# Interaction of Spin-1 Particles with Electromagnetic Field

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**Abstract**

The Foldy-Wouthuysen transformation for relativistic spin-1 particles in an electromagnetic field is performed, and the Hamilton operator in this representation is determined. Finding either the Hamilton operator, or a diagonal form of relativistic wave equations makes possible to verify spin-1 particle theories. For this purpose, spin motion equations are derived. The performed analysis shows that the first-order Proca equations and some equivalent equations are correct. Some second-order wave equations are incorrect.

# 1 Introduction

Investigations of interaction of spin-1 particles with electromagnetic field are very important for high energy physics. These investigations make possible to verify some spin-1 particle theories. There are many works where a consistency of spin-1 particle theories has been considered (see Ref. [1] and references therein). However, these works do not draw final conclusions. Here we use more exact method of verification of spin-1 particle theories. For this purpose, a particle spin motion in an electromagnetic field is analyzed.

# 2 Equations for spin-1 particles

The situation in the spin-1 particle theory differs essentially from the situation in the spin-0 and spin-1/2 particle theories. The important difference is the large number of equations, describing spin-1 particles (vector mesons). First equations for vector mesons have been found by Proca [2]. For particles in an electromagnetic field they have the form:

$$U_{\mu\nu}=D_\mu U_\nu-D_\nu U_\mu,\quad D^\mu U_{\mu\nu}=m^2U_\nu,\quad D_\mu=\partial_\mu+ieA_\mu=\left(\frac{\partial}{\partial t}+ie\Phi,\nabla-ie\mathbf{A}\right),\tag{1}$$

where $A_\mu,\Phi$, and $\mathbf{A}$ are 4-potential, scalar potential, and vector potential of the electromagnetic field, respectively.

Other equations for spin-1 particles have been proposed later. These are Duffin-Kemmer-Petiaux (DKP) [3, 4, 5], Stuckelberg [6], and multispinor Bargmann-Wigner equations [7]. The DKP equation has the form:

$$(\beta_\mu D_\mu+m)\psi=0,\tag{2}$$

The wave function $\psi$ in Eq. (2) has 10 components, and $\beta_\mu$ are the $10\times10$ matrices. They satisfy the conditions:

$$\beta_\mu\beta_\nu\beta_\lambda+\beta_\lambda\beta_\nu\beta_\mu=\beta_\mu\delta_{\nu\lambda}+\beta_\lambda\delta_{\mu\nu}.$$

Corben and Schwinger [9] showed how to include an anomalous magnetic dipole term in Proca equations, and Young and Bludman [8] took account of an electric quadrupole moment.

Many first-order equations are equivalent and they transform one into another. This refers to the Proca, DKP and Bargmann-Wigner equations [8, 10]. The Stuckelberg equations differ essentially from other equations by the inclusion of an additional scalar field. As a result, the corresponding wave function has 11 components. The wave functions of the Proca and DKP equations have 10 components.

Some of the components of the Proca equations can be expressed in terms of the others. As a result, the equations for a 10-component wave function can be reduced to the equation for a 6-component one (the generalized Sakata-Taketani equation [8, 11]). As the components of wave function are two spinors, and the wave function of the generalized Sakata-Taketani equation is bispinor, this equation is most convenient for investigations.

Besides of the first-order equations, there are also second-order wave equations. These include second-order forms of the above mentioned equations and some other equations (e.g., the Shay-Good equation [12]). The Stuckelberg and Shay-Good equations are not equivalent to the Proca theory.

# 3 Consistency of spin-1 particle theories

Soon after the appearance of the Proca theory, the problem of its consistency was stated [13]. There are many works where several difficulties of spin-1 particle theories were investigated (for example, complex energy modes for particles in a uniform magnetic field, see Refs. [1, 14, 15, 16] and references therein). In the above mentioned works the problem of consistency of the spin-1 particle theories was solved qualitatively. However, there exists a more exact criterium of correctness of any particle theory. In Refs. [17, 18] it vas shown that for particles with arbitrary spin the spin motion is described by the Bargmann-Michel-Telegdi (BMT) equation:

$$\begin{aligned}\frac{d\boldsymbol{\xi}}{dt}=\frac{e}{2m}\Biggl\{\left(g-2+\frac{2}{\gamma}\right)[\boldsymbol{\xi}\times\mathbf{H}]-(g-2)\frac{\gamma}{\gamma+1}[\boldsymbol{\xi}\times\mathbf{v}](\mathbf{v}\cdot\mathbf{H})+\\ \left(g-2+\frac{2}{\gamma+1}\right)[\boldsymbol{\xi}\times[\mathbf{E}\times\mathbf{v}]]\Biggr\},\end{aligned}\tag{3}$$

where $\boldsymbol{\xi}$ is an average spin. This equation defines essentially the Hamilton operator (see Ref. [19]).

The Foldy-Wouthuysen (FW) representation [20] is the most convenient one for finding spin motion equations. In this representation the polarization operator has the very simple form:[1)]

$$\mathbf{\Pi}=\begin{pmatrix}\mathbf{S} & 0\\ 0 & -\mathbf{S}\end{pmatrix},$$

[1)] This operator is similar to the corresponding operator for spin-1/2 particles [21].

where $\mathbf{S}$ is the $3\times3$ spin matrix for spin-1 particles. In the FW representation the Hamiltonian $\mathcal{H}$ is diagonal by two spinors.

The operator equation of spin motion is determined by the commutator:

$$\frac{d\mathbf{\Pi}}{dt} = i[\mathcal{H}, \mathbf{\Pi}]. \tag{4}$$

For finding the equation for average spin $\boldsymbol{\xi}$, Eq. (4) should be averaged.

In other representations expressions for the polarization operators are much more bulky [21]. Therefore, finding the spin motion equations in other representations is not so easy.

# 4 Foldy-Wouthuysen transformation for spin-1 particles

The operator $U$, transforming a wave function to other representation, must be pseudo-unitary:

$$U^{-1} = \rho_3 U^\dagger \rho_3, \quad U^\dagger = \rho_3 U^{-1} \rho_3.$$

Here and below $\rho_i$ $(i = 1, 2, 3)$ are the Pauli matrices. Components of these matrices act on the spinors.

The transformed Hamiltonian equals to:

$$\mathcal{H}' = U\Bigl(\mathcal{H} - i\frac{\partial}{\partial t}\Bigr)U^{-1} + i\frac{\partial}{\partial t}.$$

Let us calculate the Hamiltonian and the spin motion equation in the FW representation with allowance for first-order terms in field strengths. Second-order terms in field strengths and terms containing derivatives of field strengths will be omitted. For this approximation the initial Hamiltonian has the form (see Ref. [8]):

$$\begin{gathered}\mathcal{H} = e\Phi + \rho_3 m + i\rho_2\frac{(\mathbf{S}\cdot\mathbf{D})^2}{m} - (\rho_3 + i\rho_2)\frac{(\mathbf{D}^2 + e\mathbf{S}\cdot\mathbf{H})}{2m} - \\ (\rho_3 - i\rho_2)\frac{e\kappa\mathbf{S}\cdot\mathbf{H}}{m} - \frac{e\kappa}{2m^2}(1 + \rho_1)\left[(\mathbf{S}\cdot\mathbf{E})(\mathbf{S}\cdot\mathbf{D}) - i\mathbf{S}\cdot[\mathbf{E}\times\mathbf{D}] - \mathbf{E}\cdot\mathbf{D}\right] + \\ \frac{e\kappa}{2m^2}(1 - \rho_1)\left[(\mathbf{S}\cdot\mathbf{D})(\mathbf{S}\cdot\mathbf{E}) - i\mathbf{S}\cdot[\mathbf{D}\times\mathbf{E}] - \mathbf{D}\cdot\mathbf{E}\right],\end{gathered} \tag{5}$$

where $\kappa =$const, $\mathbf{D} = \nabla - ie\mathbf{A}$; $\mathbf{E}$ and $\mathbf{H}$ are the strengths of the electromagnetic field.

There is the generalized Sakata-Taketani equation. It can be written in the other form:

$$\mathcal{H} = \rho_3\mathcal{M} + \mathcal{E} + \mathcal{O}, \quad \rho_3\mathcal{E} = \mathcal{E}\rho_3, \quad \rho_3\mathcal{O} = -\mathcal{O}\rho_3, \tag{6}$$

where $\mathcal{E}$ and $\mathcal{O}$ are the operators, commuting and anticommuting with $\rho_3$, respectively.

The operator $\mathcal{M}$ must commute with $\mathcal{O}$. In this case

$$\begin{aligned}
\mathcal{M} &= m + \frac{\boldsymbol{\pi}^2}{2m} - \frac{e}{m}\mathbf{S}\cdot\mathbf{H}, \\
\mathcal{E} &= e\Phi + \frac{e\kappa}{4m^2}(\mathbf{S}\cdot[\boldsymbol{\pi}\times\mathbf{E}] - \mathbf{S}\cdot[\mathbf{E}\times\boldsymbol{\pi}]) - \rho_3\frac{e(\kappa-1)}{2m}\mathbf{S}\cdot\mathbf{H}, \\
\mathcal{O} &= i\rho_1\frac{e\kappa}{2m^2}\left[\boldsymbol{\pi}\cdot\mathbf{E} + \mathbf{E}\cdot\boldsymbol{\pi} - S_iS_j(E_j\pi_i + \pi_jE_i)\right] + \\
& i\rho_2\left[\frac{\boldsymbol{\pi}^2}{2m} - \frac{(\boldsymbol{\pi}\cdot\mathbf{S})^2}{m} + \frac{e(\kappa-1)}{2m}\mathbf{S}\cdot\mathbf{H}\right],
\end{aligned} \tag{7}$$

where $\boldsymbol{\pi} = -i\mathbf{D} = -i\nabla - e\mathbf{A}$ is the kinetic momentum operator.

When

$$[\mathcal{M},\mathcal{O}] = 0, \quad [\mathcal{E},\mathcal{O}] = 0, \tag{8}$$

and external field is stationary, the following property can be proved. In this case the Hamiltonian $\mathcal{H}$ can be transformed exactly to the diagonal form by the operator[2)]

$$U = \frac{\epsilon + \mathcal{M} + \rho_3\mathcal{O}}{\sqrt{2\epsilon(\epsilon+\mathcal{M})}}, \quad U^{-} = \frac{\epsilon + \mathcal{M} - \rho_3\mathcal{O}}{\sqrt{2\epsilon(\epsilon+\mathcal{M})}}, \quad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}. \tag{9}$$

The transformed Hamiltonian is equal to:

$$\mathcal{H}' = \rho_3\epsilon + \mathcal{E}.$$

In the general case an external field is not stationary, $[\mathcal{M},\mathcal{O}] = 0$, but operators $\mathcal{E}$ and $\mathcal{O}$ do not commute. In this case the transformed Hamiltonian equals to:

$$\mathcal{H}' = \rho_3\epsilon + \mathcal{E}' + \mathcal{O}', \quad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}. \tag{10}$$

Here odd terms $\mathcal{O}'$ are small as compared with both $\epsilon$ and initial Hamiltonian $\mathcal{H}$. Therefore, a usual method of the non-relativistic FW transformation can be used. The transformation operator is [20, 22, 23]:

$$U' = \exp(iS'), \quad S' = -\frac{i}{4}\rho_3\left\{\frac{1}{\epsilon},\mathcal{O}'\right\}_+ = -\frac{i}{4}\left[\frac{\rho_3}{\epsilon},\mathcal{O}'\right],$$

where $\{\ldots,\ldots\}_+$ is an anticommutator. A further transformation is similar to one used for spin-1/2 particles [20, 22, 23, 24]. As compared with Refs. [20, 24] a particle mass must be replaced by the operator $\epsilon$ [22, 23]. If only major corrections are taken into account, the transformed Hamiltonian equals to:

$$\mathcal{H}'' = \rho_3\epsilon + \mathcal{E}' + \frac{1}{4}\rho_3\left\{\frac{1}{\epsilon},\mathcal{O}'^2\right\}_+. \tag{11}$$

This is the Hamiltonian in the FW representation.

2) A similar property was proved for spin-1/2 particles in Refs. [22, 23].

## 5 Hamiltonian and spin motion equation for spin-1 particles in electromagnetic field

The operator $\mathcal{O}'$ is proportional to the field strengths, $\mathbf{E}$ and $\mathbf{H}$. After the second transformation, a contribution of this operator to the Hamiltonian $\mathcal{H}''$ is proportional to $\mathcal{O}'^2$. This is a second-order term in the field strengths. Therefore, it is negligible. Thus, the Hamiltonian in the FW representation equals to:

$$\begin{aligned}\mathcal{H}'' = \rho_3\epsilon' + e\Phi + \frac{e}{4m}\Biggl[\left\{\left(\frac{g-2}{2}+\frac{m}{\epsilon'+m}\right)\frac{1}{\epsilon'}, (\mathbf{S}\cdot[\boldsymbol{\pi}\times\mathbf{E}] - \mathbf{S}\cdot[\mathbf{E}\times\boldsymbol{\pi}])\right\}_+ - \\ \rho_3\left\{\left(g-2+\frac{2m}{\epsilon'}\right), \mathbf{S}\cdot\mathbf{H}\right\}_+ + \rho_3\left\{\frac{g-2}{2\epsilon'(\epsilon'+m)}, \{\mathbf{S}\cdot\boldsymbol{\pi}, \boldsymbol{\pi}\cdot\mathbf{H}\}_+\right\}_+\Biggr],\end{aligned} \tag{12}$$

where $\boldsymbol{\pi} = -i\mathbf{D} = -i\nabla - e\mathbf{A}$ is the kinetic momentum operator and $g = \kappa + 1$ is the factor for an anomalous magnetic moment (AMM) description. The $g$ factor $g = g_{norm} = 1$ corresponds to the particle with a normal magnetic moment (Proca particle).

For the particular case of nonrelativistic particles in a magnetic field, formula (12) complies with the Hamiltonian derived by Case [25], who also used the FW transformation. This formula is also consistent with the Hamiltonian obtained in Ref. [26] for relativistic particles in a magnetic field that are characterized by the $g$ factor of $g = 2$.

From formulae (4),(12) we find that the equation of spin motion can be represented as[3)]

$$\begin{aligned}\frac{d\mathbf{\Pi}}{dt} = \frac{e}{4m}\Biggl[\left\{\left(g-2+\frac{2m}{\epsilon}\right), [\mathbf{\Sigma}\times\mathbf{H}]\right\} - \left\{\frac{g-2}{2\epsilon(\epsilon+m)}, [\mathbf{\Sigma}\times\boldsymbol{\pi}](\boldsymbol{\pi}\cdot\mathbf{H}) + \right. \\ \left.(\boldsymbol{\pi}\cdot\mathbf{H})[\mathbf{\Sigma}\times\boldsymbol{\pi}]\right\} - \left\{\left(\frac{g-2}{2}+\frac{m}{\epsilon+m}\right)\frac{1}{\epsilon}, [\mathbf{\Pi}\times[\mathbf{E}\times\boldsymbol{\pi}]] - [\mathbf{\Pi}\times[\boldsymbol{\pi}\times\mathbf{E}]]\right\}\Biggr].\end{aligned} \tag{13}$$

Since we can recast Eq. (13) into the form

$$\frac{d\mathbf{\Pi}}{dt} = \frac{1}{2}\left([\mathbf{\Omega}\mathbf{\Pi}] - [\mathbf{\Pi}\mathbf{\Omega}]\right),$$

a transition to a description within the semiclassical approximation consists in averaging the operator of the angular velocity of spin motion,

$$\begin{aligned}\mathbf{\Omega} = \frac{e}{4m}\Biggl[-\beta\left\{\left(g-2+\frac{2m}{\epsilon}\right), \mathbf{H}\right\} + \beta\left\{\frac{g-2}{2\epsilon(\epsilon+m)}, \boldsymbol{\pi}(\boldsymbol{\pi}\cdot\mathbf{H}) + (\boldsymbol{\pi}\cdot\mathbf{H})\boldsymbol{\pi}\right\} - \\ \left\{\left(\frac{g-2}{2}+\frac{m}{\epsilon+m}\right)\frac{1}{\epsilon}, [\mathbf{E}\times\boldsymbol{\pi}] - [\boldsymbol{\pi}\times\mathbf{E}]\right\}\Biggr]\end{aligned} \tag{14}$$

over the wave functions of stationary states. For free particles, the lower spinor is equal to zero in the FW representation. For particles in external fields, the ratio of the lower and the upper spinor is at maximum on the same order of magnitude as $W_{int}/E$, where $W_{int}$ is the energy of the particle-field interaction and $E$ is the total particle energy. Thus, we

[3)] The use of the matrix $\mathbf{\Sigma} = \rho_3\mathbf{\Pi}$ for the polarization operator also leads, upon a transition to the semiclassical approximation, to formula (14) (see below) for the angular velocity of spin rotation.

have $(\chi^\dagger\chi)/(\phi^\dagger\phi) \sim (W_{int}/E)^2$. For this reason, the contribution of the lower spinor is negligible small. In the semiclassical approximation, the angular velocity of spin rotation $\boldsymbol{\omega}$ is therefore given by:

$$\boldsymbol{\omega} = \frac{e}{2m}\left\{-\left(g-2+\frac{2}{\gamma}\right)\mathbf{H} + (g-2)\frac{\gamma}{\gamma+1}\mathbf{v}(\mathbf{v}\cdot\mathbf{H}) - \left(g-2+\frac{2}{\gamma+1}\right)[\mathbf{E}\times\mathbf{v}]\right\},$$

where $\gamma = \epsilon/m$ and $\mathbf{v} = \boldsymbol{\pi}/\epsilon$. The semiclassical equation of spin motion coincides with the BMT one.

Thus, a rigorous calculation has revealed that, upot the FW transformation, the Hamiltonian determined on the basis of Proca theory (with allowance for AMM) is fully consistent with the data from [17, 18] and with the BMT equation.

# 6 Discussion and summary

Obtained results show the first-order Proca equations and some equivalent equations lead to the spin motion equation coinciding with the BMT one. Therefore, the first-order Proca equations are correct. However, some other equations are not consistent with the BMT equation.

In Refs. [14, 15, 16, 27, 28] different second-order wave equations were transformed to diagonal forms. For this purpose, the second-order forms of Proca and multispinor equations and Shay-Good equation were used. In the weak-field approximation[4)] one obtains linear equations that are consistent neither with the Lagrangian from Refs. [17, 18] nor with the BMT equation.

Diagonalized equations for particles in a uniform magnetic field derived in Refs. [14, 15, 16, 27] for three different initial second-order wave equations, can be described in the following general form:

$$E^2\phi = \left[m^2 + \boldsymbol{\pi}^2 - e(1+\kappa)\mathbf{S}\cdot\mathbf{H} + \frac{e(1-\kappa)}{2m^2}\boldsymbol{\pi}_\perp^2(\mathbf{S}\cdot\mathbf{H}) + \zeta\right]\phi, \qquad (15)$$
$$\boldsymbol{\pi}_\perp = \boldsymbol{\pi} - (\boldsymbol{\pi}\cdot\mathbf{e}_H)\mathbf{e}_H,$$

where $\zeta$ is the designation for some other terms of the first order and higher in the magnetic field strength, and $\mathbf{e}_H = \mathbf{H}/H$. The quantities $\zeta$ are different for different equations. It can be shown that after the approximate transition to first-order wave equation,[5)] Eq. (15) takes the form:

$$E\phi = \left[\epsilon' - \frac{e(1+\kappa)}{2\epsilon'}\mathbf{S}\cdot\mathbf{H} + \frac{e(1-\kappa)}{4m^2\epsilon'}\boldsymbol{\pi}_\perp^2(\mathbf{S}\cdot\mathbf{H}) + \frac{1}{2\epsilon'}\zeta\right]\phi, \qquad (16)$$
$$\epsilon' = \sqrt{m^2+\boldsymbol{\pi}^2}.$$

Eq. (16) is not consistent with the BMT equation. It follows from Eq. (16) that the angular velocity of spin precession increases when the particle energy increases. This property also contradicts the BMT equation.

4) Within this approximation, the energy of the particle-field interaction is assumed to be low in relation to the total particle energy, including the rest energy.

5) The analogous transformation was performed in Ref. [29].

In all probability, the cause of this contradiction is an incorrect method of the transition from first-order to second-order wave equations. The transition cannot be performed by a merely elimination some components of the wave function, since this leads to the emergence of non-Hermitian terms in the resulting equation. After the elimination $U_{\mu\nu}$ from the Proca equations, the term $D_\mu D^\nu$ appears. It can be shown that it is non-Hermitian:

$$(D_\mu D^\nu)^\dagger = D^\nu D_\mu = D_\mu D^\nu + g^{\nu\rho}[D_\rho, D_\mu] = D_\mu D^\nu + ieg^{\nu\rho}F_{\rho\mu} \neq D_\mu D^\nu, \tag{17}$$

where $g^{\nu\rho} = \mathrm{diag}\{1, -1, -1, -1\}$ is the metric tensor and $F_{\rho\mu}$ is the tensor of electromagnetic field. This circumstance leads to complex values of the particle energy and to the non-orthogonality of wave functions. In this connection, the presence of imaginary values in the energy spectrum of a particle in a magnetic field[6)] is quite natural. Therefore, correct wave equations must be Hermitian.

Following conclusions can be made. Finding the spin motion equations is the good method of verification of the wave equations for spin-1 particles. FW representation is very convenient for an investigation of the spin motion. The performed analysis shows the first-order Proca equations and some equivalent equations correctly determine spin motion that is defined by the BMT equation. Some second-order wave equations for spin-1 particles lead to other spin motion equations and, therefore, they cannot be correct.

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