Facts and Fancies in the Randall-Sundrum Model

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Abstract

We discuss the background and the structure of the Randall-Sundrum model with two branes. We construct the second variation Lagrangian for this model, study its gauge invariance, find the corresponding equations of motion for the metric fluctuations and decouple them. We also derive effective four-dimensional Lagrangians for the physical degrees of freedom, which include the massless graviton, massive gravitons and the massless scalar radion. It is shown that the mass spectra of these fields and their couplings to the matter are different on different branes. We argue that in order to obtain a physically acceptable model on the negative tension brane, the inverse size of the extra dimension must be of the order of $1TeV$, and not of the order of the Planck mass, as it is assumed in all the papers on the phenomenology of the Randall-Sundrum model.

1 Introduction

In the last years there was a change of the paradigm in Kaluza-Klein theories. The traditional approach to these theories is based on the assumption due to T. Kaluza and O. Klein that extra dimensions are unobservable, because they are compactified and have a very small size of the order of the Planck length $l_{Pl} = 1/M_{Pl}$. Due to this fact, it was only the zero mode sector of theories with extra dimensions that could be observable at low energies.

In 1983 Rubakov and Shaposhnikov showed [1, 2] that extra dimensions could be unobservable at low energies, even if they were infinite. It was the origin of a new scenario for Kaluza-Klein theories, which was based on the idea of localization of fields on a domain wall. In this approach the masses of the Kaluza-Klein excitations are not directly connected with the size of extra dimensions, and these excitations may be a part of the low energy theory.

Later there appeared indications that a scenario of this type can arise in the superstrings theory [3] (see Ref. [4] for a review and references). In this approach, the fields of the standard model are assumed to be trapped by some mechanism on a three-dimensional hypersurface embedded into a space-time of dimension larger than 4 (the bulk). Such hypersurfaces are called 3-branes, or just branes.

The main advantage of such scenario is a solution to the hierarchy problem. It turns out that within the framework of this scenario extra dimensions may be much larger, than the Planck length [5] (or even infinite). In this case the Planck mass in multidimensional space (in the bulk) may be much less, than the Planck mass in four-dimensional space-time M_{Pl}. In other words, gravity in multidimensional space-time becomes "strong" not

at the energies of the order of $10^{19}\,GeV$, but at much lower energies, maybe of the order of $1 \div 10\,TeV$. The effects due to interaction of this "strong" quantum gravity with the fields of the standard model can be observable already at the energies, which are available at the existing colliders.

There were numerous attempts to calculate such effects by considering linearized quantum gravity, interacting with the fields of the standard model located on a brane, in the flat background in space-time $E = M \times T^d$, T^d being a d-dimensional torus. The assumption that the multidimensional background metric can be taken to be flat means that the proper gravitational field of the brane can be neglected. The validity of the results obtained within this approach depends upon whether this approximation is good or not so good. In any case, studying the equations of motion for multidimensional gravity interacting with a brane of non-zero tension, it is not difficult to understand that if extra dimensions are compact, there should exist at least two branes, and the background metric must be essentially non-flat.

In paper [6] an exact solution for a system of two branes interacting with gravity in five-dimensional space-time was found, which allowed one to study the effect of the gravitational field induced by the branes. This model is called the Randall-Sundrum model, and it is widely discussed in the literature (see Refs. [4, 8] for reviews and references). It turned out that this model also allowed a solution to the hierarchy problem due to an exponential warp factor appearing in the metric [6, 7], and there are many papers on the effects of linearized gravity in the background of the Randall-Sundrum solution. However, there are fancies in this model, which travel from one paper to another. First of all, it concerns the size of the extra dimension. All the papers on the phenomenology of the model assume it to be of the order of the Planck mass, which is absolutely erroneous. Another fancy is the explicit identification of the physical degrees of freedom of the model. Again most of the papers on the phenomenology of the model either completely drop the radion field, or use the Gaussian normal gauge, where this degree of freedom is absorbed by the four-dimensional gravitational field. The Gaussian normal gauge being local, i.e. defined not in the whole multidimensional space-time, this approach cannot provide for a consistent quantization of all the degrees of freedom of the model.

In the present contribution we are going to dispel these fancies. To this end, we give a detailed derivation of the Lagrangian of quadratic fluctuations around the Randall-Sundrum solution and discuss its properties. In particular, we put a special emphasis on a choice of a convenient gauge. We also carry out a diagonalization of this Lagrangian and of the corresponding equations of motion, so that the physical degrees of freedom are distinguished explicitly and can be consistently quantized by the standard methods. This allows us to study the interaction of the linearized gravity with matter on the branes in more detail, than it was done in other papers.

The paper is organized as follows. In Sect. 2 we derive an expression for the Lagrangian of quadratic fluctuations in an arbitrary background relevant for our purposes. In Sect. 3 this result is used for the calculation of the Lagrangian of quadratic fluctuations in the Randall-Sundrum model. Here we also discuss the gauge transformations in the model and the identification of the physical degrees of freedom. In Sect. 4 we calculate the couplings of the fields, arising from the five-dimensional metric, to the matter. We emphasize that, for the proper interpretation of the interactions, Galilean coordinates on the branes should

be used. Sect. 5 contains some discussion of the effective theories on the branes.

2 Second variation Lagrangian in an arbitrary background

First, let us consider the standard gravitational action with cosmological constant in $(4+d)$-dimensional space-time E with coordinates $\{x^M\}, M = 0, 1, \cdots d+3$:

$$S_g = \frac{1}{16\pi\hat{G}} \int_E (\mathcal{R} - \Lambda) \sqrt{-g}\, d^{4+d}x, \tag{1}$$

where $\hat{G}$ is the multidimensional gravitational constant. The signature of the metric g_{MN} is chosen to be $(-1, 1, \cdots 1)$.

Let γ_{MN} be a fixed background metric. We denote $\hat{\kappa} = \sqrt{16\pi\hat{G}}$ and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat{\kappa} h_{MN}. \tag{2}$$

If we substitute this formula into (1) and retain only the terms of the zeroth order in $\hat{\kappa}$, we get the following Lagrangian, which is usually called the second variation Lagrangian:

$$\begin{aligned} L_g^{(2)}/\sqrt{-\gamma} &= -\frac{1}{4}\left(\nabla_R h_{MN}\nabla^R h^{MN} - \nabla_R h \nabla^R h + 2\nabla_M h^{MN}\nabla_N h - \right. \\ &- \left. 2\nabla^R h^{MN}\nabla_M h_{RN}\right) + \frac{1}{2}(\mathcal{R} - \Lambda)\left(h_{MN}h^{MN} - \frac{1}{2}hh\right) + \\ &+ 2G^{MN}h_{MR}h^R_N - G^{MN}h_{MN}h. \end{aligned} \tag{3}$$

Here $\gamma = \det\gamma_{MN}$, $h = h^M_M$, ∇_M denotes the covariant derivative with respect to metric γ_{MN}, and $G_{MN} = \mathcal{R}_{MN} - \frac{1}{2}\gamma_{MN}(\mathcal{R} - \Lambda)$. This formula in rather complicated notations can be found in [9].

Action (1) is invariant under general coordinate transformations $y^M = y^M(x)$, the corresponding transformation of the metric being

$$g'_{RS}(y)\frac{\partial y^R}{\partial x^M}\frac{\partial y^S}{\partial x^N} = g_{MN}(x). \tag{4}$$

Let us consider infinitesimal coordinate transformations

$$y^M(x) = x^M + \hat{\kappa}\xi^M(x). \tag{5}$$

Representing the initial and the transformed metrics as

$$\begin{aligned} g_{MN}(x) &= \gamma_{MN}(x) + \hat{\kappa} h_{MN}(x), \\ g'_{MN}(y) &= \gamma_{MN}(y) + \hat{\kappa} h'_{MN}(y), \end{aligned}$$

substituting these formulas into Eq. (4) and keeping the terms up to the first order in $\hat{\kappa}$, we get the transformation law for the variation h_{MN},

$$h'_{MN}(x) = h_{MN}(x) - \gamma_{MR}\partial_N\xi^R - \gamma_{NR}\partial_M\xi^R - \xi^R\partial_R\gamma_{MN},$$

which can be equivalently rewritten as [10]

$$h'_{MN}(x) = h_{MN}(x) - (\nabla_M \xi_N + \nabla_N \xi_M)\,. \tag{6}$$

It is not difficult to check that the action built with the second variation Lagrangian (3) is invariant under transformations (6). Therefore, the latter can be interpreted as the gauge transformations of the field h_{MN}.

Usually the background metric γ_{MN} is a solution of the Einstein equations

$$\mathcal{R}_{MN} - \frac{1}{2}\gamma_{MN}\left(\mathcal{R} - \Lambda\right) = 8\pi\hat{G}T_{MN} \tag{7}$$

with a certain energy-momentum tensor of the matter T_{MN}. We express G_{MN} and $(\mathcal{R} - \Lambda)$ in terms of the energy-momentum tensor T_{MN} and substitute it into (3), which gives the second variation Lagrangian in the form

$$\begin{aligned} L_g^{(2)}/\sqrt{-\gamma} &= -\frac{1}{4}\left(\nabla_R h_{MN}\nabla^R h^{MN} - \nabla_R h \nabla^R h + 2\nabla_M h^{MN}\nabla_N h - \right. \\ &- \left. 2\nabla^R h^{MN}\nabla_M h_{RN}\right) + \frac{\Lambda}{2(d+2)}\left(h_{MN}h^{MN} - \frac{1}{2}hh\right) - \\ &- \frac{4\pi\hat{G}}{d+2}T^R_R\left(h_{MN}h^{MN} - \frac{1}{2}hh\right) + \left(8\pi\hat{G}T^{MN}h_{MR}h^R_N - 4\pi\hat{G}T^{MN}h_{Mn}h\right). \end{aligned} \tag{8}$$

In the next section this expression will be calculated for the Randall-Sundrum model.

3 Second variation Lagrangian of the Randall-Sundrum model

The Randall-Sundrum model [6] describes the gravity in a five-dimensional space E with two branes embedded into it (it is often referred to as the RS1 model). We denote the coordinates by $\{x^\mu, x^4\}$, $\mu = 0, 1, 2, 3$, the coordinate $x^4 \equiv y$ parameterizing the fifth dimension. It forms the orbifold S^1/Z_2, which is the circle of the circumference $2R$ with points (y) and $(-y)$ identified. Correspondingly, the points (x, y) and $(x, -y)$ of the space-time E are also identified, and the metric g_{MN} satisfies the orbifold symmetry conditions

$$g_{\mu\nu}(x,-y) = g_{\mu\nu}(x,y), \quad g_{\mu 4}(x,-y) = -g_{\mu 4}(x,y), \quad g_{44}(x,-y) = g_{44}(x,y). \tag{9}$$

The branes are located at the fixed points of the orbifold, $y = 0$ and $y = R$. In addition we have the usual periodicity condition which identifies points (x, y) and $(x, y + 2nR)$.

The action of the model is equal to

$$S = S_g + S_1 + S_2, \tag{10}$$

where S_g is given by (1) with $d = 1$ and the action of the branes looks like

$$S_1 = V_1 \int_E \sqrt{\tilde{g}}\delta(y) d^5x, \tag{11}$$

$$S_2 = V_2 \int_E \sqrt{\tilde{g}}\delta(y - R) d^5x, \tag{12}$$

where $\tilde{g} = \det g_{\mu\nu}$ is the determinant of the induced metric and the subscripts 1 and 2 label the branes.

The Randall-Sundrum solution for the metric is given by

$$\gamma_{MN} dx^M dx^N = e^{2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + (dy)^2, \tag{13}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y)$ in the interval $-R \leq y \leq R$ is equal to $\sigma(y) = -k|y|$. This form of the metric was first put forward in paper [2].

The parameter k has the dimension of mass, and $\Lambda, V_{1,2}$ are related to it as follows:

$$\Lambda = -12k^2, \quad V_1 = -V_2 = -\frac{3k}{4\pi\hat{G}}.$$

We see that brane 1 has positive energy density, whereas brane 2 has negative one. The function σ has the properties

$$\partial_4 \sigma = -k \, sign(y), \quad \frac{\partial^2 \sigma}{\partial y^2} = -2k(\delta(y) - \delta(y - R)) \equiv -2k\tilde{\delta}. \tag{14}$$

Here and below we use the notation $\partial_4 \equiv \frac{\partial}{\partial y}$.

Now we construct the second variation Lagrangian for the Randall-Sundrum model. To this end we parameterize the metric in accordance with (2), substitute it into action (10) and keep the terms of the zeroth order in $\hat{\kappa}$. The contribution of the term S_g is given by (8) with $d = 1$ and

$$T_{MN} = -\frac{3k}{4\pi\hat{G}} \sqrt{\frac{\tilde{\gamma}}{\gamma}} \gamma_{\mu\nu} \delta^\mu_M \delta^\nu_N \tilde{\delta}, \tag{15}$$

$\tilde{\gamma}$ denoting the determinant of $\gamma_{\mu\nu}$. The contribution of the branes is given by

$$\Delta L^{(2)}_{1,2} = 3k(h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} \tilde{h}\tilde{h}) \tilde{\delta} \sqrt{-\tilde{\gamma}}, \tag{16}$$

where $\tilde{h} = \gamma^{\mu\nu} h_{\mu\nu}$.

Thus, the quadratic Lagrangian for the variation h_{MN} can be written as follows [11]:

$$\begin{aligned} L/\sqrt{-\gamma} &= -\frac{1}{4} \left(\nabla_R h_{MN} \nabla^R h^{MN} - \nabla_R h \nabla^R h + 2\nabla_M h^{MN} \nabla_N h - \right. \\ &- \left. 2\nabla^R h^{MN} \nabla_M h_{RN} \right) - 2k^2 (h_{MN} h^{MN} - \frac{1}{2} hh) + \\ &+ \left[4k(h_{MN} h^{MN} - \frac{1}{2} hh) + 3k h\tilde{h} - 6k h_{M\nu} h^{M\nu} + 3k(h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} \tilde{h}\tilde{h}) \right] \tilde{\delta}. \end{aligned} \tag{17}$$

It turns out to be convenient to transform the term $\frac{1}{2} \nabla^R h^{MN} \nabla_M h_{RN}$ to the standard Fierz-Pauli form

$$\frac{1}{2} \nabla^R h^{MN} \nabla_M h_{RN} = \frac{1}{2} \nabla_M h^{MN} \nabla^R h_{RN} + \frac{1}{2} h^{MN} h^{PQ} \mathcal{R}_{MNPQ} - \frac{1}{2} h^{MN} h_{NP} \mathcal{R}^P_M, \tag{18}$$

where $\mathcal{R}_{MNPQ}$ and $\mathcal{R}^P_M$ are the curvature and the Ricci tensors of the metric γ_{MN}. If we calculate these terms explicitly, the Lagrangian becomes

$$\begin{aligned} L/\sqrt{-\gamma} &= -\frac{1}{4}\left(\nabla_R h_{MN}\nabla^R h^{MN} - \nabla_R h \nabla_R h + 2\nabla_M h^{MN}\nabla_N h - \right. \\ &- \left. 2\nabla_M h^{MN}\nabla^R h_{RN}\right) + \frac{k^2}{2}(h_{MN}h^{MN} + hh) + \\ &+ \left[-2k h_{MN}h^{MN} + kh\tilde{h} - k h_{M\nu}h^{M\nu} + 3k(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}\tilde{h}\tilde{h})\right]\tilde{\delta}. \end{aligned} \tag{19}$$

Now let us discuss the gauge invariance of this Lagrangian. The gauge transformations (6) in the case under consideration can be found explicitly and turn out to be

$$\begin{aligned} h'_{\mu\nu}(x,y) &= h_{\mu\nu}(x,y) - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu + 2\gamma_{\mu\nu}\partial_4 \sigma \xi_4) \\ h'_{\mu 4}(x,y) &= h_{\mu 4}(x,y) - (\partial_\mu \xi_4 + \partial_4 \xi_\mu - 2\partial_4 \sigma \xi_\mu) \\ h'_{44}(x,y) &= h_{44}(x,y) - 2\partial_4 \xi_4, \end{aligned} \tag{20}$$

where the functions $\xi^M(x,y)$ satisfy the orbifold symmetry conditions

$$\begin{aligned} \xi^\mu(x,-y) &= \xi^\mu(x,y) \\ \xi^4(x,-y) &= -\xi^4(x,y). \end{aligned} \tag{21}$$

These gauge transformations in other parameterizations were discussed in papers [12] and [13]. We will use them to remove the gauge degrees of freedom of the field h_{MN}. To this end we first make a gauge transformation with

$$\xi_4(x,y) = \frac{1}{4}\int_{-y}^{y} h_{44}(x,y')\,dy' - \frac{y}{4R}\int_{-R}^{R} h_{44}(x,y')\,dy'. \tag{22}$$

One can easily see that ξ_4 satisfies the orbifold symmetry condition. After this transformation h_{44} takes the form

$$h'_{44}(x) = \frac{1}{2R}\int_{-R}^{R} h_{44}(x,y')\,dy' \tag{23}$$

and therefore it does not depend on y. Moreover, there are no remaining gauge transformations with ξ_4.

Now let us consider the components $h_{\mu 4}$. Due to orbifold symmetry

$$h_{\mu 4}(x,-y) = -h_{\mu 4}(x,y), \tag{24}$$

and the gauge transformations for $h_{\mu 4}$ read

$$h'_{\mu 4}(x,y) = h_{\mu 4}(x,y) - (\partial_4 \xi_\mu - 2\partial_4 \sigma \xi_\mu). \tag{25}$$

Let us show that we can impose the condition $h'_{\mu 4} = 0$. One can easily find that the corresponding equation for ξ_μ is

$$\partial_4\left(e^{-2\sigma}\xi_\mu\right) = e^{-2\sigma}h_{\mu 4}.$$

Thus, we have

$$\xi_\mu(x,y) = e^{2\sigma}\int_0^y e^{-2\sigma} h_{\mu 4}(x,y')\,dy', \tag{26}$$

and ξ_μ satisfies the orbifold symmetry condition (21). We will call the gauge with

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x) \tag{27}$$

the unitary gauge. After imposing this gauge, there remain gauge transformations satisfying

$$\partial_4\left(e^{-2\sigma}\xi_\mu\right) = 0. \tag{28}$$

They will be important for removing the gauge degrees of freedom of the massless mode of the gravitational field.

Varying the action corresponding to Lagrangian (19), we get the following equations of motion:

$$\begin{aligned} &\tfrac{1}{2}\nabla_R\nabla^R h_{MN} - \tfrac{1}{2}\gamma_{MN}\nabla_R\nabla^R h + \tfrac{1}{2}\nabla_M\nabla_N h + \tfrac{1}{2}\gamma_{MN}\nabla^R\nabla^S h_{RS} - \\ &-\tfrac{1}{2}\nabla_M\nabla^R h_{RN} - \tfrac{1}{2}\nabla_N\nabla^R h_{RM} + k^2(h_{MN} + \gamma_{MN}h) + [-4kh_{MN} - \\ &-k(h_{M\nu}\delta^\nu_N + h_{\mu N}\delta^\mu_M) + k\gamma_{MN}\tilde{h} + k\delta^\mu_M\delta^\nu_N\gamma_{\mu\nu}h + 6k\delta^\mu_M\delta^\nu_N(h_{\mu\nu} - \tfrac{1}{2}\gamma_{\mu\nu}\tilde{h})\Big]\,\tilde{\delta} = 0. \end{aligned} \tag{29}$$

Contracting these equations, we get a very simple equation

$$\frac{3}{2}\nabla^R\nabla^S h_{RS} - \frac{3}{2}\nabla_R\nabla^R h + 6k^2 h - 3k\tilde{h}\tilde{\delta} = 0, \tag{30}$$

which turns out to be very useful. The equations of motions for different components are derived from (29) and in the unitary gauge (27) take the form:

1) $\mu\nu$-component

$$\begin{aligned} &\frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial y^2}\right) - 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \frac{1}{2}\partial_\mu\partial_\nu\phi + \\ &+ \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \frac{\partial^2\tilde{h}}{\partial y^2} - 4\partial_4\sigma\partial_4\tilde{h} - \partial_\rho\partial^\rho\phi + 12k^2\phi\right) \\ &+ \left[2kh_{\mu\nu} - 3k\gamma_{\mu\nu}\phi\right]\tilde{\delta} = 0 \end{aligned} \tag{31}$$

2) $\mu 4$-component,

$$\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = 0, \tag{32}$$

which plays the role of a constraint,

3) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = 0. \tag{33}$$

The contracted equation can also be rewritten in this gauge to be

$$\frac{3}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{15}{2}\partial_4\sigma\partial_4\tilde{h} - \frac{3}{2}\frac{\partial^2\tilde{h}}{\partial y^2} - \frac{3}{2}\partial_\mu\partial^\mu\phi + 30k^2\phi - 12k\phi\tilde{\delta} = 0. \tag{34}$$

Let us first consider the 44-component and the contracted equation. Multiplying Eq. (33) by 3 and subtracting it from Eq. (34), we obtain the following equation, containing $\tilde{h}$ and ϕ only:

$$\frac{\partial^2 \tilde{h}}{\partial y^2} + 2\partial_4\sigma\partial_4\tilde{h} - 8k^2\phi + 8k\phi\tilde{\delta} + \partial_\mu\partial^\mu\phi = 0. \tag{35}$$

To extract the physical degrees of freedom of the model we represent the multidimensional gravitational field as

$$h_{\mu\nu} = b_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial_\nu\phi, \tag{36}$$

with $\sigma = \sigma(y)$ and c being a constant. We will see that the field $b_{\mu\nu}(x, y)$ describes a massless graviton [6, 7] and massive Kaluza-Klein spin-2 fields, whereas $\phi(x)$ describes a scalar field called the radion. Apparently, the radion field was first identified in Ref. [12]. As it will be shown, relation (36) with an appropriately chosen constant c diagonalizes the second variation Lagrangian and decouples the equations of motion (31)-(33). The form of the substitution (36) is suggested by the form of the gauge transformation that transforms the theory from the local Gaussian normal coordinates (i.e. coordinates corresponding to $g_{44} = 1$, $g_{\mu 4} = 0$) [12], where the second variation Lagrangian is diagonal in the bulk by construction, to our coordinates. We would like to note that, in fact, the equations of motion derived in Ref. [12] remain coupled at the fixed points of the orbifold, i.e. at the points where the branes are located. To decouple them the term $\sim e^{-2\sigma}$ in Eq. (36) is needed. It follows from this expression that

$$\tilde{h} = \tilde{b} + 4(\sigma - c)\phi + \frac{1}{2} + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial^\mu\phi. \tag{37}$$

Substituting (37) into Eq. (35) we get the equation

$$\partial_4(e^{2\sigma}\partial_4\tilde{b}) + 2c\partial_\mu\partial^\mu\phi + \frac{2}{k}e^{2\sigma}(\sigma - c + c\,e^{-2\sigma})\partial_\mu\partial^\mu\phi\,\tilde{\delta} = 0.$$

Choosing the constant c so that the boundary term, proportional to $\tilde{\delta}(y)$, vanishes, i.e.

$$c = \frac{kR}{e^{2kr} - 1}, \tag{38}$$

we reduce the equation to the following one:

$$\partial_4(e^{2\sigma}\partial_4\tilde{b}) + 2c\partial_\mu\partial^\mu\phi = 0. \tag{39}$$

Next, taking into account that $\gamma_{\mu\nu} = e^{2\sigma}\eta_{\mu\nu}$, we pass to the flat metric in this equation and denote $b = \eta^{\mu\nu}b_{\mu\nu}$. Then we get

$$\partial_4(e^{2\sigma}\partial_4(e^{-2\sigma}b)) + 2ce^{-2\sigma}\partial_\mu\partial^\mu\phi = 0. \tag{40}$$

Let us consider Fourier expansions of both terms in y. Since the term with the derivative ∂_4 has no zero modes, this equation implies that the radion field is a free massless field [12], i.e

$$\partial_\mu\partial^\mu\phi = 0, \tag{41}$$

and $\partial_4(e^{-2\sigma}b) = e^{-2\sigma}f(x)$. Applying the same reasoning to the last relation, we get

$$\partial_4(e^{-2\sigma}b) = 0.$$

Recall that we have at our disposal the gauge transformations, satisfying (28). With the help of these transformations, we can impose the gauge

$$\tilde{b} = b = 0. \tag{42}$$

It is easy to see that there remain gauge transformations parameterized by $\xi_\mu = e^{2\sigma}\epsilon_\mu(x)$ with $\epsilon_\mu(x)$ satisfying $\partial^\mu\epsilon_\mu = 0$. Substituting expression (36) into Eqs. (32), (33) and passing to gauge (42), we arrive at the following system of relations:

$$\partial^\mu\partial^\nu b_{\mu\nu} = 0, \tag{43}$$

$$\partial_4(e^{-2\sigma}\partial^\mu b_{\mu\nu}) = 0. \tag{44}$$

The remaining gauge transformations are sufficient to impose the condition

$$\partial^\mu b_{\mu\nu} = 0. \tag{45}$$

Conditions (42) and (45) define the gauge often called the transverse-traceless (TT) gauge. Having imposed this gauge, we are still left with residual gauge transformations

$$\xi_\mu = e^{2\sigma}\epsilon_\mu(x), \quad \partial_\rho\partial^\rho\epsilon_\mu = 0, \quad \partial^\mu\epsilon_\mu = 0, \tag{46}$$

which, as we will see shortly, are important for determining the number of degrees of freedom of the massless mode of $b_{\mu\nu}$.

In the gauge (42), (45) the constraint (32) and the 44-equation (33) are trivially satisfied. Let us turn to Eq. (31). Substituting expressions (36), (37) into it and passing to TT gauge, we transform this equation to the following form:

$$\frac{1}{2}\partial_\rho\partial^\rho b_{\mu\nu} + \frac{1}{2}\frac{\partial^2 b_{\mu\nu}}{\partial y^2} - 2k^2 b_{\mu\nu} + 2k b_{\mu\nu}\tilde{\delta} = 0. \tag{47}$$

This equation for the transverse-traceless part of the metric fluctuations was, in fact, postulated in paper [6]. Thus, we have decoupled all the equations.

To summarize, fluctuations around the Randall-Sundrum solution are described by the spin-2 field $b_{\mu\nu}(x,y)$ and the massless radion field $\phi(x)$. Their classical equations of motion are given by Eq. (47) and Eq. (41) respectively.

Now let us consider the second variation action of the theory. Substituting (36), (37) with c given by (38) into the Lagrangian (19) and taking into account the TT gauge conditions (42), (45) for $b_{\mu\nu}$, we obtain

$$L/\sqrt{-\gamma} = -\frac{1}{4}\partial_\rho b_{\mu\nu}\partial^\rho b^{\mu\nu} - \frac{1}{4}(\partial_4 b_{\mu\nu} - 2\partial_4\sigma b_{\mu\nu})(\partial_4 b^{\mu\nu} + 2\partial_4\sigma b^{\mu\nu}) - \frac{3}{4}c\, e^{-2\sigma}\partial_\mu\phi\partial^\mu\phi. \tag{48}$$

To get an effective Lagrangian for the system, it remains to decompose the field $b_{\mu\nu}$ into the modes with definite masses and to integrate L, Eq. (48), over y. After integration over y we obtain the following four-dimensional effective Lagrangian for the field ϕ:

$$L_\phi = -\frac{3}{2}\frac{kR^2}{e^{2kR}-1}\partial_\mu\phi\partial^\mu\phi, \tag{49}$$

where the index is raised with the flat metric. To bring the kinetic term to the canonical form we have to rescale the field according to

$$\phi = \sqrt{\frac{e^{2kR}-1}{3kR^2}}\varphi. \tag{50}$$

Let us turn to the mode expansion of the field $b_{\mu\nu}$. In what follows we denote $y = y$. Following Refs. [7, 14, 15], we first find eigenfunctions $\psi_n(y)$ and eigenvalues m_n of the problem

$$\left[\frac{1}{2}\frac{d^2}{dy^2} + 2k(\delta(y) - \delta(y-R)) - 2k^2\right]\psi_n(y) = -\frac{m_n^2}{2}e^{2k|y|}\psi_n(y). \tag{51}$$

The operator in the l.h.s. is a part of the equation (47). Eq. (51) can be solved exactly. To this end we first introduce the variable

$$\xi = \frac{m_n}{k}e^{k|y|}.$$

Note that the eigenvalue m_n enters the definition of ξ. Making the change of variables in Eq. (51) we transform it to

$$\left[\frac{d^2}{d\xi^2} + \frac{1}{\xi}\frac{d}{d\xi} + 1 - \frac{4}{\xi^2} + \frac{4}{\xi}\left(\delta(\xi-\xi_1) - \delta(\xi-\xi_2)\right)\right] f(\xi) = 0, \tag{52}$$

where $\xi_1 = m_n/k$ and $\xi_2 = m_n/k\exp(kR)$. This equation without the δ-function terms is just the Bessel equation, the general solution being a linear combination of Bessel and Neumann functions $J_2(\xi)$ and $Y_2(\xi)$:

$$Z_2(\xi) = aJ_2(\xi) + bY_2(\xi).$$

The term with δ-functions can be taken into account by imposing the boundary condition

$$Z_2'(\xi) + \frac{2}{\xi}Z_2(\xi) = 0$$

at $\xi = \xi_1 = m_n/k$ and $\xi = \xi_2 = m_n/k\exp(kR)$. The first boundary condition, at $\xi = \xi_1$, can be satisfied by an appropriate choice of the coefficients a and b:

$$Z_2(\xi) = Y_1\left(\frac{m_n}{k}\right)J_2(\xi) - J_1\left(\frac{m_n}{k}\right)Y_2(\xi). \tag{53}$$

The second boundary condition, at $\xi = \xi_2$, defines the mass spectrum of the theory and can be rewritten as

$$J_1(\xi_1)Y_1(t\xi_1) - J_1(t\xi_1)Y_1(\xi_1) = 0, \tag{54}$$

where $t = \exp(kR)$. There exists a theorem about such combinations of products of Bessel and Neumann functions, which asserts that for $t > 1$ this combination is an even function of ξ_1 and its zeros are real and simple [16]. Thus, the non-zero masses of tensor gravitons are defined by the positive zeros γ_n, $n = 1, 2, \cdots$ of this combination and explicitly given by the formula

$$m_n = \gamma_n k \equiv \beta_n k\exp(-kR). \tag{55}$$

As it was discussed in many papers (see Refs. [6, 7, 14, 15]), the product kR is chosen to be of the order $30 \div 35$ in order to solve the hierarchy problem. Since $\exp(-kR)$ is a tiny factor, for eigenvalues with small n, such that $\beta_n \sim \mathcal{O}(1)$, the ratio $m_n/k \ll 1$. Using the expansions for the Bessel functions for the small argument it is easy to see that such β_n are approximately given by the roots of the equation $J_1(\beta_n) = 0$.

Let us describe the complete orthonormal system $\{\psi_n(y), n = 0, 1, \cdots\}$ of solutions of equation (51), satisfying

$$\int_{-R}^{R} e^{2k|y|}\psi_m(y)\psi_n(y)dy = \delta_{mn}.$$

The normalized functions $\psi_n(y)$ corresponding to the mass eigenvalue $m_0 = 0$ and non-zero masses $m_n = \gamma_n k > 0$ are given by

$$\begin{aligned}
\psi_0(y) &= N_0\, e^{-2k|y|}, \quad N_0 = \frac{k^{\frac{1}{2}}}{(1-\exp(-2kR))^{\frac{1}{2}}} && (56)\\
\psi_n(y) &= N_n \left(Y_1(\gamma_n)\, J_2\left(\gamma_n e^{k|y|}\right) - J_1(\gamma_n)\, Y_2\left(\gamma_n e^{k|y|}\right)\right) \\
N_n &= \frac{k^{\frac{1}{2}}}{(\exp(2kR) Z_2^2(\gamma_n \exp(kR)) - Z_2^2(\gamma_n))^{\frac{1}{2}}}, \quad \gamma_n = \beta_n e^{-kR}, && (57)
\end{aligned}$$

where Z_2 was defined in (53). Now we calculate the values of the eigenfunctions for small n at $y = 0$ and $y = R$, which will be used later. Taking into account that $\exp(kR) \gg 1$ we obtain the following expressions:

$$\begin{aligned}
\psi_0(0) &= N_0 \approx \sqrt{k}; \quad \psi_n(0) \approx \frac{\sqrt{k}}{|J_2(\beta_n)|} e^{-kR}, \\
\psi_0(R) &= N_0 e^{-2kR} \approx \sqrt{k} e^{-2kR}; \quad \psi_n(R) \approx -\sqrt{k} e^{-kR}.
\end{aligned}$$

Finally, we expand $b_{\mu\nu}(x, y)$ as follows:

$$b_{\mu\nu}(x, y) = \sum_n b^n_{\mu\nu}(x)\psi_n(y), \qquad (58)$$

$b^n_{\mu\nu}(x)$ being ordinary spin-2 transverse traceless 4-dimensional fields, which can be quantized by the standard procedure. Substituting this decomposition into (48), passing to the corresponding action, integrating explicitly over the orbifold and rescaling the radion field φ in accordance with (50), we get the effective action for the physical degrees of freedom of the 5-dimensional gravitational field

$$S_{eff} = \int \left(-\frac{1}{4} \sum_n (\partial_\mu b^n_{\rho\sigma} \partial^\mu b^{n,\rho\sigma} + m_n^2 b^n_{\rho\sigma} b^{n,\rho\sigma}) - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) dx. \qquad (59)$$

Thus, we have a massless spin-2 tensor field, an infinite tower of massive spin-2 tensor fields and just one massless scalar radion. A very important point is that, due to the form of the solution with zero mass (56), the remaining gauge transformations (46) result in the following gauge transformations of the field $b^0_{\mu\nu}(x)$:

$$b'^0_{\mu\nu}(x) = b^0_{\mu\nu}(x) - (\partial_\mu \zeta_\nu(x) + \partial_\nu \zeta_\mu(x)), \quad \partial^\mu \zeta_\mu(x) = 0, \quad \partial_\mu \partial^\mu \zeta_\nu(x) = 0. \qquad (60)$$

This guarantees that the field $b^0_{\mu\nu}(x)$ has only two degrees of freedom, and, therefore, can be identified with the field of the massless graviton.

The Lagrangian in (59) is a sum of the free Lagrangians for the physical degrees of freedom and therefore it can be consistently quantized by the standard methods. To study the interaction of these quantized fields with the fields of the standard model, one has to find the corresponding interaction Lagrangians. This problem will be addressed in the next section.

4 Interaction with matter on the branes

Now we are going find the interaction of the fields, corresponding to the physical degrees of freedom of the model, with matter on the branes. The general form of this interaction is standard,

$$\frac{\hat{\kappa}}{2}\int_{B_1} h^{\mu\nu}(x,0)T^1_{\mu\nu}dx + \frac{\hat{\kappa}}{2}\int_{B_2} h^{\mu\nu}(x,R)T^2_{\mu\nu}\sqrt{-\tilde{\gamma}(R)}dx, \tag{61}$$

where $T^1_{\mu\nu}$ and $T^2_{\mu\nu}$ are energy-momentum tensors of the matter on brane 1 and brane 2 respectively:

$$T^{1,2}_{\mu\nu} = 2\frac{\delta L^{1,2}}{\delta\gamma^{\mu\nu}} - \gamma^{1,2}_{\mu\nu}L^{1,2}.$$

Substituting (36) into (61), decomposing $b_{\mu\nu}(x,y)$ according to (58) and rescaling the field φ according to (50), we find that the interaction of the graviton fields $b^n_{\mu\nu}(x)$ and the massless canonically normalized radion field φ with the matter on brane 1 is given by

$$\frac{1}{2}\int_{B_1}\left(\kappa_1\, b^0_{\mu\nu}(x)T^{\mu\nu} + \kappa_1\sum_{n=1}^{\infty}\frac{\psi_n(0)}{N_0}\, b^n_{\mu\nu}(x)T^{\mu\nu} - \frac{\kappa_2}{\sqrt{3}}\varphi\, T^\mu_\mu\right)dx. \tag{62}$$

Here

$$\kappa_1 = \hat{\kappa}N_0, \quad \kappa_2 = \kappa_1 e^{-kR} \tag{63}$$

(the normalization factor N_0 was defined in (56)). Since κ_1 is the coupling constant of the massless graviton, it can be expressed in terms of the 4-dimensional gravitational constant G_1 on brane 1 as $\kappa_1 = \sqrt{16\pi G_1}$, which gives the relation

$$G_1 = \frac{\hat{G}k}{1-e^{-2kr}}$$

between the constants on brane 1 [6, 8].

To find the interaction of the 5-dimensional gravity with the matter on brane 2 turns out to be a more complicated problem, because the coordinates $\{x^\mu\}$ in (13), in which we work, are not Galilean on this brane (coordinates are called Galilean, if $g_{\mu\nu} = diag(-1,1,1,1)$, see [17]). This fact was also noted in papers [8, 18]. In the latter paper, a covariant procedure was proposed, which consists in using covariant equations and expressing all coordinate distances in terms of proper distances. For example, within this approach the masses of brane fields, i.e. the fields localized on a brane and not propagating in the five-dimensional bulk, were determined from the rate of exponential falloff

of the 2-point function. It was shown that such masses measured by brane observers are independent of the warp factor and are given by the Lagrangian masses.

To solve this problem, we follow a simpler logic and introduce the Galilean coordinates $\{z^\mu\}$ on brane 2 related to $\{x^\mu\}$ by $x^\mu = e^{kR} z^\mu$. When we do this in formula (61), we get for brane 2

$$\frac{\hat{\kappa}}{2} \int_{B_2} h'_{\mu\nu}(z, R) T'^{2,\mu\nu} dz, \tag{64}$$

where $T'^{2,\mu\nu}$ is the canonical energy-momentum tensor of matter on brane 2, $h'_{\mu\nu}(z, R) = e^{2kR} h_{\mu\nu}(x, R)$, and the indices are raised with the flat metric. Thus, we see that in order to find the interaction of the 5-dimensional gravity with the matter on brane 2 we have to pass to the Galilean coordinates $\{z^\mu\}$ in the effective action (59) as well. This results in the following expression:

$$S_{eff} = \int \left(-\frac{1}{4} \sum_n (e^{-2kR} \partial_\mu b'^n_{\rho\sigma} \partial^\mu b'^{n,\rho\sigma} + m_n^2 b'^n_{\rho\sigma} b'^{n,\rho\sigma}) - \frac{1}{2} e^{2kR} \partial_\mu \varphi' \partial^\mu \varphi' \right) dz, \tag{65}$$

where the indices are also raised with the flat metric. To transform this expression to be a canonical action we have to rescale the fields $b'^n_{\rho\sigma}$, φ' and the masses m_n. We define the graviton fields $u^n_{\rho\sigma} = e^{-kR} b'^n_{\rho\sigma}$ and the radion field $\varphi = e^{kR} \varphi'$, as seen from brane 2, for which the expression (65) reduces to a canonical action in the Galilean coordinates $\{z^\mu\}$:

$$S_{eff} = \int \left(-\frac{1}{4} \sum_n (\partial_\mu u^n_{\rho\sigma} \partial^\mu u^{n,\rho\sigma} + (m_n e^{kR})^2 u^n_{\rho\sigma} u^{n,\rho\sigma}) - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) dz. \tag{66}$$

Now we are able to write down the interaction of these fields with the matter on brane 2, which is found to be

$$\frac{1}{2} \int_{B_2} \left(\kappa_2 \, u^0_{\mu\nu}(x) T^{\mu\nu} + \kappa_2 \sum_{n=1}^{\infty} \frac{\psi_n(R) e^{2kR}}{N_0} \, u^n_{\mu\nu}(x) T^{\mu\nu} - \frac{\kappa_1}{\sqrt{3}} \varphi \, T^\mu_\mu \right) dz. \tag{67}$$

The coupling constants κ_1 and κ_2 are defined by Eqs. (63). Since κ_2 is the coupling constant of the massless graviton, formally it gives the following relations between the 5-dimensional gravitational constant $\hat{G}$ and the four-dimensional gravitational constant G_2 on brane 2

$$G_2 = \frac{\hat{G} k}{e^{2kr} - 1}, \tag{68}$$

which coincides with the one found in [8].

Choosing the fundamental mass $M \sim k \sim 1\text{TeV}$, we get the right value of the Planck mass, if the argument in the exponential satisfies $kR \approx 30 \div 35$. Relation (67) implies that, like in the standard scenario, the coupling constants of the massive modes and of the massless radion turn out to be of the order 1 TeV^{-1} [14, 15]. In this case, the interaction Lagrangian (67) can be rewritten so that these properties are evident:

$$\frac{1}{2} \int_{B_2} \left(\frac{1}{M_{Pl}} u^0_{\mu\nu}(z) T^{\mu\nu} - \sum_{n=1}^{\infty} \frac{\omega_n}{\Lambda_\pi} u^n_{\mu\nu}(z) T^{\mu\nu} - \frac{1}{\Lambda_\pi \sqrt{3}} \varphi T^\mu_\mu \right) dz. \tag{69}$$

Here $\Lambda_\pi = M_{Pl}e^{-kR} = \sqrt{M^3/k}$, which follows from relation (68), and the parameter ω_n is of the order of unity for small n. The masses of KK-excitations, which can be found from (66), turn out to be $M_n = k\beta_n$, so that $M_n \sim k \sim 1$ TeV for small n. The existence of the massless radion with this coupling constant leads to the predictions, which are at variance with the present experimental data. To solve this problem, several mechanisms for generating a radion mass were proposed (for example, [19]).

Formulas (59), (62) and (66), (69) enable one to calculate the processes due to the interaction of the linearized quantum gravity in RS1 model with the fields of the standard model both on brane 1 and brane 2. The energy-momentum tensor of the standard model can be found, for example, in [20].

5 Conclusion and discussion

In this contribution we have presented a Lagrangian description of linearized gravity in Randall-Sundrum model with two branes, which enabled us to find easily the physical degrees of freedom of this model and to construct an effective Lagrangian for them. If we take the limit $R \to \infty$, the radion field φ drops from the Lagrangian, and we get the same degrees of freedom, as found in [13] for the case of infinite extra dimension.

A very important point of our considerations is the observation that the 5-dimensional gravity looks different, when viewed from different branes: the masses of the gravitational KK modes and the coupling constants of the massless fields differ in the exponential factor.

The case of finite extra dimension is more interesting from the point of view of possible phenomenology scenarios due to the presence of the radion field and of two coupling constants, κ_1 and κ_2. These are essentially the following two possibilities.

Let us assume that our brane is the brane with the positive energy density, in contrast to the standard RS1 scenario, so that the coupling of the massless mode $\kappa_1 \sim 1/M_{Pl}$. If we make a natural assumption that $kR >> 1$, then $\kappa_1 >> \kappa_2$, and we see from (58), (62) that the interactions of the massive KK modes and of the radion field with the matter on our brane are much weaker (exponentially suppressed) than that of the massless graviton. Therefore, the radion field does not affect Newton's gravity on our brane, which arises from the interaction of the massless graviton in (62). The massive KK modes have masses of the order of $M_{pl}e^{-kR}$. Thus, in this case the theory on brane 1 would be phenomenologically acceptable, but not interesting.

If one assumes that our brane is brane 2 with the negative energy density, then, as follows from (67), the interaction of massless radion field is not suppressed. In this case the constant G_2 can be identified with Newton's constant, $G_2 = 1/M_{Pl}^2$, provided the radion fields acquires a mass by some mechanism. Then relation (68) gives the following relation between the Planck mass and the fundamental mass scale M in the theory defined as $\hat{G} = 1/M^3$:

$$M_{Pl}^2 = \frac{M^3}{k}\left(e^{2kR} - 1\right) \approx \frac{M^3}{k}e^{2kR},$$

which can be satisfied by choosing $M \sim k \sim 1$TeV and the argument in the exponential $kR \approx 30 \div 35$. Then, as in the standard scenario, the couplings of the massive KK states

and of the radion are of the order of 1 TeV^{-1} [14, 15]. The difference between the results of these papers and ours is that in these papers $k \sim M_{Pl}$, whereas we get $k \sim 1TeV$. Nevertheless, one can check that the ratio κ_1/κ_2 on brane 2 is the same in the physical (Galilean) coordinates $\{z^\mu\}$ and in "nonphysical" coordinates $\{x^\mu\}$. Due to this fact the phenomenological predictions of papers [14, 15] remain valid.

We would like to note that the possibility of $k \sim 1TeV$, on a par with $k \sim M_{Pl}$, was mentioned in paper [6] and discussed from physical motivations in paper [18]. Our results mean that this value of the parameter k is the only one admissible for a correct physical interpretation of the theory on brane 2.

One more important result of our approach is a simultaneous description of the interaction of five-dimensional gravity with matter on both branes, which was impossible in the earlier papers using Gaussian normal coordinates. This enables us to estimate the properties of the gravitational interaction in the mirror world. Thus, if our brane is brane 2, interaction Lagrangians (62), (67) imply that the strength of the gravitational interaction in the mirror world is approximately of the same order, as the weak interaction in our world. It is quite possible that it can lead to some effect in our world, for example to those connected with the distribution of matter, because there exists gravitational interaction between two worlds. This interaction is discussed in more detail in [21].

Acknowledgments

The authors are grateful to G.Yu. Bogoslovsky, Yu.V. Grats and V.A. Rubakov for useful discussions. The work of E.B., Yu.K. and I.V. was supported by RFBR grant 00-01-00704 and the grant 990588 of the programme "Universities of Russia". E.B. was supported in part by the CERN-INTAS grant 99-0377, by the INTAS grant 00-0679, and by the Bessel Research Award of the Alexander von Humboldt Foundation. Yu.K. and I.V. were also supported in part by the programme SCOPES (Scientific co-operation between Eastern Europe and Switzerland) of the Swiss National Science Foundation, project No. 7SUPJ062239, and financed by Federal Department of Foreign Affairs.

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