}) - E(\vec{p} + \vec{q}))$$
$$\vec{v} = \frac{\vec{p}}{E}; \quad \vec{v}\,' = \frac{\vec{p}\,'}{E}; \quad |\vec{v}| = |\vec{v}\,'| = v_0; p_0 = |\vec{p}(t = 0)|. \tag{8}$$

Further we introduce variables $\tau = t_1 - t_2; t \equiv t_2$ and sum Eqs.(7) and (8). When the time of photon production $\tau \ll t$, we can neglect with τ in the equation which describes the evolution of the function $F_{\vec{k}}$ during the period $t \gg \tau$ preceding the photon emission.

Then, we find :

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau)}{\partial \tau} - i\vec{k} \cdot \vec{v} \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau)$$
$$= \frac{< {\Theta_s}^2 >}{4} \cdot \left\{ \frac{\partial^2}{\partial (\vec{\eta} - \vec{\zeta})^2} \right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau) \tag{9}$$

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau = 0)}{\partial t} - i\Big(\vec{k} \cdot \vec{v} - \vec{k} \cdot \vec{v}\,'\Big) \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau = 0) =$$
$$\frac{< {\Theta_s}^2 >}{4} \cdot \left(\frac{\partial}{\partial \vec{\eta}} + \frac{\partial}{\partial \vec{\zeta}} \right)^2 F_{\vec{k}}(\vec{p}, \vec{p}', t, \tau = 0) \tag{10}$$

In obtaining the last equations we ignore the derivative with respect to the variable $\vec{\zeta}$ in Eq.(10) because $F_{\vec{k}}$ is far more smooth function of the variable $\vec{\zeta}$ then $(\vec{\eta} - \vec{\zeta})$.

Solving Eqs.(9), (10) under the initial condition :

$$F_{\vec{k}}(\vec{p}, \vec{p}', t_1 = 0, t_2 = 0) = \frac{\delta(p - p_0)}{p^2} \cdot \delta(\vec{\eta}) \cdot \delta(\vec{\zeta}) \tag{11}$$

we will find function $F_{\vec{k}}(\vec{p}, \vec{p}', t, \tau)$. Substituting $F_{\vec{k}}(\vec{p}, \vec{p}', t, \tau)$ into the formula for the probability of photon production (1), we will go to the well-known result obtained by A.Migdal :

$$\frac{dW}{d\omega} = \frac{\alpha T}{\pi \gamma^2} \int_0^{+\infty} dz \exp\left(-\frac{z(\omega / < {\Theta_s}^2 >)^{1/2}}{2\gamma^2} \right) \cdot \left\{ \sin\left(\frac{z(\omega / < {\Theta_s}^2 >)^{1/2}}{2\gamma^2} \right) \coth z - \frac{\pi}{4} \right\} \tag{12}$$

where $\gamma = E/m$ is the Lorentz-factor of an ultrarelativistic particle; $k = \omega$.

The distinctive property of the Migdal's results is the linear dependence of the probability on the observation time T. That corresponds to the neglect with the influence of the scattering matter on the photon production during the time preceding the photon emission.

In the opposite limiting case when the scattering in the matter is rather strong so that $t\tau < {\Theta_s}^2 > k \gg 1$ the cosines in Eqs.(2), (3) are rapidly oscillating functions. Then, the terms containing these cosines in the right-hand sides of Eqs.(2), (3) are equal to zero. As a result, we have the following equations for the function $F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)$:

$$\frac{\partial F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)}{\partial t_1} - i\vec{k}\cdot\vec{v}\cdot F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2) = \frac{<{\Theta_s}^2>}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\eta}^2}\right\} F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)+ \tag{13}$$

$$\frac{\partial F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2)}{\partial t_1} + i\vec{k}\cdot\vec{v}\,'\cdot F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2) = \frac{<{\Theta_s}^2>}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\zeta}^2}\right\} F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2) \tag{14}$$

Solving them with the initial condition (11), we find :

$$\begin{aligned}F_{\vec{k}}(\vec{p},\vec{p}\,',t_1,t_2) &= \frac{\delta(p-p_0)}{p^2}\cdot\frac{\delta(p'-p_0)}{p'^2}\cdot\frac{a}{\pi<{\Theta_s}^2>\sinh(at_1)}\cdot\\ &\exp\left\{-\frac{a}{<{\Theta_s}^2>}\coth(at_1)\cdot(\vec{\eta})^2+\frac{2a}{<{\Theta_s}^2>}\tanh(\frac{at_1}{2})\cdot\right.\\ &\left.(\vec{\eta})\cdot(\vec{\theta})-\frac{2a}{<{\Theta_s}^2>}\tanh(\frac{at_1}{2})\cdot(\vec{\theta})^2+ikvt_1\right\}\cdot\\ &\frac{b}{\pi<{\Theta_s}^2>\sinh(bt_2)}\cdot\exp\left\{-\frac{b}{<{\Theta_s}^2>}\coth(bt_2)\cdot(\vec{\zeta})^2+\right.\\ &\left.\frac{2b}{<{\Theta_s}^2>}\tanh(\frac{bt_2}{2})\cdot(\vec{\zeta})\cdot(\vec{\theta})-\frac{2b}{<{\Theta_s}^2>}\tanh(\frac{bt_2}{2})\cdot(\vec{\theta})^2-ikvt_2\right\}\end{aligned} \tag{15}$$

where $a, b, v, \vec{\theta}$ are given by the expressions :

$$a=\frac{(1+i)\sqrt{kv<{\Theta_s}^2>}}{2};\qquad b=\frac{(1-i)\sqrt{kv<{\Theta_s}^2>}}{2};\qquad v=\frac{p_0}{E(\vec{p}_0)}$$

$$\vec{k}=k\vec{e}_z(1-\frac{\vec{\theta}^2}{2})+k\vec{\theta};\qquad \vec{\theta}\perp\vec{e}_z;\qquad |\,\vec{\theta}\,|\ll 1 \tag{16}$$

The initial condition (11) means that the particle begins to move in the matter at time $t=0$. On the other hand it has moved as a free particle before this moment. To take into account of this fact [6] we rewrite Eq.(1) as follows :

$$dW = \frac{\alpha}{4\pi^2\omega E^2}\int_0^T dt_1\int_0^T dt_2\int d^3\vec{p}\int d^3\vec{p}\,'(\vec{k}\times\vec{p})\cdot(\vec{k}\times\vec{p}\,')\cdot\Big\{\exp(i(t_1-t_2))\cdot$$

$$\Big[F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2)+F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2;a=b=0)\Big]-$$

$$2Re\Big[F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2;a;b=0)\cdot\exp(i(t_1-t_2))\Big]\Big\}d^3\vec{k} \qquad (17)$$

where α is the fine structure constant; T is the observation time.

Substituting $F_{\vec{k}}$ given by Eq.(15) into Eq.(17) and doing needed integrations, we find :

$$\frac{dW}{d\omega} = \frac{\alpha\omega}{2\pi\gamma^2}Re\Big\{\int_0^T dt_1\int_0^T dt_2\exp\Big(-\frac{i(t_1-t_2)\omega}{2\gamma^2}\Big)$$

$$\Big[\frac{2}{\Big(\frac{a}{<\Theta_s{}^2>}\tanh at_1-\frac{i\omega t_2}{2}\Big)\cosh^2 at_1}-$$

$$\frac{1}{\Big(\frac{a}{<\Theta_s{}^2>}\tanh at_1+\frac{b}{<\Theta_s{}^2>}\tanh bt_2\Big)\cosh^2 at_1}+$$

$$\frac{2\alpha a}{\pi\omega}\int_0^T dt\exp\left(-\frac{it\omega}{2\gamma^2}\right)\left[\frac{1}{at}-\frac{1}{\sinh at\cosh at}\right]\Big\} \qquad (18)$$

where $k=\omega$; $\gamma=E/m$ is the Lorentz-factor of an ultrarelativistic particle.

Let us analyse the probability of the photon production (18).

In the range of soft photons $\omega \ll <\Theta_s{}^2>^{-1}T^{-2}$ the probability is given by the formula :

$$\frac{dW}{d\omega} = \frac{\alpha T}{\pi}\sqrt{\frac{<\Theta_s{}^2>}{\omega}} \qquad (19)$$

The last expression coincides with the results obtained by A.Migdal in the long wave range of the emission spectrum. This takes place because very soft photons are formed over the whole trajectory of the particle motion in the matter. Therefore the influence of the multiple collisions on the photon production during the time preceding the photon generation is negligible. As a result, the probability is the linear function of the observation time T for such energies of photons.

In the intermediate range of the spectrum $<\Theta_s{}^2>^{-1}T^{-2} \ll \omega \ll <\Theta_s{}^2>\gamma^4$ we have :

$$\frac{dW}{d\omega} = -\frac{2\alpha}{\omega}\ln\left(\frac{1}{2\gamma^2}\sqrt{\frac{\omega}{<\Theta_s{}^2>}}\right) \qquad (20)$$

The probability of the production of the hard photons, so that $\omega \gg< \Theta_s{}^2 > \gamma^4$, is given by the expression :

$$\frac{dW}{d\omega} = \frac{\alpha}{\pi\omega}\left(\frac{3}{8} + \frac{16 < \Theta_s{}^2 > \gamma^8}{\omega^2}\right) \tag{21}$$

It follows from Eqs.(19) - (21) that the emission spectrum in such mode (the strong scattering matter) is rearranged in comparison with the case has been studied by A.Migdal (see Eq.(12)). This manifests itself in the dependence of the probability of the photon production both on the observation time T and on the photon energy ω. First, the probability dW is not linear function of the observation time. Besides dW depends logarithmically on the photon energy in the range of rather hard photons. We should note that the probability of the photon production appear to be suppressed in the short wave region of spectrum as compared with mode has been studied by A.Migdal [3,4]. It can be explained by the fact that in the case of the rather strong scattering the collisions of the particle in the matter break the coherence of the photon production more dramatically then in the situation considered in the papers [3,4].

3 Conclusion

On a basis of the formalism for two-particle Green's functions bremsstrahlung by high energy particles scattered multiply elastically in the static matter the nonchronological Green's functions is studied. The obtained functions completely determines the dynamics of the formation of bremsstrahlung by the ultrarelativistic particles scattered multiple elastically in the matter. The probability of the bremsstrahlung is found. It is shown that in the case of weak scattering matter when $t\tau < \Theta_s{}^2 > k \ll 1$ the emission spectrum is given by the formulae of the papers [3,4]. These inequality determines the application limits of the results of Migdal's theory of bremsstrahlung by ultrarelativistic particle in the matter.

When the scattering in the matter is strong so that $t\tau < \Theta_s{}^2 > k \gg 1$ emission spectrum is rearranged (see Eqs.(19)-(21)) as compared with one (see (12)) calculated in the papers [3,4]. This rearrangement manifests itself in the dependence of the probability of the photon production on all parameters of the solved problem - the observation time, the photon energy, the energy of the particle. The probability becomes the sufficiently nonlinear function of the time spent by the in the matter. It also depends logarithmically on the photon energy in intermediate of the emission spectrum.

References

[1] Ł.D.Landau, I.Ya.Pomeranchuk, Dokl. Akad. Nauk SSSR **92** (1953) 535

[2] Ł.D.Landau, I.Ya.Pomeranchuk, Dokl. Akad. Nauk SSSR **92** (1953) 735

[3] A.B.Migdal, Dokl. Akad. Nauk SSSR **96** (1954) 49

[4] A.B.Migdal, Phys. Rev. **103** (1956) 1811

[5] V.M.Galitskii, V.V.Yakimetsl, Zh. Eksp. Teor. Fiz. **46** (1964) 1066 (Engl. Transl. Sov.Phys. JETP **19** (1964) 723

[6] I.I.Gol'dman, Zh. Eksp. Teor. Fiz. **38** (1960) 1866 (Engl. Transl. Sov.Phys. JETP **11** (1960) 1341

[7] R.Baier, Yu.L.Dokshitzer, A.H.Muller, D.Schiff, Nucl.Phys **B478** (1996) 577

[8] S.Klein, Suppression of Bremsstrahlung and Pair Production due to Environmental Factors, Rev. Mod. Phys. **77** (1998) 1501

[9] A.V.Koshelkin, J. Exper. Theor. Phys. **91** (2000) 265