Leptonic Decays of the Heavy–Light Pseudoscalar Mesons in the Instant Hamiltonian Dynamics

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Abstract

Calculation of the lepton decay constants of the heavy–light pseudoscalar mesons B , B_s , D , D_s is performed in the framework of instant form of Relativistic Hamiltonian Dynamics (RHD). Wave functions in the sense of RHD are calculated approximately by the variational method. Results are in good agreement with lattice calculations and experiments.

It is well known that the lepton decays of the heavy mesons are the important source of information about the parameters of Standard Model (SM) (e.g. Cabbibo– Kobayashi–Maskawa matrix elements), and also can serve for searches of physics beyond the SM (see e.g. [1]). The retrieval of this information calls for the precise calculations of the lepton decay constants. The values of these constants are determined by the structure of mesons, and therefore the non-perturbative approaches are necessary for the calculations.

There exist different approaches to this problem, for example, the calculations on lattices [2, 3, 4, 5, 6, 7, 8, 9, 10], QCD sum rules (see e.g. [11, 12, 13]), and potential approaches (see e.g. review [14] and references therein, [15]).

The results of computation in these approaches are in rather poor agreement with each other and often have a sufficiently large theoretical uncertainties. So, development of the new approaches to this problem and performance of new calculations are necessary nowadays. In particular, the calculations have to include in correct way the relativistic effects.

Among the relativistic approaches the different forms of relativistic constituent quark model hold an important position, for example, quasipotential approach [16], dispersion relations [17, 18], and different forms of Relativistic Hamiltonian Dynamics (RHD): light-front dynamics [19, 20] and point dynamics [21].

In this Letter we use the instant form of RHD in the version developed by the authors [22, 23, 24, 25] for the calculation of the lepton decay constants of the heavy–light pseudoscalar mesons f_B , f_{Bs} , f_D , f_{Ds} (general features of RHD are discussed in greater detail in references [26, 27, 28] and references therein).

The distinctive property of our version of the instant RHD is the method of construction of current transition matrix elements [24, 29] which includes the relativistic covariance conditions. This approach was used successfully for the description of electromagnetic properties of light mesons [22, 24] and semileptonic decays of light and heavy–light mesons [29, 30].

Our method gives the following integral representation for the lepton decay constant of the pseudoscalar mesons f_P [24, 25]:

$$\int d\sqrt{s}\, G_0(s)\varphi(s) = f_P \ , \qquad (1)$$

$$G_0(s) = \frac{n_c\sqrt{(p_{q0}+m_q)(p_{Q0}+m_Q)}}{2\sqrt{2}\pi\,(p_{Q0}+p_{q0})}\left[1 - \frac{k^2}{(p_{q0}+m_q)\,(p_{Q0}+m_Q)}\right],$$

$$p_{i0} = \sqrt{k^2+m_i^2}, \qquad i = q\,, Q\;, \qquad k = \frac{\left[s^2 - 2s\left(m_q^2+m_Q^2\right)+\eta^2\right]^{1/2}}{2\sqrt{s}}\;.$$

m_q , m_Q are mass of light and heavy quark respectively, $n_c = 3$ is the number of quark colors, $\eta = m_Q^2 - m_q^2$, $\varphi(s)$ is the wave function in sense of RHD.

Let us remark that the derived expression for the lepton decay constant of pseudoscalar mesons (1) coincides with those in the light–front form [19], point form [21] of RHD and in the dispersion relations approach [18]. However, for example, the expressions for electromagnetic form factors are different.

Let us discuss the calculation of the wave function in RHD [26, 27, 28]. In RHD the wave function of the system of interacting particles is the eigenfunction of a complete set of commuting operators. In the instant form this set is:

$$\hat{M}_I = \hat{M}_0 + \hat{V}\;, \qquad \hat{J}^2\;, \qquad \hat{J}_3\;, \qquad \hat{\vec{P}}\;. \tag{2}$$

Here $\hat{M}_I$ is the mass operator of the system of interacting particles, $\hat{M}_0$ is the mass operator of the system without interaction, $\hat{V}$ is the operator of interaction, $\hat{J}^2$ is the operator of the square of the total angular momentum, $\hat{J}_3$ is the operator of the third component of the total angular momentum, $\hat{\vec{P}}$ is the total momentum operator.

In the instant form the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ coincide with those for the free system. So, in (2) only the operator $\hat{M}_I$ depends on the interaction.

To find the eigenfunctions for the system (2) one has first to construct the adequate basis in the state space of composite system. In the case of two-particle system (for example, quark-antiquark system $q\,\bar{Q}$) the Hilbert space in RHD is the direct product of two one-particle Hilbert spaces: $\mathcal{H}_{q\bar{Q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{Q}}$.

As a basis in $\mathcal{H}_{q\bar{Q}}$ one can choose the following set of two-particle state vectors:

$$|\,\vec{p}_1\,, m_1;\,\vec{p}_2\,, m_2\rangle = |\,\vec{p}_1\,m_1\rangle \otimes |\,\vec{p}_1\,m_2\,\rangle\;, \tag{3}$$

$$\langle\vec{p}\,, m\,|\,\vec{p}\,'\,m'\rangle = 2p_0\,\delta(\vec{p}-\vec{p}\,')\,\delta_{mm'}\;.$$

Here $\vec{p}_1$, $\vec{p}_2$ are 3–momenta of particles, m_1 , m_2 are spin projections on the axis z, $p_0 = \sqrt{\vec{p}^{\,2}+M^2}$, M is the constituent mass.

One can choose another basis, where the motion of the two-particle center of mass is separated and where three operators of the set (2) are diagonal:

$$|\,\vec{P},\ \sqrt{s},\ J,\ l,\ S,\ m_J\,\rangle, \tag{4}$$

Here $P_\mu = (p_1+p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the two-particle system, l is the orbital angular momentum in the center–of–mass frame (C.M.S.), $\vec{S}^{\,2} = (\vec{S}_1+\vec{S}_2)^2 = S(S+1)$, S is the total spin in C.M.S., J is the total angular momentum with the projection m_J.

The basis (4) is connected with (3) through the Clebsh–Gordan decomposition for the Poincareé group (see e.g. [23]). As in the basis (4) the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ in (2) are diagonal, in order to obtain the system wave function. one needs to diagonalize only the operator $\hat{M}_I^2$ in(2).

The corresponding composite–particle wave function has the form [24]:

$$\langle \vec{P}\,',\, \sqrt{s'},\, J',\, l',\, S',\, m'_J|\,\Psi\rangle = N_C\,\delta(\vec{P}\,' - \vec{p}_c)\delta_{JJ'}\delta_{m_J m'_J}\,\varphi^{J'}_{l'S'}(k')\;, \tag{5}$$

Here J , m_J are quantum numbers in state $|\,\Psi\rangle$. The explicit form of N_C will not be used.

In the case of pseudoscalar mesons $J = l = S = 0$, and we use in equation (1) for simplicity the notation: $\varphi^J_{lS}(k) \;\;\rightarrow\;\; \varphi(k)$, $\varphi(k)$ is a phenomenological wave function normalized with the account of relativistic density of states [24]:

$$\varphi(k(s)) = \sqrt{\sqrt{s}(1-\eta^2/s^2)}u(k)k\;, \qquad n_c\int u^2(k)k^2\,dk = 1\,. \tag{6}$$

In this work we use the variational method for the approximate calculation of the function $u(k)$ in equations (5), (6) (see [31] too). It is a common knowledge that this method is reduced to calculation of minimum of the following functional:

$$M_P(\alpha\,,\,\beta\,,\,\gamma\,,\ldots) = \langle\Psi|\hat{M}_I|\Psi\rangle = \langle\Psi|\hat{M}_0|\Psi\rangle + \langle\Psi|\hat{V}|\Psi\rangle\;. \tag{7}$$

Here $\alpha\,,\,\beta\,,\,\gamma\,,\ldots$ are parameters of trial function, M_P is meson mass.

As a trial function one can use the wave function of the ground state of harmonic oscillator which contains one parameter β only:

$$u(k) = \left(\frac{4}{\sqrt{\pi}\,\beta^3}\right)^{1/2}\,\exp\left(-\frac{k^2}{2\,\beta^2}\right)\;. \tag{8}$$

The matrix elements of $\hat{M}_0$ and $\hat{V}$ are calculated in momentum and coordinate representations respectively:

$$\langle\Psi|\hat{M}_0|\Psi\rangle = \int k^2dk|u(k)|^2\left(\sqrt{k^2+m_q^2} + \sqrt{k^2+m_{\bar{Q}}^2}\right)\;, \tag{9}$$

$$\langle\Psi|\hat{V}|\Psi\rangle = \int r^2dr|u(r)|^2V(r)\;. \tag{10}$$

For the interaction operator in coordinate representation we use the operator with Coulomb behavior at small distances and linear confinement:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + \omega_0 + \frac{8\pi}{3}\,\frac{\alpha_s}{m_q\,m_{\bar{Q}}}\,\left(\vec{S}_q\,\vec{S}_{\bar{Q}}\right)\,\delta^3(\vec{r})\;. \tag{11}$$

Here $\vec{S}_q$ and $\vec{S}_{\bar{Q}}$ are spin operators of light and heavy quarks respectively. We choose the parameters in (11) as in the work [32], where meson spectra are calculated: $\alpha_s = 0.18$, $\sigma = 0.18$ GeV^2.

Now let us discuss the values of quark masses $m_u = m_d$, m_s , m_b , m_c. The mass of light quark $m_u = m_d$ can be fixed from the description of pion form factor in rather model-independent way [33]. As shown in reference [33], the value $m_u = 0.25$ GeV is the best suited for the existing experimental data for the pion form factor.

Table 1: The lepton decay constants of heavy–light pseudoscalar mesons. The values are given in MeV. The theoretical uncertainty of the calculations of the present Letter is not more than 5%. The starred theoretical values are averages over different calculations. The starred experimental value is a current world average.

	f_B	f_{Bs}	f_D	f_{Ds}
Present Letter	175	185	254	235
Lattice	160^{+59}_{-25} [2]	185^{+46}_{-10} [2]	194^{+68}_{-11} [2]	212^{+50}_{-11} [2]
calculations	147^{+34}_{-38} [3]	175^{+40}_{-36} [3]	192^{+42}_{-19} [4]	210^{+51}_{-19} [4]
	157^{+69}_{-20} [4]	171^{+71}_{-21} [4]	211^{+16}_{-26} [6]	240^{+30}_{-25} [5]
	179^{+52}_{-27} [6]	204^{+52}_{-16} [6]	$225\pm30^*$ [9, 10]	231^{+20}_{-13} [6]
	204^{+81}_{-37} [8]	187 ± 34 [7]		223^{+110}_{-108} [7]
	$200\pm30^*$ [9, 10]	242^{+81}_{-43} [8]		
Sum rules	$178\pm42^*$ [12]	232 ± 25 [13]	$188\pm48^*$ [12]	231 ± 24 [11]
			195 ± 20 [12]	
Experiments			$300^{+180+80}_{-150-40}$ [35]	$280\pm48^*$ [36]
				$286\pm44\pm41$ [37]

The mass of constituent s–quark m_s is obtained from the model- independent estimation of $SU(3)$–breaking parameter [34]: $(m_s - m_u)/m_u \simeq 0.37$.

The analogous hard restrictions on the heavy quark masses are not generally accepted. The following values for these masses are the most–used in current calculations: $1.60 \leq m_c \leq 1.88$ GeV, $4.60 \leq m_b \leq 5.28$ GeV. This uncertainty is the only source of the theoretical uncertainty in the calculation of the lepton decay constants in our work.

The parameter β in equation (8) is determined from the minimum conditions for the functional (7):

$$\min\, M_P\,(\beta) = M_P^{exp}, \tag{12}$$

where M_P^{exp}- experimental value of the pseudoscalar meson mass.

So, all parameters of our model are fixed. The results of our calculation of the lepton decay constants f_B, f_{Bs}, f_D, f_{Ds} are indicated in Table 1.

Let us discuss the obtained results. The firm experimental data exist for the value of f_{Ds} only. Therefore calculation of this value can be used to test our theoretical predictions for other lepton constants. This statement is particularly important for the constant f_B, because the measurement of f_B from $B \to l\,\bar{\nu}_l$ decay is currently not feasible [37]. Our result for f_{Ds} coincides with experiments within the experimental errors, but is far enough from the mean experimental values. At the same time it should be remarked that our f_{Ds} is in good agreement with the lattice and sum rules calculations.

The last remark is true for the constant f_B obtained in this Letter.

Our constant f_{Bs} is in reasonably good agreement with lattice calculations, but our result correlates poorly with the recent sum rules calculation from [13].

Finally, our value of f_D agrees closely with an average over different calculations on the lattice [9, 10], but it is in a rather poor agreement with those done with sum rules [12]. We can remark that the only experimental result for f_D [35] has large errors and thus of minor importance for existing calculations. Our theoretical uncertainty in the leptonic decay constant is not more than 5%.

Let us discuss briefly the relativistic effects contained in the equation (1). The non-relativistic expression for the lepton decay constant can be obtained from the equation (1) by taking the non-relativistic limit which gives the standard form in terms of the coordinate space wave function at zero value. The relativistic effects are very significant and have the following values for the calculated constants: for f_B and $f_{Bs} \simeq 40\%$ and 30%, for the more light mesons f_D and $f_{Ds} \simeq 80\%$ and 60%.

For the estimation of the power corrections in $1/m_Q$ (see [38] also) it is instructive to calculate the lepton decay constants in the limit $m_Q \to \infty$. The corresponding asymptotic estimation of the integral representation (1) with the wave function (8) gives the following expression for the leading term:

$$f_P \sim \frac{\sqrt{2\,n_c}}{\pi^{3/4}} \frac{b^{3/2}}{m_Q^{1/2}} \,. \tag{13}$$

The estimation of lepton decays constant from the asymptotic expression (13) is performed with the values of parameters listed above. The calculations give the following values for the lepton decay constants: $f_B = 177$ MeV, $f_{Bs} = 180$ MeV, $f_D = 360$ MeV, $f_{Ds} = 297$ MeV. One can see that the leading term gives good description for B, B_s– mesons, and the power corrections in $1/m_Q$ are large for D, D_s– mesons (compare with 1).

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