The Structure Functions F_2^c and F_L^c in the Framework of the k_T Factorization

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Abstract

We present the perturbative parts of the structure functions F_2^c and F_L^c for a gluon target having nonzero transverse momentum squared at order α_s. The results of the double convolution (with respect to the Bjorken variable x and the transverse momentum) of the perturbative part and unintegrated gluon densities are compared with HERA experimental data for F_2^c. The contribution from F_L^c structure function ranges $10 \div 30\%$ of that of F_2^c at the kinematic range of HERA experiments.

1 Introduction

Recently there have been obtained important new data on the charm structure function (SF) F_2^c, of the proton from the H1 [1] and ZEUS [2] Collaborations at HERA, which have probed the small-x region down to $x = 8 \times 10^{-4}$ and $x = 2 \times 10^{-4}$, respectively. At these values of x, the charm contribution to the total proton SF, F_2^p, is found to be around 25%, which is a considerably larger fraction than that found by the European Muon Collaboration at CERN [3] at larger x, where it was only $\sim 1\%$ of F_2^p. Extensive theoretical analyses in recent years have generally served to confirm that the F_2^c data can be described through perturbative generation of charm within QCD (see, for example, the review in Ref. [4] and references therein).

We note, that perhaps more relevant analyses of the HERA data, where the x values are quite small, are those based on BFKL dynamics [5], because the leading $ln(1/x)$ contributions are summed. The basic dynamical quantity in BFKL approach is the unintegrated gluon distribution $\varphi_g(x, k_\perp^2)$ [1] (f_g is the (integrated) gluon distribution multiplied by x and $k_\perp$ is the transverse momentum)

$$f_g(x, Q^2) \;=\; f_g(x, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, \varphi_g(x, k_\perp^2), \tag{1}$$

which satisfies the BFKL equation. The integral is divergent at the lower limit and it is necessary to consider the difference $f_g(x, Q^2) - f_g(x, Q_0^2)$ with some nonzero Q_0^2.

Then, in the BFKL approach, the SFs $F_{2,L}^c(x, Q^2)$ are driven at small x by gluons and are related in the following way to the unintegrated distribution $\varphi_g(x, k_\perp^2)$:

$$F_{2,L}^c(x, Q^2) \;=\; \int_x^1 \frac{dz}{z} \int \frac{dk_\perp^2}{k_\perp^2}\, C_{2,L}^g(z, Q^2, m_c^2, k_\perp^2)\; \varphi_g(x/z, k_\perp^2), \tag{2}$$

[1] Hereafter p^μ and k^μ are the hadron and the gluon 4-momentums, respectively, and q^μ is the photon 4-momentum.

The functions $C^g_{2,L}(x,Q^2,m_c^2,k_\perp^2)$ may be regarded as the structure functions of the off-shell gluons with virtuality $k_\perp^2$ (hereafter we call them as *hard structure functions*[2]). They are described by the quark box (and crossed box) diagram contribution to the photon-gluon interaction (see Fig. 1 in [8]).

The purpose of the article is to present the results for these hard SF $C^g_{2,L}(x,Q^2,m_c^2,k_\perp^2)$ and to analyze experimental data for $F_2^c(x,Q^2)$ by applying Eq. (2) with different sets of unintegrated gluon densities (see Ref. [9]) and to give predictions for the longitudinal SF $F_L^c(x_B,Q^2)$.

It is instructive to note that the results should be similar to those of the photon-photon scattering process. The corresponding QED contributions have been calculated many years ago in Ref. [10] (see also the beautiful review in Ref. [11]). Our results have been calculated independently in [8] (based on approaches of [12, 13]) and they are in full agreement with Ref. [10]. However, we hope that our formulas which are given in a more simple form could be useful for others.

2 Hard structure functions

The gluon polarization tensor (hereafter the indices α and β are connected with gluons), which gives the main contribution at high energy limit, has the form:

$$\hat{P}^{\alpha\beta}_{BFKL} \;=\; \frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2} \;=\; \frac{x^2}{-k^2}p^\alpha p^\beta \;=\; -\frac{1}{2}\frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 g^{\alpha\beta} - 12bx^2\frac{q^\alpha q^\beta}{Q^2}\right], \tag{3}$$

where $\tilde{\beta}^2 = 1-4bx^2, \quad b=-k^2/Q^2 \equiv k_\perp^2/Q^2 > 0,$

Contracting the corresponding photon projectors, we have:

$$C_2^g(x) = \frac{\mathcal{K}}{\tilde{\beta}^2}\left[f^{(1)}_{BFKL} + \frac{3}{2\tilde{\beta}^2}\, f^{(2)}_{BFKL}\right],\; C_L^g(x) = \frac{\mathcal{K}}{\tilde{\beta}^2}\left[4bx^2 f^{(1)}_{BFKL} + \frac{(1+2bx^2)}{\tilde{\beta}^2}\, f^{(2)}_{BFKL}\right] \tag{4}$$

$$f^{(1)}_{BFKL} = \frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 f^{(1)} \;-\; 3bx^2\tilde{f}^{(1)}\right], \quad f^{(2)}_{BFKL} \;=\; \frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 f^{(2)} \;-\; 3bx^2\tilde{f}^{(2)}\right],$$

where the normalization factor $\mathcal{K} = e_c^2\,\alpha_s(Q^2)/(4\pi)\;x,\;\; a=m^2/Q^2$ and

$$\begin{aligned} f^{(1)} \;&=\; -2\beta\Big[1-\Big(1-2x(1+b-2a)\,[1-x(1+b+2a)]\Big)\,f_1 \\ &+\; (2a-b)(1-2a)x^2\,f_2\Big], \end{aligned} \tag{5}$$

$$\begin{aligned} f^{(2)} \;&=\; 8x\,\beta\Big[(1-(1+b)x) - 2x\Big(bx(1-(1+b)x)(1+b-2a) + a\tilde{\beta}^2\Big)\,f_1 \\ &+\; bx^2(1-(1+b)x)(2a-b)\,f_2\Big], \end{aligned} \tag{6}$$

[2]This notation reflects the fact that structure functions $F^c_{2,L}$ connect with the functions $C^g_{2,L}$ at the same form as cross-sections connect with hard ones (see [6, 7]).

$$\tilde{f}^{(1)} = -\beta\Bigg[\frac{1-x(1+b)}{x} - 2\Big(x(1-x(1+b))(1+b-2a) + a\tilde{\beta}^2\Big)\ f_1 - x(1-x(1+b))(1-2a)\ f_2\Bigg], \quad (7)$$

$$\tilde{f}^{(2)} = 4\ \beta\ (1-(1+b)x)^2\Bigg[2-(1+2bx^2)\ f_1 - bx^2\ f_2\Bigg], \quad (8)$$

with

$$\beta^2 = 1 - \frac{4ax}{(1-(1+b)x)}, \qquad f_1 = \frac{1}{\tilde{\beta}\beta}\ ln\frac{1+\beta\tilde{\beta}}{1-\beta\tilde{\beta}}, \qquad f_2 = \frac{-4}{1-\beta^2\tilde{\beta}^2}$$

For the important regimes when $k^2 = 0$, $m^2 = 0$ and $Q^2 = 0$, the results coincide with one of Refs. [6, 14]. Notice that our results in Eq. (4) should also agree with those in Ref. [15], but a direct comparison is quite difficult, because the structure of their results is quite cumbersome (see Appendix A in Ref. [15]). We have found numerical agreement in the case of $F_2(x, Q^2)$ (see Fig.4 in [8]).

3 Comparison with F_2^c experimental data and predictions for F_L^c

With the help of the results obtained in the previous Section we have analyzed HERA data for SF F_2^c from ZEUS [2] collaboration.

Notice that in Ref. [6] the $k_\perp^2$-integral has been evaluated using the BFKL results for the Mellin transform of the unintegrated gluon distribution and the Wilson coefficient functions have been calculated for the full perturbative series at asymptotically small x values. Since we would like to analyze experimental data for the SF F_2^c, we have an interest to obtain results at quite broad range of small values of x. For this reason we need a parameterization of unintegrated gluon distribution function.

We consider two different parametrizations for the unintegrated gluon distribution (see [9]): the Ryskin-Shabelski (RS) one [16] and the Blumlein one [17].

In Fig. 1 we show the SF F_2^c as a function of x for different values of Q^2 in comparison with ZEUS [2] experimental data. We see that at large Q^2 ($Q^2 \geq 10$ GeV^2) the SF F_2^c obtained in the k_T factorization approach is higher than the SF obtained in the standard parton model with the GRV [18] gluon density at the LO approximation (see curve 1) and has a more rapid growth in comparison with the standard parton model results, especially at $Q^2 \sim 130$ GeV^2 [19]. At $Q^2 \leq 10$ GeV^2 the predictions from perturbative QCD (in GRV approach) and those based on the k_T factorization approach are very similar [3] and show the disagreement with the data below $Q^2 = 7$ GeV^2 [4]. Unfortunately the available experimental data do not permit yet to distinguish the k_T factorization effects from those due to boundary conditions [16].

[3] This fact is due to the quite large value of $Q_0^2 = 4$ GeV^2 chosen here.

[4] A similar disagreement with data at $Q^2 \leq 2$ GeV^2 has been observed for the complete structure function F_2 (see, for example, the discussion in Ref. [20] and reference therein). We note that the insertion of higher-twist corrections in the framework of usual perturbative QCD improves the agreement with data (see Ref. [21]) at quite low values of Q^2.

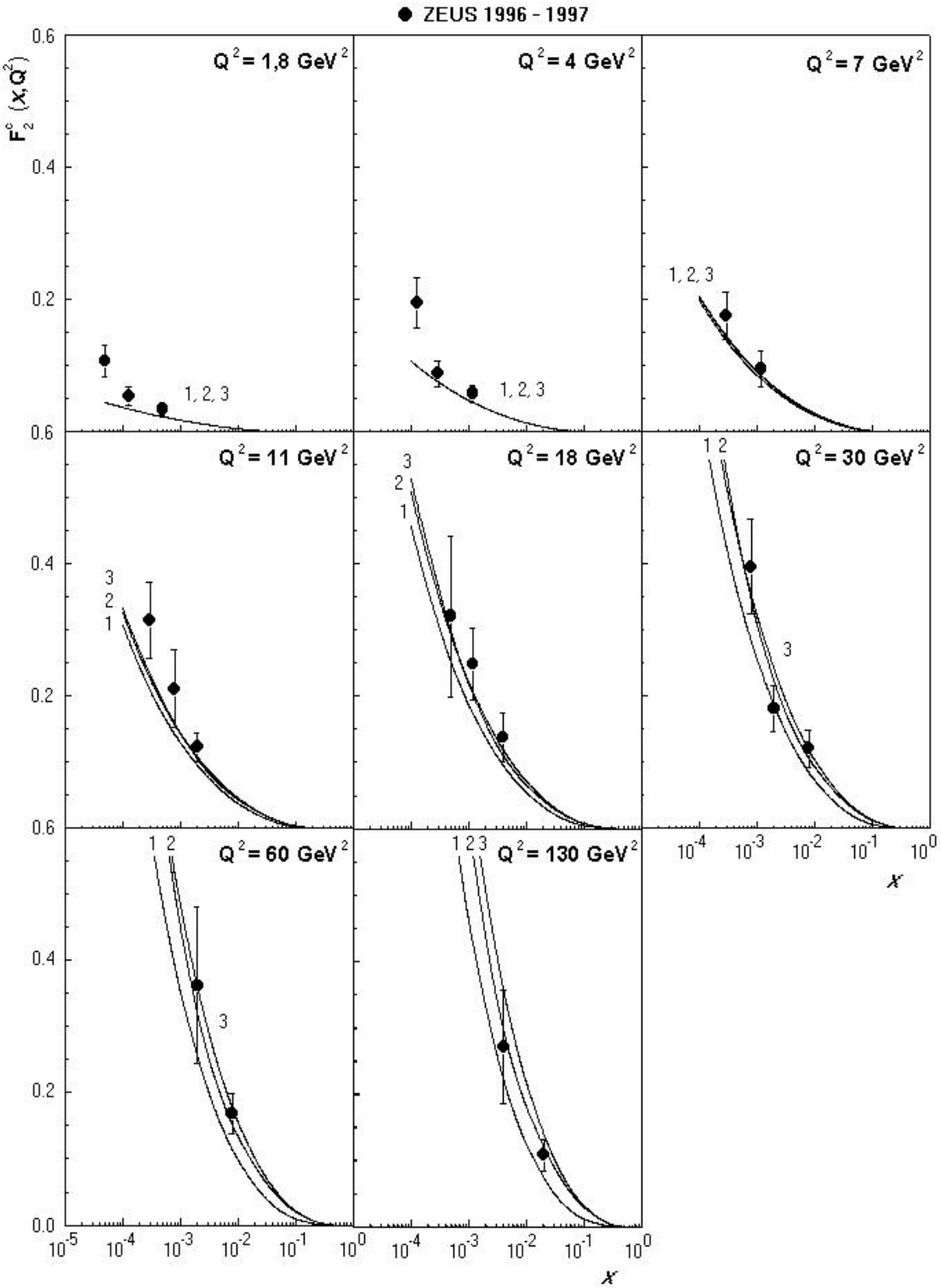

Figure 1: The structure function $F_2^c(x, Q^2)$ as a function of x for different values of Q^2 compared to ZEUS data [2]. Curves 1, 2 and 3 correspond to SF obtained in the standard parton model with the GRV [18] gluon density at the leading order approximation and to SF obtained in the k_T factorization approach with RS [16] and Blumlein (at $Q_0^2 = 4$ GeV^2) [17] parameterizations of unintegrated gluon distribution.

The results for the SF F_L^c obtained in perturbative QCD and from the k_T factorization approach are quite similar to the F_2^c case discussed above. The ratio $R^c = F_L^c/F_2^c$ is shown in Fig. 2. We see $R^c \approx 0.1 \div 0.3$ in a wide region of Q^2. The estimation of R^c is very close to the results for $R = F_L/(F_2 - F_L)$ ratio (see Refs. [22]-[24]). We would like to note that these values of R^c contradict the estimation obtained in Refs. [2, 1]. The effect of R^c on the corresponding differential cross-section should be considered in the extraction of F_2^c from future more precise measurements.

For the ratio R^c we found quite flat x-behavior at low x in the low Q^2 region (see Fig. 2), where approaches based on perturbative QCD and on k_T factorization give similar predictions (see Fig 1). It is in agreement with the corresponding behaviour of the ratio $R = F_L/(F_2 - F_L)$ (see Ref. [22]) at quite large values of Δ_P [5] ($\Delta_P > 0.2 - 0.3$). The low x rise of R^c at high Q^2 disagrees with early calculations [22] in the framework of perturbative QCD. It could be due to the small x resummation, which is important at high Q^2 (see Fig 1). We plan to study this effect on R in the framework of k_T factorization in future.

4 Conclusions

We have presented the results for the perturbative parts of the SFs F_2^c and F_L^c for a gluon target having nonzero momentum squared, in the process of photon-gluon fusion.

We have applied the results in the framework of k_T factorization approach to the analysis of present data for the charm contribution to F_2 (F_2^c) and we have given the predictions for F_L^c. The analysis has been performed with two parameterizations of unintegrated gluon distributions for comparison. We have found good agreement of our results with experimental F_2^c HERA data, except at low Q^2 ($Q^2 \leq 7$ GeV^2) [6]. We have also obtained quite large contribution of the SF F_L^c at low x and high Q^2 ($Q^2 \geq 30$ GeV^2).

We would like to note the good agreement between our results for F_2^c and the ones obtained in Ref. [26] by Monte-Carlo studies. Moreover, we have also good agreement with fits of H1 and ZEUS data for F_2^c (see recent reviews in Ref. [27] and references therein) based on perturbative QCD calculations. But unlike to these fits, our analysis uses universal unintegrated gluon distribution which gives in the simplest way the main contribution to the cross-section in the high-energy limit.

It could be also very useful to evaluate the complete F_2 itself and the derivatives of F_2 with respect to the logarithms of $1/x$ and Q^2 with our expressions using the unintegrated gluons. We are considering to present this work and also the predictions for F_L in a forthcoming article.

The consideration of the SF F_2 in the framework of the leading-twist approximation of perturbative QCD (i.e. for "pure" perturbative QCD) leads to very good agreement (see Ref. [20] and references therein) with HERA data at low x and $Q^2 \geq 1.5$ GeV^2. The agreement improves at lower Q^2 when higher twist terms are taken into account [21]. As it has been studied in Refs. [20, 21], the SF F_2 at low Q^2 is sensitive to the small-x behavior of quark distributions. Thus, the analysis of F_2 in a broader Q^2 range should

[5] At small values of Δ_P, i.e. when $x^{-\Delta_P} \sim Const$, the ratio R tends to zero at $x \to 0$ (see Ref. [25]).

[6] It must be noted that the cross section of inelastic $c\bar{c}-$ and $b\bar{b}-$pair photoproduction at HERA are described by the Blumlein parametrization at a smaller value of Q_0^2 ($Q_0^2 = 1$ GeV^2) (see [9]).

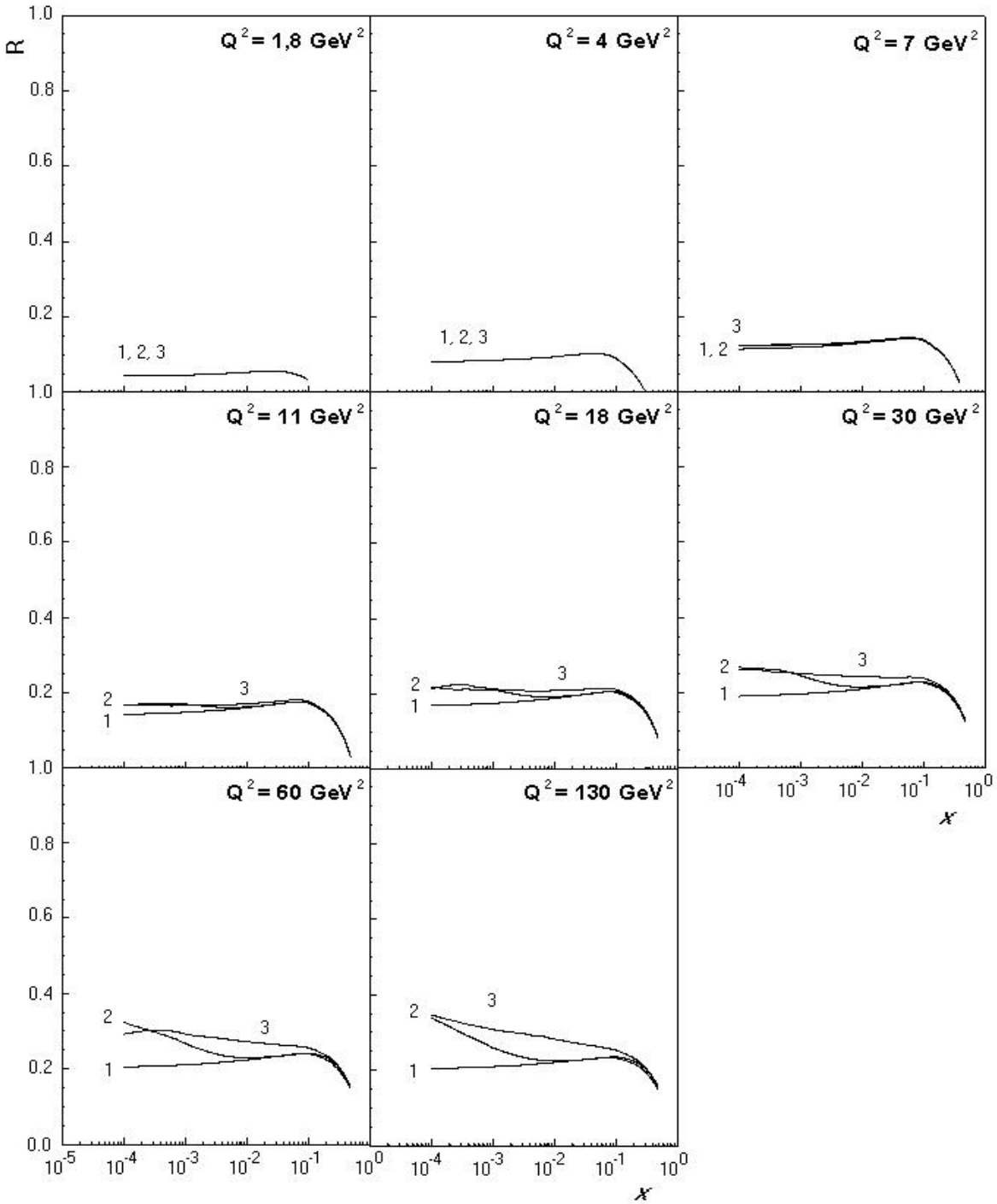

Figure 2: The ratio $R^c = F_L^c(x, Q^2)/F_2^c$ as a function of x for different values of Q^2. Curves 1, 2 and 3 are as in Fig. 1.

require the incorporation of parametrizations for unintegrated quark densities introduced recently (see Ref. [28] and references therein).

The study of the complete SF F_L should also be very interesting. The structure function F_L depends strongly on the gluon distribution (see, for example, Ref. [29]), which in turn is determined [30] by the derivative $dF_2/dlnQ^2$. Thus, in the framework of perturbative QCD at low x, the relation between F_L, F_2 and $dF_2/dlnQ^2$ could be violated by non-perturbative contributions which are expected to be important in the F_L case (see Ref. [31]). The application of present analysis to F_L will give a "non pure" perturbative QCD predictions for the structure function that should be compared with data [24] and with the "pure" perturbative results of Ref. [22].

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References

[1] H1 Coll.: S. Aid et al., Z.Phys. C72 (1996) 593; Nucl.Phys. B545 (1999) 21

[2] ZEUS Coll.: J. Breitweg et al., Phys.Lett. B407 (1997)402; Eur.Phys.J. C12 (2000) 35

[3] EM Coll.: J.J. Aubert et al., Nucl.Phys. B213 (1983) 31; Phys.Lett. B94 (1980) 96; B110 (1983) 72

[4] A.M. Cooper-Sarkar et al., Int.J.Mod.Phys. A13 (1998) 3385.

[5] L.N. Lipatov, Sov.J.Nucl.Phys. 23 (1976) 642; E.A. Kuraev et al., Sov.Phys.JETP 44 (1976) 45, 45 (1977) 199; Ya.Ya. Balitzki, L.N. Lipatov, Sov.J.Nucl.Phys. 28 (1978) 822; L.N. Lipatov, JETP 63 (1986) 904

[6] S. Catani et al., Phys.Lett. B242 (1990) 97; Nucl.Phys. B366 (1991) 135.

[7] J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3;

[8] A.V. Kotikov et al., Preprint US-FT/7-01 (hep-ph/0107135)

[9] A. V. Lipatov et al., Mod.Phys.Lett. A15 (2000) 695; 1727

[10] V.N. Baier et al., JETP 50 (1966) 156; V.G. Zima, Yad.Fiz. 16 (1972) 1051

[11] V.M. Budnev et al., Phys.Rept. 15 (1975) 181

[12] D.I. Kazakov, A.V. Kotikov, Nucl.Phys. B307 (1988) 721.

[13] A.V. Kotikov, Theor.Math.Phys. 78 (1989) 134; Phys.Lett. B375 (1996) 240; B254 (1991) 158; B259 (1991) 314; B267 (1991) 123

[14] E. Witten, Nucl.Phys. B104 (1976) 445; M. Gluck, E. Reya, Phys.Lett. B83 (1979) 98; F.M. Steffens et al., Eur.Phys.J. C11 (1999) 673

[15] G. Bottazzi et al., JHEP 9812 (1998) 011

[16] M.G. Ryskin, Yu.M. Shabelski, Z.Phys. C61 (1994) 517; C66 (1995) 151

[17] J. Blumlein, Preprints DESY 95-121 (hep-ph/9506403).

[18] M. Gluck et al., Z. Phys. C67 (1995) 433

[19] A.V. Lipatov and N.P. Zotov, *in* Proc. of the 8th Int. Workshop DIS 2000 (2000), World Scientific, p. 157.

[20] A.V. Kotikov, G. Parente, Nucl.Phys. B549 (1999) 242;

[21] A.V. Kotikov, G. Parente, hep-ph/0106175

[22] A.V. Kotikov, JETP 80 (1995) 979; A.V. Kotikov, G. Parente, Mod.Phys.Lett. A12 (1997) 963; JETP 85 (1997) 17;

[23] R.S. Thorne, Phys.Lett. B418 (1998) 371

[24] H1 Coll.: S. Aid et al., Phys.Lett. B393 (1997) 452; CCFR/NuTeV Coll.: U.K. Yang et al., hep-ex/0010001; A. Bodek, hep-ex/00105067

[25] S. Keller et al., Phys.Lett. B270 (1990) 61; L.H. Orr, W.J. Stirling, Phys.Rev.Lett. B66 (1991) 1673; E. Berger, R. Meng, Phys.Lett. B304 (1993) 318; A.V. Kotikov, JETP Lett. 59 (1994) 1; Phys.Lett. B338 (1994) 349

[26] H. Jung, Nucl.Phys.(Proc.Suppl.) 79 (1999) 429

[27] G. Wolf, Preprint DESY 01-058 (hep-ex/0105055);

[28] M.A. Kimber et al., Phys.Rev. D63 (2001) 114027

[29] A.M. Cooper-Sarkar et al., Z.Phys. C39 (1988) 281; A.V. Kotikov, Phys.Atom.Nucl. 57 (1994) 133; Phys.Rev. D49 (1994) 5746.

[30] K. Prytz, Phys.Lett. B311 (1993) 286; A.V. Kotikov, JETP Lett. 59 (1994) 667; A.V. Kotikov, G. Parente, Phys.Lett. B379 (1996) 195.

[31] J. Bartels et al., Eur.Phys.J. C17 (2000) 121; G. Parente at al., Phys.Lett. B333 (1994) 190; Z. Phys. C58 (1993) 465; A.V. Kotikov, V.G. Krivokhijine, Dubna preprint E2-2001-190 (hep-ph/0108224)