A Study of QCD Coupling Constant from Fixed Target Deep Inelastic Measurements

V.G. Krivokhijine and A.V. Kotikov

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Abstract

We reanalyze deep inelastic scattering data of BCDMS Collaboration by including proper cuts of ranges with large systematic errors. We perform also the fits of high statistic deep inelastic scattering data of BCDMS, SLAC, NM and BFP Collaborations taking the data separately and in combined way and find good agreement between these analyses. We extract the values of both the QCD coupling constant $\alpha_s(M_Z^2)$ up to NLO level. The fits of the combined data for the non-singlet part of the structure function F_2 predict the coupling constant value $\alpha_s(M_Z^2) = 0.1174 \pm 0.0007$ (stat) ± 0.0019 (syst) ± 0.0010 (normalization). The fits of the combined data for both: the non-singlet part and the singlet one, lead to the values $\alpha_s(M_Z^2) = 0.1177 \pm 0.0007$ (stat) ± 0.0021 (syst) ± 0.0009 (normalization). Both above values are in very good agreement with each other.

1 Introduction

Deep inelastic scattering (DIS) of leptons on hadrons is the basic process to study the values of the parton distribution functions (PDF) which are universal (after choosing the factorization and renormalization schemes) and can be used in other processes. The accuracy of the present data for deep inelastic structure functions (SF) reached the level at which the Q^2-dependence of logarithmic QCD-motivated and power-like ones may be studied separately (for a review, see the recent papers [1] and references therein).

In the present paper we review the results of our analysis [2] at the next-to-leading (NLO) order [1] of perturbative QCD for the most known DIS SF $F_2(x,Q^2)$ taking into account SLAC, NMC, BCDMS and BFP experimental data [4]-[7]. We stress the power-like effects, so-called twist-4 (i.e. $\sim 1/Q^2$) contributions. To our purposes we represent the SF $F_2(x,Q^2)$ as the contribution of the leading twist part $F_2^{pQCD}(x,Q^2)$ described by perturbative QCD and the non-perturbative part (twist-four terms $\sim 1/Q^2$):

$$F_2(x,Q^2) \equiv F_2^{full}(x,Q^2) = F_2^{pQCD}(x,Q^2) \left(1 + \frac{\tilde{h}_4(x)}{Q^2}\right) \qquad (1)$$

The SF $F_2^{pQCD}(x,Q^2)$ obeys the (leading twist) perturbative QCD dynamics including the target mass corrections (and coincides with $F_2^{tw2}(x,Q^2)$ when the target mass corrections are withdrawn).

[1] The evaluation of $\alpha_s^3(Q^2)$ corrections to anomalous dimensions of Wilson operators, that will be done in nearest future by Vermaseren and his coauthors (see discussions in [3]), gives a possibility to apply many modern programs to perform fits of data at next-next-to-leading order (NNLO) of perturbative theory (see detail discussions in [2]).

Eq.(1) allows us to separate pure kinematic power corrections, i.e. so-called target mass corrections, so that the function $\tilde{h}_4(x)$ corresponds to "dynamical" contribution of the twist-four operators. The parameterization (1) implies [2] that the anomalous dimensions of the twist-two and twist-four operators are equal to each other, that is not correct in principle. Meanwhile, in view of limited precision of the data, the approximations (1) and one in the footnote 2 give rather good predictions (see discussions in [8]).

Contrary to standard fits (see, for example, [8]- [10]) where the direct numerical calculations based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [11] are used to evaluate structure functions, we use the exact solution of DGLAP equation for the Mellin moments $M_n^{tw2}(Q^2)$ of SF $F_2^{tw2}(x,Q^2)$:

$$M_n^k(Q^2) = \int_0^1 x^{n-2}\, F_2^k(x,Q^2)\, dx \qquad \text{(hereafter } k = full, pQCD, tw2, ...\text{)} \qquad (2)$$

and the subsequent reproduction of $F_2^{full}(x,Q^2)$, $F_2^{pQCD}(x,Q^2)$ and/or $F_2^{tw2}(x,Q^2)$ at every needed Q^2-value with the help of the Jacobi Polynomial expansion method [12, 13] (see similar analyses at the NLO level [13, 14] and at the NNLO level and above [15].

In this paper we consider in detail only the fits of the non-singlet part (NS) of SF F_2 and only review final results in general case, that can be found in [2]. Moreover, we do not present exact formulas of Q^2-dependence of SF F_2 which are also given in [2]. We note only that the moments $M_{NS}(n,Q^2)$ at some Q_0^2 is a theoretical input of our analysis and the twist-four term $\tilde{h}_4(x)$ is considered as a set free parameters at each x_i bin. The set has the form $\tilde{h}_4^{free}(x) = \sum_{i=1}^{I} \tilde{h}_4(x_i)$, where I is the number of bins. The constants $\tilde{h}_4(x_i)$ (one per x-bin) parameterize x-dependence of $\tilde{h}_4^{free}(x)$.

2 Fits of F_2: procedure

Having the QCD expressions for the Mellin moments $M_n^k(Q^2)$, we can reconstruct the SF $F_2^k(x,Q^2)$ as

$$F_2^{k,N_{max}}(x,Q^2) = x^a(1-x)^b \sum_{n=0}^{N_{max}} \Theta_n^{a,b}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}^k\left(Q^2\right), \qquad (3)$$

where $\Theta_n^{a,}$ are the Jacobi polynomials [3] and a, b are the parameters, fitted by the requirement of the minimization of the error of the reconstruction of the structure functions (see Ref.[13] for details).

First of all, we choose the cut $Q^2 \geq 1$ GeV^2 in all our studies. For $Q^2 < 1$ GeV^2, the applicability of the twist expansion is very questionable.

Secondly, we choose quite large values of the normalization point Q_0^2. There are several reasons of this choice:

- Our perturbative formulas should be applicable at the value of Q_0^2. Moreover, the higher order corrections $\sim \alpha_s^n(Q_0^2)$ $(n \geq 2)$, coming from normalization conditions of PDF, are less important at higher Q_0^2 values.

[2] The r.h.s. of the Eq.(1) is represented sometimes as $F_2^{pQCD}(x,Q^2) + \overline{h}_4(x)/Q^2$. It implies that the anomalous dimensions of the twist-four operators are equal to zero.

[3] We would like to note here that there is similar method [16], based on Bernstein polynomials. The method has been used in the analyses at the NLO level in [17] and at the NNLO level in [18].

- To reach $Q^2 = M_Z^2$, the point of QCD coupling constant normalization, it is necessary to cross less heavy quark thresholds.

- It is better to have the value of Q_0^2 around the middle point of *logarithmic* range of considered Q^2 values. In this case the higher order corrections $\sim (\alpha_s(Q^2) - \alpha_s(Q_0^2))^n$ $(n \geq 2)$ are less important.

We use MINUIT program [19] for minimization of two χ^2 values:

$$\chi^2(F_2) = \left| \frac{F_2^{exp} - F_2^{theor}}{\Delta F_2^{exp}} \right|^2$$

We would like to apply the following procedure: we study the dependence of χ^2/DOF value on value of Q^2 cuts for various sets of experimental data. The study will be done for both cases: including higher twists corrections and without them.

We use free normalizations of data for different experiments. For the reference, we use the most stable deuterium BCDMS data at the energy $E_0 = 200$ GeV [4]. Using other types of data as reference gives negligible changes in our results. The use of fixed normalization for all data leads to fits with a bit worse χ^2.

3 Results of fits

Hereafter we choose $Q_0^2 = 90$ GeV^2 that is in good agreement with above conditions. We use also $N_{max} = 8$, the cut $0.25 \leq x \leq 0.8$, where the non-singlet evolution is dominant.

3.1 BCDMS $C^{12} + H_2 + D_2$ data

We start our analysis with the most precise experimental data [6] obtained by BCDMS muon scattering experiment at the high Q^2 values. The full set of data is 607 points.

It is well known that the original analyses given by BCDMS Collaboration itself (see also Ref. [9]) lead to quite small values of $\alpha_s(M_Z^2) = 0.113$. Although in some recent papers (see, for example, [8, 20]) more higher values of $\alpha_s(M_Z^2)$ have been considered, we think that an additional reanalysis of BCDMS data should be very useful.

Based on the study [21] (see also [20]) in [2] we propose that the reason for small values of $\alpha_s(M_Z^2)$ coming from BCDMS data is the existence of a subset of the data having large systematic errors. We study this subject by introducing several so-called Y-cuts [5] (see [21]). Excluding this set of data with large systematic errors leads to essentially larger values of $\alpha_s(M_Z^2)$ and very slow dependence of the values on the concrete choice of the Y-cut (see below).

We have studied influence of the experimental systematic errors on the results of the QCD analysis as a function of Y_{cut3}, Y_{cut4} and Y_{cut5} applied to the data. We use the following x-dependent y-cuts:

$$y \geq 0.14 \quad \text{when} \quad 0.3 < x \leq 0.4$$

[4] E_0 is the initial energy of the lepton beam.

[5] Hereafter we use the kinematic variable $Y = (E_0 - E)/E_0$, where E_0 and E are initial and scattering energies of lepton, respectively.

$$
\begin{aligned}
y &\geq 0.16 \quad \text{when} \quad 0.4 < x \leq 0.5 \\
y &\geq Y_{cut3} \quad \text{when} \quad 0.5 < x \leq 0.6 \\
y &\geq Y_{cut4} \quad \text{when} \quad 0.6 < x \leq 0.7 \\
y &\geq Y_{cut5} \quad \text{when} \quad 0.7 < x \leq 0.8
\end{aligned} \tag{4}
$$

We use several sets N of the values for the cuts at $0.5 < x \leq 0.8$ which are given in the Table 1.

Table 1. The values of Y_{cut3}, Y_{cut4} and Y_{cut5}.

N	0	1	2	3	4	5	6
Y_{cut3}	0	0.14	0.16	0.16	0.18	0.22	0.23
Y_{cut4}	0	0.16	0.18	0.20	0.20	0.23	0.24
Y_{cut5}	0	0.20	0.20	0.22	0.22	0.24	0.25

The systematic errors for BCDMS data are given [6] as multiplicative factors to be applied to $F_2(x, Q^2)$: f_r, f_b, f_s, f_d and f_h are the uncertainties due to spectrometer resolution, beam momentum, calibration, spectrometer magnetic field calibration, detector inefficiencies and energy normalization, respectively.

For this study each experimental point of the undistorted set was multiplied by a factor characterizing a given type of uncertainties and a new (distorted) data set was fitted again in agreement with our procedure considered in the previous section. The factors $(f_r, f_b, f_s, f_d, f_h)$ were taken from papers [6] (see CERN preprint versions in [6]). The absolute differences between the values of α_s for the distorted and undistorted sets of data are given in Fig. 1 as the total systematic error of α_s estimated in quadratures. The number of the experimental points and the value of α_s for the undistorted set of F_2 are also presented in Fig. 1.

From Fig. 1 we can see that the α_s values obtained for $N = 1 \div 6$ of Y_{cut3}, Y_{cut4} and Y_{cut5} are very stable and statistically consistent. The case $N = 6$ reduces the systematic error in α_s by the factor 1.8 and increases the value of α_s, while increasing the statistical error on the 30%.

After the cuts have been implemented (hereafter we use the set $N = 6$), we have 452 points in the analysis. Fitting them in agreement with the same procedure as considered in previous Section, we obtain the following results:

$$
\begin{aligned}
\alpha_s(M_Z^2) &= 0.1153 \pm 0.0013 \text{ (stat)} \pm 0.0022 \text{ (syst)} \pm 0.0012 \text{ (norm)}, \\
&= 0.1153 \pm 0.0028 \text{ (total experimental error)}
\end{aligned}
$$

where hereafter the symbol "norm" marks the error of normalization of experimental data. The total experimental error is the squared root of the sum of squares of statistical, systematic and normalization errors.

3.2 SLAC, BCDMS, NMC and BFP data

After these cuts have been incorporated (with $N = 6$) for BCDMS data, the full set of data combine 797 points.

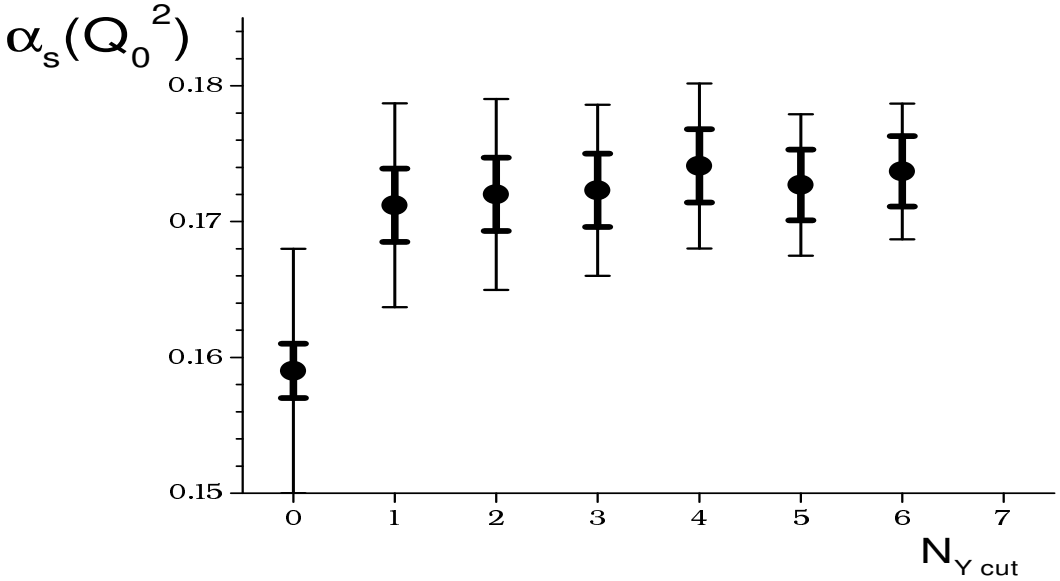

Figure 1: The study of systimatics at different Y_{cut} values in the fits based on non-singlet evolution. The QCD analysis of BCDMS C^{12}, H_2, D_2 data (non-singlet case): $x_{cut} = 0.25$, $Q_0^2 = 90$ GeV^2. Thresholds of c and b quarks are chosen at $Q^2 = 9$ GeV^2 and $Q^2 = 80$ GeV^2, respectively. The inner (outer) error-bars show statistical (systematic) errors.

To verify the range of applicability of perturbative QCD, we analyze firstly the data without a contribution of twist-four terms, i.e. when $F_2 = F_2^{pQCD}$. We do several fits using the cut $Q^2 \geq Q^2_{cut}$ and increase the value Q^2_{cut} step by step. We observe good agreement of the fits with the data when $Q^2_{cut} \geq 10$ GeV^2 (see the Fig. 2).

Later we add the twist-four corrections and fit the data with the usual cut $Q^2 \geq 1$ GeV^2. We have found very good agreement with the data. Moreover, the predictions for $\alpha_s(M_Z^2)$ in both above procedures are very similar (see the Fig. 2).

The analysis of combined SLAC, NMC, BCDMS and BFP data have given the following results:

- When twist-four corrections are not included and the cut of Q^2 is 10 GeV^2 at the free normalization

$$\chi^2/\mathrm{DOF} \;=\; 0.98 \quad \text{and} \quad \alpha_s(M_Z^2) \;=\; 0.1170 \pm 0.0009\ (\mathrm{stat}) \tag{5}$$

- When twist-four corrections are included and the cut of Q^2 is 1 GeV^2

$$\chi^2/\mathrm{DOF} \;=\; 0.97 \quad \text{and} \quad \alpha_s(M_Z^2) \;=\; 0.1174 \pm 0.0010\ (\mathrm{stat}) \tag{6}$$

Thus, as it follows from non-singlet fits of experimental data, perturbative QCD works rather well at $Q^2 \geq 10$ GeV^2.

4 Summary

We have demonstrated several steps of our study [2] of the Q^2-evolution of DIS structure function F_2 fitting all modern experimental data existing at the values of Bjorken variable

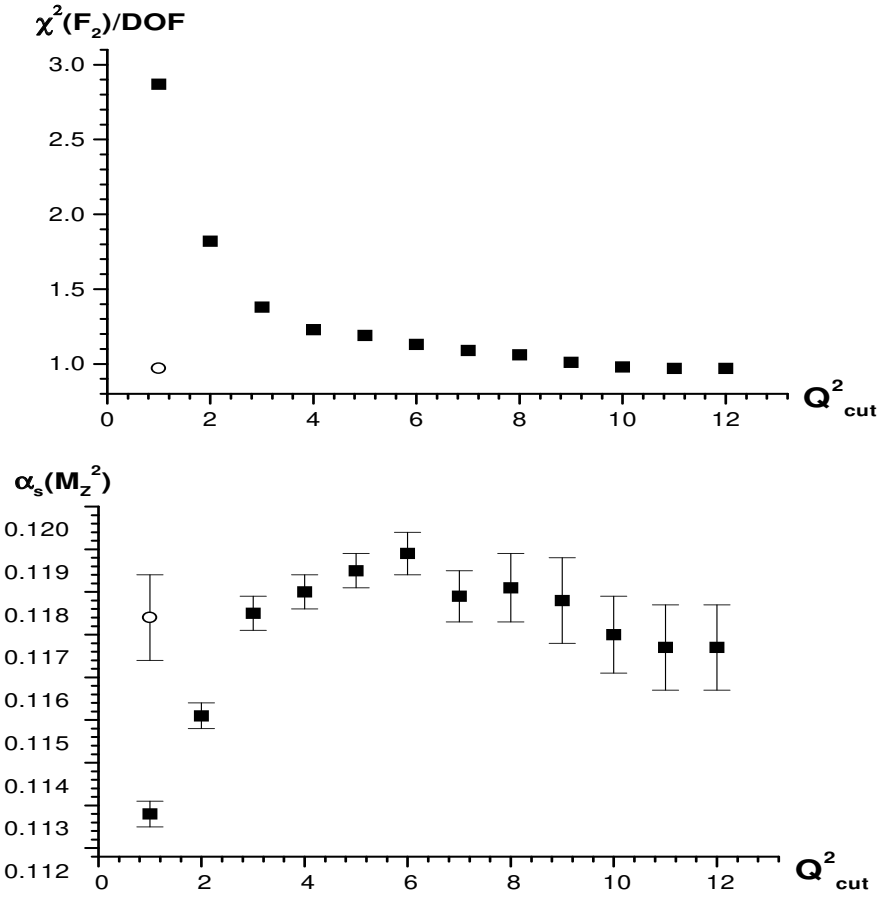

Figure 2: The values of $\alpha_s(M_Z^2)$ and χ^2 at different Q^2-values of data cuts in the fits based on non-singlet evolution. The black points show the analyses of data without twist-four contributions. The white point corresponds to the case where twist-four contributions were added. Only statistical errors are shown.

$x \geq 10^{-2}$.

From the fits we have obtained the value of the normalization, $\alpha_s(M_Z^2)$, of QCD coupling constant. First of all, we have reanalyzed the BCDMS data cutting the range with large systematic errors. As it is possible to see from subsection 3.1 (see also Fig. 1), the values of $\alpha_s(M_Z^2)$ rise strongly if the cuts of systematics are incorporated. On the other hand, the values of $\alpha_s(M_Z^2)$ do not dependent on the concrete type of the cut within modern statistical errors.

We have found that at $Q^2 \geq 10 \div 15$ GeV^2 the formulas of pure perturbative QCD (i.e. twist-two approximation together with target mass corrections) are in good agreement with all data. The results for $\alpha_s(M_Z^2)$ are very similar (see [2]) for both types of analyses: that based on non-singlet evolution, and that based on combined singlet and non-singlet evolution. They have the following form:

- from fits, based on non-singlet evolution:

$$\alpha_s(M_Z^2) \;=\; 0.1170 \pm 0.0009\ (\text{stat}) \pm 0.0019\ (\text{syst}) \pm 0.0010\ (\text{norm}), \qquad (7)$$

- from fits, based on combined singlet and non-singlet evolution:

$$\alpha_s(M_Z^2) \;=\; 0.1180 \pm 0.0013\ (\text{stat}) \pm 0.0021\ (\text{syst}) \pm 0.0009\ (\text{norm}), \qquad (8)$$

When we have added twist-four corrections, we have very good agreement between QCD (i.e. the first two coefficients of the Wilson expansion) and the data starting already from $Q^2 = 1$ GeV^2, where the Wilson expansion should begin to be applicable. The results for $\alpha_s(M_Z^2)$ coincide for both types of analyses: that based on non-singlet evolution, and that based on combined singlet and non-singlet evolution. They have the following form:

- from the fits, based on non-singlet evolution:

$$\alpha_s(M_Z^2) \;=\; 0.1174 \pm 0.0007\ (\text{stat}) \pm 0.0019\ (\text{syst}) \pm 0.0010\ (\text{norm}), \qquad (9)$$

- from the fits, based on combined singlet and non-singlet evolutions:

$$\alpha_s(M_Z^2) \;=\; 0.1177 \pm 0.0007\ (\text{stat}) \pm 0.0021\ (\text{syst}) \pm 0.0009\ (\text{norm}), \qquad (10)$$

Thus, there is a very good agreement (see Eqs. (7), (8), (9) and (9)) between the results based on the pure perturbative QCD at large Q^2 values (i.e. at $Q^2 \geq 10 \div 15$ GeV^2) and the results based on the first two twist terms of Wilson expansion (at $Q^2 \geq 1$ GeV^2, where the Wilson expansion should be applicable).

We would like to note that we have good agreement also with the analysis [20] of combined H1 and BCDMS data which has been given by H1 Collaboration very recently. Our results for $\alpha_s(M_Z^2)$ are in good agreement also with the average value for the coupling constant presented in the recent studies (see [8, 22, 18, 23] and references therein) and in the famous Bethke review [24].

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