Newtonian Limit and the Light Deflection for RS1 Model in the Zero Mode Approximation

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Abstract

We decouple the equations of motion for linearized gravity in the Randall-Sundrum model with matter on the branes and calculate the Newtonian limit and light deflection in the zero mode approximation. We also consider the effects of "shadow" matter, which lives on the other brane, compare them with those of ordinary matter for both positive and negative tension branes and explicitly distinguish the contribution of the radion field. An important feature of our approach is the use of Galilean coordinates on the negative tension brane.

The present-day approach to Kaluza-Klein theories is based on the idea due to Rubakov and Shaposhnikov of localization of fields on a domain wall [1, 2]. A particular realization of this scenario was proposed in paper [3]. In this paper, an exact solution for a system of two branes interacting with gravity in a five-dimensions space-time was found. This model is called the Randall-Sundrum model (usually abbreviated as RS1 model), and it is widely discussed in the literature (see Refs. [5, 4] for reviews and references). A consistent analysis of this model without matter on the branes was made in [7]. However, the equations of motion for metric fluctuations, when there is matter located on the branes, have not been studied in detail yet. In the present contribution, we will decouple the equations of motion for this case and calculate the Newtonian limit of the RS1 model and the light deflection by a point-like mass in the zero mode approximation.

The Randall-Sundrum solution for the background metric looks like [3] (see also [8])

$$ds^2 = \gamma_{MN} d\hat{x}^M d\hat{x}^N = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{1}$$

where $\sigma(y) = -k|y|$ in the range $-R \leq y \leq R$ and R is the size of extra dimension. The orbifold structure of the space of extra dimension implies that the metric fluctuation field h_{MN} must satisfy certain symmetry conditions under the reflection $y \to -y$.

The second variation action of this model is invariant under gauge transformations

$$h'_{MN}(x, y) = h_{MN}(x, y) - (\nabla_M \xi_N(x, y) + \nabla_N \xi_M(x, y)), \tag{2}$$

where ∇_M is the covariant derivative with respect to γ_{MN} and the functions $\xi_N(x, y)$ satisfy the orbifold symmetry conditions. With the help of these gauge transformations we can impose the gauge

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x), \tag{3}$$

which will be called the *unitary gauge* (see [7, 8]).

The equations of motion for different components of the metric fluctuations in the unitary gauge take the form:

1) $\mu\nu$-component

$$\frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial x^{4\,2}}\right) - 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \frac{1}{2}\partial_\mu\partial_\nu\phi + \tag{4}$$

$$+ \quad \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \frac{\partial^2\tilde{h}}{\partial x^{4\,2}} - 4\partial_4\sigma\partial_4\tilde{h} - \partial_\rho\partial^\rho\phi + 12k^2\phi\right)$$

$$+ \quad [2kh_{\mu\nu} - 3k\gamma_{\mu\nu}\phi]\,\tilde{\delta} = -\frac{\hat{\kappa}}{2}T_{\mu\nu},$$

$T_{\mu\nu}$ being the energy-momentum tensor of the matter;

2) $\mu 4$-component,

$$\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = 0, \tag{5}$$

which plays the role of a constraint;

3) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = 0. \tag{6}$$

In what follows, we will also use an auxiliary equation, which is obtained by multiplying the equation for 44-component by 2 and subtracting it from the contracted equation for $\mu\nu$-component. This equation contains $\tilde{h}$ and ϕ only and has the form:

$$\frac{\partial^2\tilde{h}}{\partial x^{4\,2}} + 2\partial_4\sigma\partial_4\tilde{h} - 8k^2\phi + 8k\phi\tilde{\delta} + \partial_\mu\partial^\mu\phi = -\frac{\hat{\kappa}}{2}T^\mu_\mu. \tag{7}$$

If $T_{\mu\nu} = 0$, the physical degrees of freedom of the model can be extracted by the substitution [7]

$$h_{\mu\nu} = b_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial_\nu\phi \tag{8}$$

with $c = \frac{kR}{e^{2kR}-1}$. It turns out that the field $b_{\mu\nu}(x^\mu, x^4)$ describes the massless graviton [3, 9] and massive Kaluza-Klein spin-2 fields, whereas $\phi(x)$ describes a scalar field called the radion.

However, the situation is rather different, when there is matter on the branes. Let us first consider the case, where matter is located on the brane at 0, i.e. the energy-momentum tensor is of the form $T_{\mu\nu} = t_{\mu\nu}(x)\delta(y)$. The substitution, which allows one to decouple the equations, looks like

$$h_{\mu\nu} = u_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma + kR + \frac{1}{2}\right)\partial_\mu\partial_\nu\phi. \tag{9}$$

After this substitution equations (5), (6) and (7), rewritten in the flat metric (i.e. $u = \eta^{\mu\nu}u_{\mu\nu}$) take the form:

$$\partial_4(e^{-2\sigma}(\partial^\nu u_{\mu\nu} - \partial_\mu u)) = 0, \tag{10}$$

$$e^{-4\sigma}(\partial^\mu\partial^\nu u_{\mu\nu} - \Box u) - 3\partial_4\sigma\partial_4(e^{-2\sigma}u) + 3ce^{2kR-2\sigma}\Box\phi = 0, \tag{11}$$

$$\partial_4(e^{2\sigma}\partial_4(e^{-2\sigma}u)) + 2\frac{c}{k}\delta(y)\left[e^{2kR}-1\right]\Box\phi = \frac{\kappa}{3}t^\mu_\mu\,\delta(y). \tag{12}$$

Let us consider Fourier expansion of all terms of equation (12). Since the term with the derivative ∂_4 has no zero mode, this equation implies that

$$\Box\phi = \frac{\hat\kappa}{6R}t^\mu_\mu. \tag{13}$$

It is well known that the field $u_{\mu\nu}$ in the presence of matter is a combination of zero and massive modes [11]. In particular, zero mode can be represented as $u^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu}$, where $\alpha_{\mu\nu}$ depends on x only. It also means that it is possible to impose the gauge condition

$$\partial^\nu u^m_{\mu\nu} = u^{m\mu}_\mu = 0 \tag{14}$$

for the massive modes $u^m_{\mu\nu}$. It is easy to see that there remain gauge transformations of the form $\xi_\mu = e^{2\sigma}\epsilon_\mu(x)$. These gauge transformations allow us to impose the de Donder gauge condition for the zero mode

$$\partial^\nu\left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) = 0. \tag{15}$$

Having imposed this gauge, we are still left with residual gauge transformation

$$\xi_\mu = e^{2\sigma}\epsilon_\mu(x), \quad \Box\epsilon_\mu = 0. \tag{16}$$

The gauge transformations with ξ_μ satisfying these conditions are important for determining the number of degrees of freedom of the massless graviton. It follows from equations (11), (13), (14), (15) that

$$\Box\alpha = \frac{\hat\kappa c e^{2kR}}{R}t^\mu_\mu. \tag{17}$$

Now let us consider equation (4). Making substitution (9) with condition (13) and passing to the gauge (14), (15) we get

$$\begin{aligned}\frac{1}{2}\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) + \frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = \\ = -\frac{\hat\kappa}{2}t_{\mu\nu}\delta(y) + \frac{\hat\kappa k e^{2kR}}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho - \frac{\hat\kappa}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho\,\delta(y).\end{aligned} \tag{18}$$

Since we are going to calculate the Newtonian limit and light deflection in the zero mode approximation, we have to find an equation for the field $\alpha_{\mu\nu}$. If we multiply equation (18) by $e^{2\sigma}$, integrate it over y and take into account the orthonormality condition for the wave functions of the modes, we get

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat\kappa k}{(1-e^{-2kR})}t_{\mu\nu}. \tag{19}$$

The equation for massive modes takes the form

$$\frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2 u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = -\frac{\hat{\kappa}}{2}t_{\mu\nu}\delta(y) + \tag{20}$$
$$+\frac{\hat{\kappa}ke^{2kR}}{2(e^{2kR}-1)}t_{\mu\nu} + \frac{\hat{\kappa}ke^{2kR}}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho - \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho\,\delta(y).$$

In principle, it is possible to solve equation (18) exactly. In the case $R = \infty$ (one brane) this was done in paper [11]. For a finite R all calculations can be made analogously. But it seems to be impossible to sum up analytically the series, which appears in the expression for the solution of equation (18) even for a simple form of $t_{\mu\nu}$ (for example, for a static point mass).

When matter is on brane 2 (at $y = R$), all the reasonings are the same, as presented above. The substitution looks like

$$h_{\mu\nu} = u_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma + \frac{1}{2}\right)\partial_\mu\partial_\nu\phi. \tag{21}$$

The gauge conditions are the same too. The equations of motion for the fields $\alpha_{\mu\nu}$, ϕ in the presence of matter on brane 2 can be derived analogously and read

$$\frac{1}{2}\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) + \frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2 u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = \tag{22}$$
$$= -\frac{\hat{\kappa}}{2}t_{\mu\nu}\delta(y-R) + \frac{\hat{\kappa}k}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho - \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho\,\delta(y-R),$$

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat{\kappa}k}{(e^{2kR}-1)}t_{\mu\nu}, \tag{23}$$

$$\Box\phi = \frac{\hat{\kappa}}{6R}t^\mu_\mu. \tag{24}$$

An important point is that these equations are written in the coordinates $\{x^\mu\}$, which are *Galilean on brane 1* and are inappropriate for studying physical effects on brane 2.

Obviously, when there is matter on both branes, the solution for the fluctuation of metric is just the sum of solutions for each brane separately, which follows from the linearity of the equations.

Now we are ready to find the Newtonian limit and light deflection by a point-like static mass. Let us make it for brane 1 first. The substitution (9) in the zero mode approximation is

$$h^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu} - \frac{kR}{e^{2kR}-1}\eta_{\mu\nu}\phi + \frac{R}{2k}\partial_\mu\partial_\nu\phi + \frac{1}{4k^2}\partial_\mu\partial_\nu\phi. \tag{25}$$

The equation for $h_{\mu\nu}$ on brane 1 in the zero mode approximation looks like

$$\Box h^0_{\mu\nu} = -\frac{\hat{\kappa}ke^{2kR}}{e^{2kR}-1}\left(t_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}t^\rho_\rho\right) - \frac{\hat{\kappa}k}{6(e^{2kR}-1)}\eta_{\mu\nu}t^\rho_\rho + \frac{\hat{\kappa}}{12k^2R}\left(kR + \frac{1}{2}\right)\partial_\mu\partial_\nu t^\rho_\rho. \tag{26}$$

Let us consider a static point mass with energy-momentum tensor $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$, M denoting the inertial mass. Then we can find a solution of equation (26) for h_{00}-component

$$h_{00} = \frac{\hat{\kappa}k(e^{2kR} + \frac{1}{3})}{8\pi(e^{2kR} - 1)}\frac{M}{r}, \tag{27}$$

where $r = |\vec{x}|$.

The gravitational potential V is defined by [12]

$$g_{00} = -1 - 2V, \tag{28}$$

so that $V = -\frac{\hat{\kappa}}{2}h_{00}$, and we get

$$V = -\hat{G}k\frac{(e^{2kR} + \frac{1}{3})}{(e^{2kR} - 1)}\frac{M}{r} = -G_1\left(1 + \frac{1}{3}e^{-2kR}\right)\frac{M}{r}. \tag{29}$$

Now let us show that the Randall-Sundrum model in the zero mode approximation is equivalent to the linearized Brans-Dicke theory (apparently, it was first noted in [11]). Appropriate formulas for the light deflection in the Brans-Dicke theory are well known and can be found , for example, in [12].

The equation for the fluctuations of metric in the linearized Brans-Dicke theory looks like

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G\left(t_{\mu\nu} - \frac{1}{2\omega + 3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right), \tag{30}$$

where ω is the BD-parameter. It is easy to see that $h_{\mu\nu}$ satisfies the de Donder gauge condition $\partial^\nu\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = 0$. The light deflection angle is given by [12]

$$\Delta\varphi \approx \frac{4MG}{r_0}\left(\frac{2\omega + 3}{2\omega + 4}\right), \tag{31}$$

with r_0 being the impact parameter and M being the mass of the static point-like source.

Now let us examine the four-dimensional effective theory on brane 1. It is easy to see that $h^0_{\mu\nu}$ (25) does not satisfy the de Donder gauge condition. The residual gauge transformations (16) are not sufficient to pass to this gauge. But since we consider only the effective theory on brane 1, we can drop the $\Box\epsilon_\mu = 0$ condition, which fixes the gauge for the field $\alpha_{\mu\nu}$. Then we can pass to the de Donder gauge condition for the field $h^0_{\mu\nu}$ *on brane 1* with the help of the gauge functions of the form

$$\epsilon_\mu = \frac{1}{2}\left(\frac{R}{2k} + \frac{1}{4k^2}\right)\partial_\mu\phi + \frac{kR}{e^{2kR} - 1}\frac{\partial_\mu}{\Box}\,\phi. \tag{32}$$

In this gauge equation (26) takes the form

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G_1\left(t_{\mu\nu} - \frac{e^{-2kR}}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{33}$$

Here and in what follows we drop the superscript 0 of the zero mode approximation. By comparing (33) with (30), we get $\omega = \frac{3}{2}\left(e^{2kR} - 1\right)$. Substituting this into (31), we find the angle of light deflection by a static point-like source on brane 1

$$\Delta\varphi \approx \frac{4MG_1}{r_0}\left(\frac{1}{1+\frac{1}{3}e^{-2kR}}\right). \tag{34}$$

The second term in the denominator of formula (34) and the second term in formula (29) correspond to the contribution of the radion. One can see that the contribution of the radion is e^{2kR} times smaller than the contribution of the massless graviton.

Thus, we have examined the case of the mass and the observer being located on brane 1. But there are three more possible cases to be examined. It is the case of "shadow" matter, when the mass is located on brane 2 and the observer is located on brane 1, the case of the mass and the observer being located on brane 2, and the case of the mass being located on brane 1, whereas the observer being located on brane 2 ("shadow" matter effect as well). All the calculations in these cases are the same, as described above. But if the observer is located on brane 2, it is necessary to pass to Galilean coordinates on brane 2 to get the correct result. This problem was discussed in paper [7]. Below the Galilean coordinates are denoted by x^μ, or simply by x. The energy-momentum tensor $t_{\mu\nu}$ takes the form $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$ in the Galilean coordinates on the brane, the mass is located on. In the formulas presented below (in the Galilean coordinates on the observer's brane) the energy-momentum tensor always has this canonical form and passing to the Galilean coordinates is taken into account by a factor in front of the brackets (see, for example, (35)).

Direct calculations lead us to the following results:

1) We live on brane 1, the mass is located on brane 2 ("shadow" matter)

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G_1 e^{-kR}\left(t_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{35}$$

The light deflection angle is

$$\Delta\varphi \approx \frac{3MG_1e^{-kR}}{r_0} = 4MG_1e^{-kR}\left(\frac{1}{1+\frac{1}{3}}\right)\frac{1}{r_0}. \tag{36}$$

Newton's Law looks like

$$V = -\frac{4}{3}G_1e^{-kR}\frac{M}{r} = -\left(1+\frac{1}{3}\right)G_1e^{-kR}\frac{M}{r}. \tag{37}$$

The second term in the denominator of formula (36) and the second term in formula (37) correspond to the contribution of the radion. One can see that the contribution of the radion is of the same order, as contribution of the massless graviton, but the effect of the "shadow" matter is e^{kR} times smaller, than the effect of the ordinary matter (compare with (34) and (29)). It means that the effective theory on brane 1 is phenomenologically acceptable, but not interesting (see [7, 8]).

2) We live on brane 2, and the mass is located on the same brane

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2\left(t_{\mu\nu}-\frac{e^{2kR}}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right) \tag{38}$$

with $G_2=\hat{G}k\frac{1}{e^{2kR}-1}$.

The light deflection angle is

$$\Delta\varphi\approx\frac{4MG_2}{r_0}\left(\frac{1}{1+\frac{e^{2kR}}{3}}\right). \tag{39}$$

Newton's Law looks like

$$V=-G_2\left(1+\frac{e^{2kR}}{3}\right)\frac{M}{r}. \tag{40}$$

The second term in the denominator of formula (39) and the second term in formula (40) correspond to the contribution of the radion. Now the contribution of the radion is e^{2kR} times stronger than the contribution of the massless graviton. It means that, in the case of the massless radion, scalar gravity is realized on brane 2. However, if one assumes some mechanism for generating the radion mass, for example, the Goldberger-Wise mechanism [13], gravity in the zero mode approximation becomes tensor and hierarchy problem solves.

3) We live on brane 2, the mass is located on brane 1 (also "shadow" matter)

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2e^{kR}\left(t_{\mu\nu}-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{41}$$

The light deflection angle is

$$\Delta\varphi\approx\frac{4MG_2e^{kR}}{r_0}\left(\frac{1}{1+\frac{1}{3}}\right)=\frac{3MG_1e^{-kR}}{r_0}. \tag{42}$$

Newton's Law looks like

$$V=-\left(1+\frac{1}{3}\right)G_2e^{kR}\frac{M}{r}=-\frac{4}{3}G_1e^{-kR}\frac{M}{r}. \tag{43}$$

As before, the second term in the denominator of formula (42) and the second term in formula (43) correspond to the contribution of the radion. We can see that the contribution of the radion is of the same order, as contribution of the massless graviton, but the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times smaller, than the contribution of the ordinary matter, because of the interaction with the massless radion (compare with (40)). But if the radion is massive (otherwise we have scalar gravity in "our" world on brane 2), the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times stronger, than the contribution of the ordinary matter. It could lead to some observable effects, resulting from the distribution of matter in the "mirror" world (brane 1).

We would like to note that in paper [11] the effects of the "shadow" matter were also considered. If we compare the effects of the "shadow" and the ordinary matter, as it was made in [11] (the deflection angles by the same Newtonian masses), we get the following results: 40% instead of 25% in [11] for brane 1 and the difference of the order of e^{4kR} instead of 25% in [11] for brane 2. This discrepancy appears because we use the Galilean coordinates on brane 2. In the case of the massive radion there is no difference in the zero mode approximation at all for both branes.

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