Quantum Mechanics as the Objective Local Theory

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Abstract

In the work it is shown that the principles "the objective local theory" and corollaries of the standard quantum mechanics are not in such antagonistic inconsistency as it is usually supposed. In the framework of algebraic approach, the postulates are formulated which allow constructing the updated mathematical scheme of quantum mechanics. This scheme incorporates the standard mathematical apparatus of quantum mechanics. Simultaneously, in it there is a mathematical object, which adequately describes individual experiment.

In works of Bell [1, 2] the concept of "the objective local theory" was introduced, in which the properties of a physical system exist objectively, irrespective of measurement and are fulfilled the requirements: 1) every system is characterized by some variables, probably, correlated for two systems; 2) the results of measurement of one system do not depend on whether measurements on other system is made; 3) the characteristics of statistical ensembles depends only on conditions in earlier times, "the retrospective causality" is impossible.

Actually, these requirements are the further development of the concept of "the physical reality", formulated in the famous work of Einstein-Podolsky-Rosen [3]: a) every element of the physical reality must have a counter-part in the complete physical theory; b) if, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity.

All these statements seem quite reasonable. However, they have one essential drawback — they badly agree with the basic conceptions of the standard quantum mechanics (theory of Bohr, Heisenberg, Dirac, von Neumann). On the other hand, the corollaries of the standard quantum mechanics perfectly agree with enormous range of phenomena.

In proposed work I want to show that the statements, formulated by Bell and Einstein-Podolsky-Rosen, and corollaries of the standard quantum mechanics are not in such antagonistic inconsistency, as it seems at the first glance. Slightly loosening initial suppositions of the standard quantum mechanics, not changing its corollaries, it is possible to reconcile them with the basic principles of "the complete objective local theory".

It is suggested to begin relaxing suppositions by abandoning the postulate of the standard quantum mechanics: self-adjoint linear operators in Hilbert space correspond to observable quantities.

Instead of this supposition we shall accept much weaker postulate of the algebraic approach to quantum theory: the elements of some algebra correspond to observable quantities. This phrase denotes only that observable quantities can be summed up, multiplied together, and multiplied by numbers. It is convenient to leave the framework of directly observable quantities and to consider their complex combinations, which further will be referred to as dynamical quantities.

Correspondingly we shall accept *the first postulate.*

To dynamical quantities there correspond elements of an involutive associative (generally) noncommutative algebra $\mathfrak{A}$*, obeying: (i) for each element* $\hat{R} \in \mathfrak{A}$ *there is the hermitian element* $\hat{A}$ ($\hat{A}^* = \hat{A}$) *such that* $\hat{R}^*\hat{R} = \hat{A}^2$*; (ii) if* $\hat{R}^*\hat{R} = 0$*, then* $\hat{R} = 0$.

Here and everywhere further it is supposed that the requirement of presence of the unity element $\hat{I}$ is included in the definition of the algebra. The hermitian elements of algebra $\mathfrak{A}$ correspond to observable quantities. The set of these elements will be designated $\mathfrak{A}_+$.

The so-called simultaneously measurable observables, for which there are measuring apparatuses (system of apparatuses), permitting in principle to measure them simultaneously to any desired degree of precision, play a preferential role in quantum mechanics. Strictly speaking, simultaneity is not a determinative fact. Therefore, it is better to call these observables compatible. Peculiarity of these observables is that for them there is a system of measuring apparatuses which allow repeatedly to measure these observables in an arbitrary sequence and the results for repeated measured observables do not vary.

We shall borrow *the second postulate* from the standard quantum mechanics.

Mutually commuting elements of the set $\mathfrak{A}_+$ *correspond to compatible observables.*

In connection with this postulate commutative subalgebras of the algebra $\mathfrak{A}$ will play essential role in the following. Let us designate a maximal real commutative subalgebra of the algebra $\mathfrak{A}$ by $\mathfrak{Q}_\xi$ ($\mathfrak{Q}_\xi \equiv \{\hat{Q}\}_\xi \in \mathfrak{A}_+$). It is an algebra of the simultaneously measurable observables.

The subscript ξ ($\xi \in \Xi$) distinguishes one such subalgebra from the other. If the algebra $\mathfrak{A}$ is commutative (algebra of classical dynamical quantities), the set Ξ comprises one element. If the algebra $\mathfrak{A}$ is noncommutative (algebra of quantum dynamical quantities), the set Ξ has potency of continuum.

The hermitian elements of the algebra $\mathfrak{A}$ are latent form of the observable quantities. The explicit form of an observable should be some number. In other words, for the representation of the explicit form of observables on hermitian elements of the algebra $\mathfrak{A}$ some functional φ should be set: $\varphi(\hat{A}) = A$ is a real number.

Physically, the latent form of an observable $\hat{A}$ becomes explicit as a measurement result. This means that the functional $\varphi(\hat{A})$ gives the value of the observable $\hat{A}$ which is obtained in a *concrete (individual)* measurement. We shall call this functional the physical state of the considered quantum object.

In an individual experiment can be measured only mutually commuting elements. The sum and the product of observed data should correspond to the sum and product of observables: $\hat{A}_1 + \hat{A}_2 \to A_1 + A_2$, $\hat{A}_1\hat{A}_2 \to A_1A_2$.

Let us use the following definition. Let $\mathfrak{B}$ be a complex (real) commutative algebra and φ is a linear functional on $\mathfrak{B}$. If

$$\varphi(\hat{B}_1\hat{B}_2) = \varphi(\hat{B}_1)\varphi(\hat{B}_2) \tag{1}$$

for all $\hat{B}_1 \in \mathfrak{B}$ and $\hat{B}_2 \in \mathfrak{B}$, then the functional φ refers to as by a complex (real) homomorphism on algebra $\mathfrak{B}$. The functional satisfying the equality (1) also is referred to as a multiplicative functional.

Let us accept *the third postulate.*

The physical state of a quantum object, appearing in the individual measurement, is described by a (generally, multi-valued) functional $\varphi(\hat{A})$ ($\hat{A} \in \mathfrak{A}_+$), for which the restriction ($\varphi_\xi(\hat{A})$) on any subalgebra $\mathfrak{Q}_\xi$ is single-valued and is a real homomorphism ($\varphi_\xi(\hat{A}) = A$ is real number).

The multi-valuedness of the functional φ is caused by the fact that the result of measurement can depend not only on the measured quantum object, but also on the nature of the measuring device. Let us say that the device, measuring an observable $\hat{A}$, is coordinated with subalgebra of observables $\mathfrak{Q}_\xi$ ($\hat{A} \in \mathfrak{Q}_\xi$), if for any physical state φ the measurement outcome is $\varphi_\xi(\hat{A})$. The coordination of the device with this or that subalgebra $\mathfrak{Q}_\xi$ is determined by its classical characteristics, i.e. by the construction, position in space and so forth.

Thus, the functional $\varphi(\ )$ does not describe the value of an observable $\hat{A}$ in a particular physical state. It describes reaction of the particular type of measuring device to the observable $\hat{A}$. Correspondingly a physical reality is not the value of an observable $\hat{A}$ in the considered physical state, but a reaction of the measuring device to this state.

If the functional $\varphi(\ )$ is single-valued in the point $\hat{A}$, we shall say that the corresponding physical state φ is stable on the observable $\hat{A}$.

It is possible to show [4] that the functionals, appearing in the third postulate, have the properties:

$$
\begin{aligned}
&/1/ \quad \varphi_\xi(0) = 0; \\
&/2/ \quad \varphi_\xi(\hat{I}) = 1; \\
&/3/ \quad \varphi_\xi(\hat{A}^2) \geq 0; \\
&/4/ \quad \text{if } \lambda = \varphi_\xi(\hat{A}), \text{ than } \lambda \in \sigma(\hat{A}); \\
&/5/ \quad \text{if } \lambda \in \sigma(\hat{A}) \text{ than } \lambda = \varphi_\xi(\hat{A}) \text{ for some } \varphi_\xi(\hat{A}).
\end{aligned}
\tag{2}
$$

Here $\sigma(\hat{A})$ is a spectrum of the element $\hat{A}$ in algebra $\mathfrak{A}$. In the standard quantum mechanics the corresponding properties of individual measurements are postulated, here they are consequences of the third postulate.

Multi-valuedness of the functional $\varphi(\ )$ allows to introduce it in a consistent manner. It is possible to make it by direct construction. Due to multi-valuedness of the functional φ, the conditions of the Cohen-Specker no-go theorem [5] are not fulfilled for it.

It seems natural to require additionally existence of an experiment (i.e. a functional $\varphi(\ )$), which would separate various observables. In other words, it is valid *the fourth postulate*

$$\varphi(\hat{A}_1) = \varphi(\hat{A}_2) \textit{ for all } \varphi, \textit{ iff } \hat{A}_1 = \hat{A}_2.$$

In the standard quantum mechanics more strong supposition is done.

Now we shall introduce a construction, which corresponds to a pure state in the standard quantum mechanics. The functional φ maps a set $\mathfrak{Q}_\xi = \{\hat{Q}\}_\xi$ (maximal commutative subalgebra) into the set of real numbers:

$$\{\hat{Q}\}_\xi \xrightarrow{\varphi} \{Q = \varphi(\hat{Q})\}_\xi.$$

For different functionals $\varphi_i(\ )$, $\varphi_j(\ )$ the sets $\{\varphi_i(\hat{Q})\}$, $\{\varphi_j(\hat{Q})\}$ can either differ or coincide. If for all $\hat{Q} \in \{\hat{Q}\}$ is valid $\varphi_i(\hat{Q}) = \varphi_j(\hat{Q}) = Q$, then we shall call physical

functionals $\varphi_i(\ )$ and $\varphi_j(\ )$ as $\{Q\}$-equivalent functionals. Let $\{\varphi\}_Q$ be the set of all $\{Q\}$-equivalent functionals, stable on observables of the subalgebra $\{\hat{Q}\}_\xi$. The set of corresponding physical states we shall call a quantum (pure) state, and we shall designate it by Ψ_Q.

The physical state cannot be uniquely fixed. Really, in one experiment we can measure only mutually commuting observables. All these observables are elements of a certain subalgebra $\mathfrak{Q}_\xi$. But all $\{Q\}$-equivalent functionals have identical values on any observable $\hat{Q} \in \{Q\}_\xi$. Therefore, performing an experiment we cannot distinguish one physical state from another, if both of them belong to the set $\{\varphi\}_Q$.

Strictly speaking, the above definition of quantum state is valid only for a physical system in which there are no identical particles. For description of identical particles it is necessary to extend definition of quantum state [6].

The set of physical states belonging to one quantum state has potency of continuum. Beforehand we have no special possibilities to organize an experiment so that the preassigned physical state appeared in it. Therefore, probability of appearance of the investigated quantum object in a preassigned physical state is equal to zero. It follows from this that the probability to meet two identical physical states is equal to zero. Moreover, in any finite or denumerable set of experiments the probability of meeting two identical physical states is equal to zero. Certainly, it does not signify that such event is impossible. Nevertheless, we can consider that in different experiments we always deal with different physical states.

We use certain definitions of the probability theory. Let us consider a set of physical states as a general set and each measurement of an observable as a trial. We shall consider experiment in which the measured value of the observable $\hat{A}$ is no more $\tilde{A}$, i.e. $\varphi(\hat{A}) = A \leq \tilde{A}$, as event $\tilde{A}$.

This event is not unconditioned. In quantum theory there is a specific constraint: one trial is not to be an event for measurement of two noncommuting observables. Probability of event $\tilde{A}$ is determined by the structure of the set of physical states and the mentioned constraint. Let this probability be equal to $P(\tilde{A})$.

Let us consider a sample of n mutually independent random trials. By definition, the probability of appearance of an event $\tilde{A}$ in each of these trials is equal to $P(\tilde{A})$. Let the event $\tilde{A}$ occur in this sample k_n times. Then, by definition, k_n/n is a probability of appearance of the event $\tilde{A}$ in the sample. By Bernoulli theorem (see [7]) the quantity k_n/n converges in probability to $P(\tilde{A})$ as $n \to \infty$.

Thus, the quantity to which k_n/n at $n \to \infty$ converges in probability is a probability of appearance of the event $\tilde{A}$ in any denumerable sample of independent random trials. This probability is identical for all such samples and is equal to $P(\tilde{A})$. It determines the probability measure $\mu(\varphi) \qquad (\varphi(\hat{A}) \leq \tilde{A})$ on any such sample.

Let us use the Khintchine's theorem (see [7]), formulated in the terms, which are used in the present work.

Let $A_1 = \varphi_1(\hat{A}), \ldots, A_n = \varphi_n(\hat{A})$ be a sequence of values of an observable $\hat{A}$ in a sample of physical states. Every A_i has the same distribution $(P(\tilde{A}))$ and the finite expectation $< A >$. Then the aleatory variable $\bar{A}_n = (A_1 + \ldots + A_n)/n$ converges in probability to $< A >$ as $n \to \infty$.

Thus,

$$< A >= \text{P-}\lim_{n\to\infty}\frac{1}{n}\Big(\varphi_1(\hat{A}) + \ldots + \varphi_n(\hat{A})\Big) = \int d\mu(\varphi)\,\varphi(\hat{A}). \qquad (3)$$

In the right-hand side it is necessary to integrate over denumerable random sample.

Denumerable random sample, which contains all experiments for the measurement of a magnitude of an observable $\hat{A}$, we shall call a set of relevant states for $\hat{A}$ and we shall designate as $[\varphi]^{\hat{A}}$. Similarly, we shall define a set of relevant states for an observable $\hat{B}$. Let $[\hat{A},\hat{B}] \neq 0$, then the observables $\hat{A}$ and $\hat{B}$ should be measured in different experiments. As the sets $[\varphi]^{\hat{A}}$ and $[\varphi]^{\hat{B}}$ are denumerable, the probability of their intersection is equal to zero. Therefore, for relevant states the constraint is fulfilled automatically with the probability equal to unity.

All reasoning remain in force, if the events satisfy a certain additional constraint. In particular, (it is of special interest): all physical states belong to a certain quantum state $\Psi_Q = \{\varphi\}_Q$.

In this case the formula (3) will be rewritten as follows

$$\Psi_Q(\hat{A}) = \text{P-}\lim_{n\to\infty}\frac{1}{n}\Big(\varphi_1(\hat{A}) + \ldots + \varphi_n(\hat{A})\Big) = \int_{\{\varphi\}_Q^{\hat{A}}} d\mu_Q(\varphi)\,\varphi(\hat{A}). \qquad (4)$$

Here $\{\varphi\}_Q^{\hat{A}} = \{\varphi\}_Q \cap [\varphi]^{\hat{A}} \qquad \varphi_i \in \{\varphi\}_Q^{\hat{A}}$.

The formula (4) defines a functional (quantum average) on the set $\mathfrak{A}_+$. The totality of quantum experiments uniquely testifies in favor of what we should accept *the fifth postulate.*

A functional $\Psi_Q(\ )$ *is linear on the set* $\mathfrak{A}_+$.
This means

$$\Psi_Q(\hat{A}+\hat{B}) = \Psi_Q(\hat{A}) + \Psi_Q(\hat{B}) \text{ and in the case, if } [\hat{A},\hat{B}] \neq 0.$$

Any element $\hat{R}$ of the algebra $\mathfrak{A}$ can uniquely be represented in form $\hat{R} = \hat{A} + i\hat{B}$, where $\hat{A},\hat{B} \in \mathfrak{A}_+$. Therefore, it is possible to extend the functional $\Psi_Q(\ )$ up to the linear functional on the algebra $\mathfrak{A}$: $\Psi_Q(\hat{R}) = \Psi_Q(\hat{A}) + i\Psi_Q(\hat{B})$.

The equality $\|\hat{R}\|^2 \equiv \sup_Q \Psi_Q(\hat{R}^*\hat{R})$ defines a norm of an element $\hat{R} \in \mathfrak{A}$. This norm has properties $\|\hat{R}^*\| = \|\hat{R}\|$, $\|\hat{R}^*\hat{R}\| = \|\hat{R}\|^2$. With such norm the algebra $\mathfrak{A}$ is C^*-algebra. Therefore, according to the Gelfand-Naumark-Segal construction (see, for example, [8]), the functional $\Psi_Q(\ )$ canonically generates a Hilbert space and the representation of the algebra $\mathfrak{A}$ by linear operators in this Hilbert space. In other words, in the proposed approach it is possible to reproduce the mathematical formalism of the standard quantum mechanics completely.

At the same time, such only mathematical notions as vectors of Hilbert space and operators in Hilbert space are not the primary elements of the theory in the proposed approach. They arise only at the second stage. The primary elements are the observables and physical states which are related directly to the results of the experiment. It is possible to say that they are connected with the material structure of the investigated quantum object and do not depend on the observer.

The physical state is "a physical reality", which Einstein did not see in the standard quantum mechanics, and so he considered that the quantum mechanics is the incomplete theory.

At the same time, the physical states can play a role of the Bell's variables which characterize physical system. Here, however, it is necessary to make an essential remark. Bell implied that these variables are certain numerical parameters (practically hidden parameters). On their basis he has obtained the famous inequality, which contradicts corollaries of the standard quantum mechanics and experiments, that is especially essential.

In the proposed approach physical states are nonlinear functionals (many-valued). The Bell's inequality does not follow from existence of such specific "hidden parameters". See my report at our Workshop of the previous year [9]. In this report it was shown that for such "hidden parameters" the reasoning of von Neumann [10] on impossibility of existence of hidden parameters is not valid.

Here, the basic fact is the nonlinearity of the functional describing the physical state. Actually von Neumann has shown that the linearity of a state is in the conflict with causality and the hypothesis about the hidden parameters. from this he has made a deduction that causality is absent at the microscopic level, and the causality occurs due to averaging over large number of noncausal events at the macroscopic level.

The approach formulated in the present work allows to solve the same conflict in the opposite way. It is possible to suppose that there is causality at the level of a single microscopic phenomenon, and the linearity is absent. The linearity of the (quantum) state occurs due to averaging over quantum ensemble. The transition from a single phenomenon to the quantum ensemble replaces the initial determinism by probabilistic interpretation.

The hypothesis about the physical states allows to solve the problem of a choice. The standard quantum mechanics considers only probability of this or that event. At the same time, in each concrete experiment we deal with a particular result. Who does the corresponding choice? The God playing dice? Abstract ego? Human consciousness? Mind of the observer? Any of the above variants does not satisfy me.

It seems that the physical state which is determined by the material structure of a concrete explored object allows quite objective (and materialistic) to solve the problem of choice.

At the same time, the proposed approach reserve to observer certain freedom of choice. The matter is that one physical state can belong to different sets $\{\varphi\}_Q$ and $\{\varphi\}_P$. Here in $\{\varphi\}_Q$ for definition of equivalence is used the set $\{\hat{Q}\}$ of mutually commuting observables $\hat{Q}$, and in $\{\varphi\}_P$ are used the observables $\hat{P} \in \{\hat{P}\}$, where observables $\hat{Q}$ and $\hat{P}$ do not commute among themselves. Then depending on a set ($\{\hat{Q}\}$ or $\{\hat{P}\}$), i.e. measuring apparatus, which the observer will choose (the freewill) for classification, the physical state φ will be referred either to the quantum state $\{\varphi\}_Q$ or to the quantum state $\{\varphi\}_P$.

Thus, in contrast to the physical state, the quantum state is not the quite objective characteristic of a quantum object. To my opinion, it explains a hypertrophied role which is ascribed to the observer in the standard quantum mechanics.

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