Soft supersymmetry breaking and explicit CP violation in the two-Higgs-doublet model

E.N. Akhmetzyanova, M.V. Dolgopolov, M.N. Dubinin

Dept. of General and Theoretical Physics, Samara State University,
Russia, 443011, Samara, Akad. Pavlov st. 1.;
D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University,
Russia, 119992, Moscow, Vorobyevi Gory.

Abstract

The effective potential approach for the two-Higgs-doublet sector of the minimal supersymmetric model is used to express the parameters of the effective Higgs potential in terms of the parameters describing the Higgs boson interactions with scalar quarks. The complex parameters of the scalar quark-Higgs boson soft supersymmetry breaking sector induce the complex parameters in the effective Higgs potential of the minimal supersymmetry, thus radiatively breaking its CP invariance.

1 Introduction

It is well-known that in the Standard Model (SM) the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1] originates from the Lagrangian terms describing Higgs boson interactions with quarks (Yukawa terms)

$$L \;=\; g^u_{ij}\bar{\psi}^{i\prime}_L u^{j\prime}_R H + g^d_{ij}\bar{\psi}^{i\prime}_L d^{j\prime}_R \tilde{H} + h.c.,$$

where $\bar{\psi}^{1\prime}_L = (\bar{u}', \bar{d}')_L$, $\bar{\psi}^{2\prime}_L = (\bar{c}', \bar{s}')_L$, $\bar{\psi}^{3\prime}_L = (\bar{t}', \bar{b}')_L$, $u^{1\prime}_R = u'_R$, $u^{2\prime}_R = c'_R$, $u^{3\prime}_R = t'_R$, $d^{1\prime}_R = d'_R$, $d^{2\prime}_R = s'_R$, $d^{3\prime}_R = b'_R$, H denotes the doublet of scalar complex fields, $\tilde{H}_k = \epsilon_{kl} H^*_l$ and g^u_{ij}, g^d_{ij} are 3×3 matrices with, generally speaking, *complex* matrix elements. After the symmetry breaking in order to diagonalize the Yukawa mass terms the quark fields u' and d' are rotated by the four unitary matrices to define a new (unprimed) fields $u_{L,R} = U_{L,R}\, u'_{L,R}$, $d_{L,R} = D_{L,R}\, d'_{L,R}$ and the corresponding quark mass matrix becomes diagonal. However, after this diagonalization the matrices U_L and D_L appear in the charged currents interaction $g\bar{u}'_L\gamma_\mu d'_L W^\mu = g\bar{u}_L\gamma_\mu U^\dagger_L D_L d_L W^\mu$, their product is the CKM matrix $V = U^\dagger_L D_L$. The phenomenology of CP violation in meson decays is based on the CKM matrix with complex matrix elements. In this picture the origin of CP violation is hidden behind the observation that the number of quark generations is exactly three.

At the same time it is possible to introduce hermitian Lagrangians, which explicitly violate CP invariance, in other extended sectors of the SM [2]. The simplest example is the Lagrangian and its CP transformed, $L \to L^{CP}$, for a scalar particle decaying into two scalar particles

$$L = \lambda\varphi_1\varphi^*_2\varphi^*_3 + \lambda^*\varphi^*_1\varphi_2\varphi_3, \qquad L^{CP} = \lambda\varphi^*_1\varphi_2\varphi_3 + \lambda^*\varphi_1\varphi^*_2\varphi^*_3$$

We can observe that L is identical to L^{CP} only if the parameter λ is real. Otherwise the CP transformed Lagrangian differs from L in the sign of the imaginary part of λ. In this trivial example the phase of λ can be of course rotated away from the theory using the hypercharge global transformation of the scalar field. However, if the number of fields is greater than three or their interactions are decribed by an additional higher dimension terms, it is obviously not the case and we have explicitly CP violating Lagrangians, which are reduced to CP invariant only if some specific 'fine-tuning' conditions for the phases are introduced. From this point of view the models with extended Higgs sector are of a major interest. Their minimal version, the two-doublet Higgs potential of the minimal supersymmetry (MSSM), includes (if the spontaneous CP violation [3] is not accounted for) nine parameters of the effective potential, four of them are, generally speaking, complex [4, 5].

2 The two-doublet Higgs potential

The Higgs potential of the two-Higgs-doublet model (THDM) is constructed using $SU(2)$ doublets of complex scalar fields:

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix}, \qquad \Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix}, \tag{1}$$

with nonzero vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}. \tag{2}$$

Here v_1 and v_2 are real. In the general case the phases ζ and ξ can be introduced [6]. In this proceeding we consider the case $\zeta = \xi = 0$.

The general renormalizable hermitian $SU(2)\times U(1)$ invariant form of the Lagrangian is $\mathcal{L}_H = (\mathcal{D}_\nu\Phi_1)^\dagger\mathcal{D}^\nu\Phi_1 + (\mathcal{D}_\nu\Phi_2)^\dagger\mathcal{D}^\nu\Phi_2 + \kappa\,(\mathcal{D}_\nu\Phi_1)^\dagger\mathcal{D}^\nu\Phi_2 + \overset{*}{\kappa}\,(\mathcal{D}_\nu\Phi_2)^\dagger\mathcal{D}^\nu\Phi_1 - U(\Phi_1,\Phi_2)$, where potential[1]

$$\begin{aligned} U(\Phi_1,\Phi_2) = &- \mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu}{}_{12}^2\,(\Phi_2^\dagger\Phi_1) + \\ &\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \\ &\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \\ &\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_6\,(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7\,(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \end{aligned} \tag{3}$$

The potential (3) can be obtained in the MSSM after the integration of the superpotential over Grassman variables and the introduction of soft supersymmetry breaking terms. The parameters λ_1, ..., λ_4 and μ_1^2, μ_2^2 are taken real. In the MSSM one can show that they remain real in the effective potential after the integration over the degrees of freedom of massive scalar quarks.

[1] Note for the comparison, that all λ in [4] differ by sign and λ_5 is used instead of our $\lambda_5/2$. The self-couplings $\lambda_{1,2}/2$ are used for the CP conserving case in [7, 8] in comparison with our $\lambda_{1,2}$.

Additional global or discrete symmetries have been discussed, the abelian Peccei–Quinn symmetry $U(1)_{PQ}$ for the two-doublet scalar field system [9] and the discrete symmetry Z_2, introduced to remove the flavor-changing neutral currents (FCNC) in the Higgs sector, $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$ or vice versa. The potential (3) can be rewritten as the sum of three parts [9], subject to additional symmetries,

$$\begin{aligned} U_1 = & -{\mu_1}^2({\Phi_1}^\dagger\Phi_1) - {\mu_2}^2({\Phi_2}^\dagger\Phi_2) + \lambda_1({\Phi_1}^\dagger\Phi_1)^2 + \\ & + \lambda_2({\Phi_2}^\dagger\Phi_2)^2 + \lambda_3({\Phi_1}^\dagger\Phi_1)({\Phi_2}^\dagger\Phi_2) + \lambda_4({\Phi_1}^\dagger\Phi_2)({\Phi_2}^\dagger\Phi_1)\,, \end{aligned} \tag{4}$$

$$U_2 = \frac{1}{2}|\,\lambda_5|\,e^{i\delta_5}({\Phi_1}^\dagger\Phi_2)^2 + \text{h.c.}, \tag{5}$$

$$\begin{aligned} U_3 = & -|\,\mu_{12}^2|\,e^{i\delta_{12}}(\Phi_1^\dagger\Phi_2) + |\,\lambda_6|\,e^{i\delta_6}({\Phi_1}^\dagger\Phi_1)({\Phi_1}^\dagger\Phi_2) + \\ & + |\,\lambda_7|\,e^{i\delta_7}({\Phi_2}^\dagger\Phi_2)({\Phi_1}^\dagger\Phi_2) + \text{h.c.}, \end{aligned} \tag{6}$$

with CP violating phases δ_5, δ_6, δ_7, δ_{12} which are involved in the terms with $(\Phi_1^\dagger\Phi_2)$ or conjugated $(\Phi_2^\dagger\Phi_1)$. One can see that the U_1 term is $SU(2)\times U(1)\times U(1)_{PQ}\times Z_2$ invariant, U_2 is $SU(2)\times U(1)\times Z_2$ invariant and the U_3 term is $SU(2)\times U(1)$ invariant. If the potential U is $SU(2)\times U(1)$ invariant, then all the four phases appear. If the $U(1)_{PQ}$ invariance is required, then $U \equiv U_1$ and all possible CP violating phases of the Higgs sector disappear. If in addition to $SU(2)\times U(1)$ we introduce only the Z_2 symmetry, then $U \equiv U_1 + U_2$ and only one phase δ_5 remains.

In the MSSM the following relations (at the tree level) between the potential parameters λ_i and the $SU(2)\times U(1)$ gauge coupling constants take place [10]

$$\begin{gathered} \lambda_1^{SUSY} = \lambda_2^{SUSY} = \tfrac{1}{8}\,(g_2^2 + g_1^2), \quad \lambda_3^{SUSY} = \tfrac{1}{4}\,(g_2^2 - g_1^2), \quad \lambda_4^{SUSY} = -\,\tfrac{1}{2}g_2^2, \\ \lambda_5^{SUSY} = \lambda_6^{SUSY} = \lambda_7^{SUSY} = 0 \end{gathered} \tag{7}$$

at the energy scale M_{SUSY}. So in the tree level potential the four masses of the Higgs bosons and the two mixing angles in the Higgs sector are defined by the two independent parameters (e.g. $\tan\beta$, m_A). However the parameters of any model, generally speaking, depend on the energy scale at which they are measured or fixed. This dependence is described by the renormalization group equations (RGE). At the energy smaller than M_{SUSY} the relations (7) are violated by the quantum effect, caused by the Higgs boson interactions with the third generation squarks. It will be shown in the next section that the complex parameters $\lambda_{5,6,7}$ are radiatively induced in the effective potential (3) by the scalar quark $\tilde{t}$, $\tilde{b}$ interactions with Higgs fields.

3 The one-loop effective potential with CP violation

The radiatively corrected Higgs sector of the supersymmetric models has been intensively studied during the last years using three main tools: diagrammatic calculations, effective potential techniques and renormalization group methods. The third method is based on the effective field theory approach: the low-energy effective theory at the scale lower than $M_{SUSY} \gg M_Z$ can be represented by just the (nonsupersymmetric) THDM or SM (in the particular $m_A \sim M_{\widetilde{Q}} \sim M_{SUSY}$) and the supersymmetric partners, playing the role of Pauli–Villars regulators [11]. Above the scale M_{SUSY} the supersymmetry is restored, imposing the conditions (7) on the parameters λ at the renormalization scale $\sigma = M_{SUSY}$. Thus, the conditions (7) play the role of boundary conditions for the RGE. The evolution of the Higgs potential parameters λ, the Yukawa couplings $h_{t,b}$ and the gauge couplings g_i ($i = 1, 2, 3$) is defined by the RGE. The solution of the RGE allows to consider the evolution of the complex parameters λ_5, λ_6, λ_7 from the high energy scale down to the m_Z scale. In our analysis the Higgs boson interaction with the third generation scalar quarks is accounted for in the boundary condition for the RGE.

3.1 The soft supersymmetry breaking terms

The scalar quark-Higgs boson interaction Lagrangian (soft SUSY breaking terms) has the form [7]

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,, \tag{8}$$

where

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,, \tag{9}$$

$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*\,, \tag{10}$$

$$\begin{aligned}\mathcal{V}_\Lambda = {} & \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] + \\ & + \overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + h.c.\right]\,, \qquad i, j, k, l = 1, 2\end{aligned} \tag{11}$$

and $\mathcal{V}_{\widetilde{Q}}$ denotes the quartic scalar quarks interaction terms, Pauli matrix $\sigma_2 \equiv \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$. The Yukawa couplings for the third generation of scalar quarks are defined in a standard way $h_t = \frac{\sqrt{2} m_t}{v \sin\beta}$, $h_b = \frac{\sqrt{2} m_b}{v \cos\beta}$. Following [12][2]:

$$\Gamma_{\{1;2\}}^U = h_U \{-\mu^*; A_U\}, \qquad \Gamma_{\{1;2\}}^D = h_D \{A_D\,; -\mu^*\}, \tag{12}$$

now they are complex. The trilinear parameters A_t, A_b and the higgsino-neutralino mass parameter μ in the sector of scalar quark interactions with the Higgs fields can be complex, when an analogue of the CKM mixing naturally exists in this sector.

The corrections to the quartic couplings for Φ are obtained from two sources: F and D terms. F-terms are in the trilinear interaction terms in (10). The contributions of the D-terms of interaction with scalar quarks in (11)[3] correspond to the gauge group $SU(2) \times U(1)$. Thus namely the trilinear interaction F-terms could be the radiative CP-violating sources in the effective Higgs sector.

[2]For CP conserving case in [7] the trilinear couplings in (10) are real and denoted as $\Gamma_{\{1;2\}}^U \equiv h_U \{-\mu; A_U\}$, $\Gamma_{\{1;2\}}^D \equiv h_D \{A_D\,; -\mu\}$.

[3]The couplings in (11) are consistent with [7, 12].

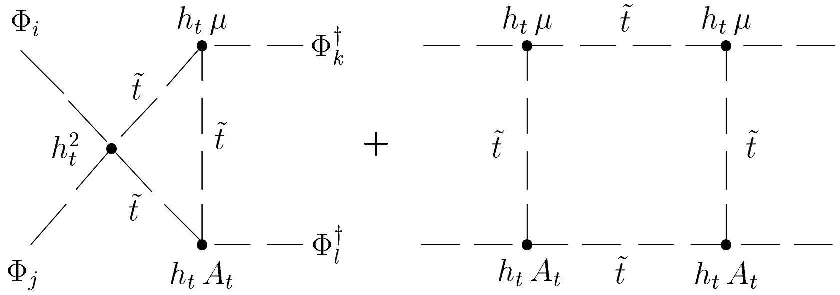
3.2 The effective potential

The one-loop effective potential [13] (the effective action at zero external momentum) $\mathcal{V}$ can be obtained using the method of functional integration (in the Landau gauge using the dimensional reduction and the modified scheme of the minimal subtraction ($\overline{\text{MS}}$) [7]):

$$\mathcal{V} = \mathcal{V}^0 + \frac{N_C}{32\pi^2}\text{tr}\mathcal{M}^4\left[\log\left(\frac{\mathcal{M}^2}{\sigma^2}\right) - \frac{3}{2}\right]. \tag{13}$$

The Higgs self-interaction part of $\mathcal{V}^0$ coincides with the tree-level potential (3) if (7) are accounted for, $\mathcal{M}^2$ is the matrix with the scalar quarks squared masses, σ is the renormalization scale and "tr" denotes the operator trace. Here $N_C = 3$ colors and a factor 2 is included since the squarks fields are complex, as in [7].

Schematically the leading one-loop CP violating contributions to the effective potential can be presented as triangle and box diagrams



It is clear that the complex μ and $A_{t,b}$ induce the complex parameters $\lambda_{5,6,7}$ in the effective potential.

3.3 The scalar quarks mass matrix

The squared mass matrix of the scalar quarks can be presented in the form [7]

$$\mathcal{M}^2 = \mathcal{M}^2_M + \mathcal{M}^2_\Gamma + \mathcal{M}^2_\Lambda \tag{14}$$

convenient for the following calculations. Each term in the expression above can be obtained by the differentiation of the corresponding term in the supersymmetric potential (8)

$$(\mathcal{M}^2_X)_{ab} \equiv \frac{\partial^2 \mathcal{V}_X}{\partial\widetilde{\Psi}_a\,\partial\,\widetilde{\Psi}^*_b} \tag{15}$$

where $X \equiv \{M,\,\Gamma,\,\Lambda\}$, and $\widetilde{\Psi} \equiv \{\widetilde{Q}_{up}, \widetilde{Q}_{down}, \widetilde{U}^*, \widetilde{D}^*\}$, $\widetilde{\Psi}^* \equiv \{\widetilde{Q}^*_{up}, \widetilde{Q}^*_{down}, \widetilde{U}, \widetilde{D}\}$ are the collections of the scalar quark fields.

In this way, consistent with [12][4], we have the following hermitian mass matrices for the real neutral components of the complex scalar doublets ($\varphi^0_{1,2}$ is the real neutral

[4]In [12] Pauli matrix $\sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is used (see also previous note about the minus sign in [4]), and the matrix $\mathcal{M}^2_\Gamma$ is transposed to our but it is not important. In this article we write only the mass matrices for the neutral components of the Higgs doublets which is enough for extracting directly $\lambda_{1,2,5,6,7}$. The extracting of all λ from the effective potential was done by several ways for the check. In this way it is convenient to rewrite (3) in terms of up and down components of the Higgs doublets. All mass matrix elements coincided with [12].

component of a column $\Phi_{1,2}$, ${\varphi^0_{1,2}}^*$ is the real neutral component of a conjugated line)

$$\mathcal{M}^2_\Gamma = \begin{pmatrix} 0 & 0 & i\,\Gamma^U_i \Phi^T_i \sigma_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} & 0 \\ 0 & 0 & 0 & \Gamma^D_i \Phi^\dagger_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ -i\,\overset{*}{\Gamma^U_i}\,(1\,0)\,\sigma_2 \Phi^*_i & 0 & 0 & 0 \\ 0 & \overset{*}{\Gamma^D_i}\,(0\,1)\,\Phi_i & 0 & 0 \end{pmatrix} = \tag{16}$$

$$= \begin{pmatrix} 0 & 0 & h_t(A_t\varphi^0_2 - \overset{*}{\mu}\,\varphi^0_1) & 0 \\ 0 & 0 & 0 & h_b(A_b{\varphi^0_1}^* - \overset{*}{\mu}\,{\varphi^0_2}^*) \\ h_t(\overset{*}{A_t}\,{\varphi^0_2}^* - \mu{\varphi^0_1}^*) & 0 & 0 & 0 \\ 0 & h_b(\overset{*}{A_b}\,\varphi^0_1 - \mu\varphi^0_2) & 0 & 0 \end{pmatrix} \tag{17}$$

$$\mathcal{M}^2_\Lambda = \begin{pmatrix} \mathcal{M}^2_{\Lambda 11} & 0 & 0 & 0 \\ 0 & \mathcal{M}^2_{\Lambda 22} & 0 & 0 \\ 0 & 0 & \mathcal{M}^2_{\Lambda 33} & 0 \\ 0 & 0 & 0 & \mathcal{M}^2_{\Lambda 44} \end{pmatrix} \tag{18}$$

where the matrix elements $\mathcal{M}^2_\Lambda$ have the form

$$\mathcal{M}^2_{\Lambda 11} = |h_t|^2\,{\varphi^0_2}^*\varphi^0_2 + \left(\frac{1}{4}g^2_2 - \frac{1}{12}g^2_1\right)\left({\varphi^0_1}^*\varphi^0_1 - {\varphi^0_2}^*\varphi^0_2\right), \tag{19}$$

$$\mathcal{M}^2_{\Lambda 22} = |h_b|^2\,{\varphi^0_1}^*\varphi^0_1 - \left(\frac{1}{4}g^2_2 + \frac{1}{12}g^2_1\right)\left({\varphi^0_1}^*\varphi^0_1 - {\varphi^0_2}^*\varphi^0_2\right), \tag{20}$$

$$\mathcal{M}^2_{\Lambda 33} = |h_t|^2\,{\varphi^0_2}^*\varphi^0_2 + \frac{1}{3}g^2_1\left({\varphi^0_1}^*\varphi^0_1 - {\varphi^0_2}^*\varphi^0_2\right), \tag{21}$$

$$\mathcal{M}^2_{\Lambda 44} = |h_b|^2\,{\varphi^0_1}^*\varphi^0_1 - \frac{1}{6}g^2_1\left({\varphi^0_1}^*\varphi^0_1 - {\varphi^0_2}^*\varphi^0_2\right), \tag{22}$$

which coincide with the corresponding expressions in [12].

3.4 The effective boundary conditions and self-couplings

In the special case $M_{\widetilde{U}} = M_{\widetilde{D}} = M_{\widetilde{Q}} \equiv M_{\rm SUSY}$ (so we have $\mathcal{M}^2_M = \hat{1}M^2_{\rm SUSY}$) we can decompose the effective potential (13) in the inverse powers of $M^2_{\rm SUSY}$. In order to get the effective λ_i it is necessary to pick out of this decomposition the terms containing the fourth degree in Higgs doublet Φ. Insofar as $\mathcal{M}_\Gamma$ contains the field Φ in the first power and $\mathcal{M}_\Lambda$ contains Φ in the second power, we get the following quartic interaction effective potential

$$V_{quartic} = \Lambda^{jl}_{ik}(\Phi^\dagger_i\Phi_j)(\Phi^\dagger_k\Phi_l) + \frac{N_C}{32\pi^2}\left\{\log\left(\frac{M^2_{\rm SUSY}}{\sigma^2}\right)\,{\rm tr}(\mathcal{M}^2_\Lambda)^2 + \right.$$
$$\left. + \frac{1}{M^2_{\rm SUSY}}\,{\rm tr}(\mathcal{M}^2_\Gamma)^2\,\mathcal{M}^2_\Lambda - \frac{1}{12M^4_{\rm SUSY}}\,{\rm tr}(\mathcal{M}^2_\Gamma)^4\right\}. \tag{23}$$

The last two terms give the contributions to λ_6, λ_7 and only the last term gives the contribution to λ_5. Note that they do not contain the renormalisation scale σ.

Substituting the mass matrix of scalar quarks to the one-loop effective quartic potential (23) we obtain the effective real parameters λ_i $(i=1,2,3,4)$ and complex parameters λ_i $(i=5,6,7)$ at the scale below M_{SUSY}.

The effective potential method does not pick up the terms arising from corrections $\Delta\lambda_i^{field}$, which derive from one-loop wave-function renormalization [7]:

$$\Delta\,\lambda_1^{field}=\frac{1}{2}(g_1^2+g_2^2)A'_{11},\qquad \Delta\,\lambda_2^{field}=\frac{1}{2}(g_1^2+g_2^2)A'_{22},$$
$$\Delta\,\lambda_3^{field}=-\,\frac{1}{4}(g_1^2-g_2^2)(A'_{11}+A'_{22}),\qquad \Delta\,\lambda_4^{field}=-\,\frac{1}{2}g_2^2(A'_{11}+A'_{22}),\tag{24}$$
$$\Delta\,\lambda_5^{field}=0,$$
$$\Delta\,\lambda_6^{field}=\frac{1}{8}(g_1^2+g_2^2)(A'_{12}-A'^{\,*}_{21})=0,\qquad \Delta\,\lambda_7^{field}=\frac{1}{8}(g_1^2+g_2^2)(A'_{21}-A'^{\,*}_{12})=0.$$

Note the difference in comparison with finite corrections of CP-conserving THDM considered in [7] (see (12) and footnote). Also we incorporated the logarithmic correction (see in (27)) depending on

$$l\equiv\log\left(\frac{M^2_{\rm SUSY}}{\sigma^2}\right)\,.\tag{25}$$

In our CP-violating case we have

$$A'_{ij}=-\,\frac{3}{96\,\pi^2\,M^2_{SUSY}}\left[h_U^2\left[\begin{array}{cc}|\mu|^2 & -\mu^* A_U^*\\ -\mu A_U & |A_U|^2\end{array}\right]+\,h_D^2\left[\begin{array}{cc}|A_D|^2 & -\mu^* A_D^*\\ -\mu A_D & |\mu|^2\end{array}\right]\right]\times\tag{26}$$
$$\times\,\left(1-\,\frac{1}{2}\,l\right)\,.\tag{27}$$

We see that there is not any CP-violating contribution to self-couplings from the one-loop field renormalization.

Taking into account the field renormalization contributions (24) we can denote the deviation from the coupling λ_i^{SUSY} at the MSSM scale (7) by $\Delta\lambda_i$ as in [5]:

$$\lambda_{1,2}\equiv\lambda_{1,2}^{SUSY}-\Delta\lambda_{1,2}/2,\quad \lambda_{3,4}\equiv\lambda_{3,4}^{SUSY}-\Delta\lambda_{3,4},\quad \lambda_{5,6,7}\equiv-\,\Delta\lambda_{5,6,7},\tag{28}$$

where [5] $$\Delta\lambda_i\equiv\Delta\lambda_i^{eff.pot.}-\Delta\lambda_i^{field}\,,\qquad \Delta\,\lambda_i^{\{eff.pot.;\,field\}}\equiv\Delta\lambda_i^{log}+\Delta\lambda_i^{finite}\,,\tag{29}$$

$$\text{note}\qquad \Delta\,\lambda_{5,6,7}^{log}=0\,,\qquad \Delta\,\lambda_{5,6,7}^{field}=0\,.\tag{30}$$

These running effective self-couplings λ_i (28) satisfy the boundary conditions determined by the running gauge coupling constants (in $\lambda_{1,2,3,4}^{SUSY}$) and finite power [11, 7] terms from the soft SUSY-breaking interactions of Higgs bosons with scalar quarks ($\Delta\lambda_{1,2,3,4}^{finite}$ and finite $\Delta\lambda_{5,6,7}$). The logarithmic dependence $\log\left(\frac{M^2_{\rm SUSY}}{\sigma^2}\right)$ corresponds to the running

[5]We have to take $\frac{1}{2}\,\Delta\,\lambda_{1,\,2}$ in the left side of the first (29) due to its notation in (28).

of λ from $M^2_{\rm SUSY}$ to σ (down to m_t) and the leading finite m_t^4 dependence corresponds to the dominant piece of the beta-function for λ.

Thus

$$\lambda_5 = -\,\Delta\lambda_5 = -\,\frac{3}{96\,\pi^2}\left(h_t^4\left(\frac{\mu A_t}{M^2_{\rm SUSY}}\right)^2 + h_b^4\left(\frac{\mu A_b}{M^2_{\rm SUSY}}\right)^2\right), \tag{31}$$

$$\lambda_6 = -\,\Delta\lambda_6 = \frac{3}{96\,\pi^2}\left[h_t^4\,\frac{|\mu|^2\mu A_t}{M^4_{\rm SUSY}} - h_b^4\,\frac{\mu A_b}{M^2_{\rm SUSY}}\left(6 - \frac{|A_b|^2}{M^2_{\rm SUSY}}\right) + \right. \tag{32}$$

$$\left. + \,(h_b^2 A_b - h_t^2 A_t)\,\frac{3\,\mu}{M^2_{\rm SUSY}}\,\frac{g_2^2\, +\, g_1^2}{4}\right], \tag{33}$$

$$\lambda_7 = -\,\Delta\lambda_7 = \frac{3}{96\,\pi^2}\left[h_b^4\,\frac{|\mu|^2\mu A_b}{M^4_{\rm SUSY}} - h_t^4\,\frac{\mu A_t}{M^2_{\rm SUSY}}\left(6 - \frac{|A_t|^2}{M^2_{\rm SUSY}}\right) + \right. \tag{34}$$

$$\left. + \,(h_t^2 A_t - h_b^2 A_b)\,\frac{3\,\mu}{M^2_{\rm SUSY}}\,\frac{g_2^2\, +\, g_1^2}{4}\right]. \tag{35}$$

Comparison of the expressions (31)-(35) for the parameters λ_i ($i = 5, 6, 7$) with [4] (the case of CP violation) and [7, 14] (the theory with CP conservation) shows that the results are consistent. D-term contributions (due to the interaction terms in (11) with gauge couplings g_1 and g_2) (33), (35) are neglected in [4, 14]. With this note the analytical expressions (31)-(35) coincide in the CP conserving limit with the one-loop contributions in [7, 14] exactly.

For phenomenological considerations at $A_t = A_b \equiv A$ the universal phase of μA is usually introduced:

$$\lambda_5 = |\,\lambda_5|\,\exp\left[i\,2\arg(\mu A)\right], \quad \lambda_6 = |\,\lambda_6|\,\exp\left[i\arg(\mu A)\right], \quad \lambda_7 = |\,\lambda_7|\,\exp\left[i\arg(\mu A)\right].$$

Let us note that the corrections to the parameters λ_i ($i = 1, 2, 3, 4$) do not contain phases up to the two-loop approximation and coincide with corresponding terms [4] in the one-loop approximation (neglect D-term contributions), being consistent with [14] and finite power terms in [7] in the CP conserving limit:

$$\lambda_1 = \frac{g_2^2 + g_1^2}{8} + \frac{3}{32\pi^2}\left[h_b^4\frac{|A_b|^2}{M^2_{\rm SUSY}}\left(2 - \frac{|A_b|^2}{6M^2_{\rm SUSY}}\right) - h_t^4\frac{|\,\mu|^4}{6M^4_{\rm SUSY}} + 2h_b^4 l \right. \tag{36}$$

$$\left. + \frac{g_2^2 + g_1^2}{4\,M^2_{\rm SUSY}}(h_t^2|\,\mu|^2 - h_b^2|A_b|^2)\right] + \Delta\,\lambda_1^{field} + \tag{37}$$

$$+ \frac{1}{768\pi^2}\left(11g_1^4 + 9g_2^4 - 36\,(g_1^2 + g_2^2)\,h_b^2\right)l\,, \tag{38}$$

$$\lambda_2 = \lambda_1\ (\textit{replace indices}\ \ t \longleftrightarrow b)\,, \tag{39}$$

$$\lambda_3 = \frac{g_2^2 - g_1^2}{4}\left[1 - \frac{3}{16\pi^2}\,(h_t^2 + h_b^2)\,l\right] + \frac{3}{8\pi^2}\,h_t^2 h_b^2\left[l + \frac{1}{2}X_{tb}\right] +$$

$$+\frac{3}{96\pi^2}\frac{|\mu|^2}{M_{\rm SUSY}^2}\left[h_t^4\left(3-\frac{|A_t|^2}{M_{\rm SUSY}^2}\right)+h_b^4\left(3-\frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right]+ \tag{40}$$

$$+\frac{3(g_2^2-g_1^2)\,[h_b^2(|\mu|^2-|A_b|^2)+h_t^2(|\mu|^2-|A_t|^2)]}{128\,\pi^2 M_{\rm SUSY}^2}+\Delta\,\lambda_3^{field}+ \tag{41}$$

$$+\frac{9g_2^4-11g_1^4}{384\,\pi^2}\,l\,, \tag{42}$$

$$\lambda_4=-\frac{g_2^2}{2}\left[1-\frac{3}{16\pi^2}\,(h_t^2+h_b^2)\,l\right]-\frac{3}{8\pi^2}\,h_t^2h_b^2\left[l+\frac{1}{2}X_{tb}\right]+$$

$$+\frac{3}{96\pi^2}\frac{|\mu|^2}{M_{\rm SUSY}^2}\left[h_t^4\left(3-\frac{|A_t|^2}{M_{\rm SUSY}^2}\right)+h_b^4\left(3-\frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right]- \tag{43}$$

$$-\frac{3g_2^2\,[h_b^2(|\mu|^2-|A_b|^2)+h_t^2(|\mu|^2-|A_t|^2)]}{64\pi^2 M_{\rm SUSY}^2}+\Delta\,\lambda_4^{field}- \tag{44}$$

$$-\frac{3g_2^4}{64\pi^2}\,l\,. \tag{45}$$

where

$$X_{tb}\equiv\frac{|A_t|^2+|A_b|^2+2\Re{\rm e}(A_b^*A_t)}{2M_{\rm SUSY}^2}-\frac{|\mu|^2}{M_{\rm SUSY}^2}-\frac{||\mu|^2-A_b^*A_t|^2}{6M_{\rm SUSY}^4}\,. \tag{46}$$

Again D-term contributions are neglected in [4]. And thus one-loop contributions for $\lambda_{1,2,3,4}$ in [4] coincide exactly with only parts of our one-loop expressions (36), (40), (43) and corresponding part of (39).

Neglecting the D-term contributions (from interaction terms in (11) with gauge couplings), the field renormalization contributions $\Delta\,\lambda_i^{field}$ and bottom Yukawa coupling effects we can rewrite[6] (31)-(45) in the leading $\mathcal{O}(h_t^4)$ one-loop approximation ($A_t\equiv A$):

$$\lambda_1\approx\frac{g_2^2+g_1^2}{8}-\frac{3}{192\,\pi^2}\,h_t^4\frac{|\,\mu|^4}{M_{\rm SUSY}^4}\,, \tag{47}$$

$$\lambda_2\approx\frac{g_2^2+g_1^2}{8}+\frac{3}{32\,\pi^2}\left[h_t^4\frac{|A|^2}{M_{\rm SUSY}^2}\left(2-\frac{|A|^2}{6M_{\rm SUSY}^2}\right)+2h_t^4 l\right]\,, \tag{48}$$

$$\lambda_3\approx\frac{g_2^2-g_1^2}{4}+\frac{3}{96\,\pi^2}\frac{|\mu|^2}{M_{\rm SUSY}^2}\,h_t^4\left(3-\frac{|A|^2}{M_{\rm SUSY}^2}\right)\,, \tag{49}$$

$$\lambda_4\approx-\frac{g_2^2}{2}+\frac{3}{96\,\pi^2}\frac{|\mu|^2}{M_{\rm SUSY}^2}\,h_t^4\left(3-\frac{|A|^2}{M_{\rm SUSY}^2}\right)\,. \tag{50}$$

$$\lambda_5\approx-\frac{3}{96\,\pi^2}\,h_t^4\left(\frac{\mu A}{M_{\rm SUSY}^2}\right)^2\,, \tag{51}$$

$$\lambda_6\approx\frac{3}{96\,\pi^2}\,h_t^4\,\frac{|\mu|^2\mu A}{M_{\rm SUSY}^4}\,, \tag{52}$$

[6]These results are consistent ((51)-(53) in the CP conserving limit) with [8], replace our notation $\lambda_{1,2}$ to $\lambda_{1,2}/2$ in [8].

$$\lambda_7 \approx \frac{3}{96\,\pi^2}\left[-\,h_t^4\,\frac{\mu A_t}{M_{\rm SUSY}^2}\left(6\,-\,\frac{|A_t|^2}{M_{\rm SUSY}^2}\right)\right]. \tag{53}$$

It is interesting to note that only the beta-function for λ_2 (see (48)) gets constant negative value $-6h_t^4$, which coincides with the result of [7].

The expressions (47)-(53) for λ_i can be used for obtaining approximate theoretical predictions. Numerical analysis demonstrates that the contributions from the D-terms and the wave-function renormalization can be of the same order as the leading two-loop contributions presented in [4]. In some regions of the MSSM parameter space they play more significant role than the leading two-loop contributions. So for the phenomenological applications it is better to use the expressions (31)-(45). For the precise two-loop approximation it is necessary to incorporate also the two-loop contributions from D-terms and the wave-function renormalization which comes from the interaction with massive virtual scalar quarks fields.

Summary

The effective potential approach for the two-Higgs-doublet sector of the minimal supersymmetric model was used to express the complex parameters of the effective Higgs potential in terms of the parameters describing the Higgs boson interactions with scalar quarks. The complex parameters of the scalar quark-Higgs boson soft supersymmetry breaking sector induce the complex parameters in the effective Higgs potential of the minimal supersymmetry, thus radiatively breaking its CP invariance.

The invariant effective parameters (charges) λ_1, ..., λ_7 (36)-(45) and (31)-(35) obtained by neans of the effective theory approach represent the evolution of the effective Higgs potential from the energy scale M_{SUSY} to electroweak scale of order m_t. They take into account the interaction between Higgs bosons and massive scalar quarks fields. We can incorporate these results into the masses of physical Higgs states after the diagonalization in the local minimum in the THDM with CP violation [5]. Also we can incorporate these results into the renormalization group analysis for computing RGE due to the scalar quark sector by demanding that the one-loop renormalized effective potential is independent on the arbitrary renormalization scale σ and obtaining running self-couplings at M_Z energy scales by solving RGE including the contributions from other particles.

The authors thank very much I.F. Ginzburg and M. Krawczyk for useful discussions. E.N. Akhmetzyanova thanks the foundation "Dynasty" and ICFPM for the financial support. The work of M.V. Dolgopolov and M.N. Dubinin was partially supported by RFBR 04-02-17448. The work of M.N. Dubinin was partially supported by INTAS 03-51-4007, UR 02.03.028 and NS 1685.2003.2.

References

[1] M. Kobayashi, T. Maskawa, Prog.Theor.Phys. 49 (1973) 652

[2] H. Georgi, Hadr.J.Phys. 1 (1978) 155

[3] T.D. Lee, Phys.Rev. D8 (1973) 1226

[4] A. Pilaftsis, C.E.M. Wagner, Nucl.Phys. B553 (1999) 3

[5] M.N. Dubinin, A.V. Semenov, Eur.J.Phys. C28 (2003) 223

[6] E.N. Akhmetzyanova, M.V. Dolgopolov, M.N. Dubinin, Vestnik of Samara State University. № 4(30) (2003) 147 (see `http://www.ssu.samara.ru/~vestnik`)

[7] H.E. Haber, R. Hempfling, Phys.Rev. D48 (1993) 4280

[8] M. Quiros, In: Perspectives on Higgs physics II, ed. by Kane G.L. World Scientific. (1998)

[9] R.D. Peccei, E-preprint hep-ph/0209245 (2002)

[10] K. Inoue, A. Kakuto, H. Komatsu, S. Takeshita, Progr.Theor.Phys. 67 No.6 (1982) 1889; Progr.Theor.Phys. 68. No.3 (1982) 927
R.A. Flores, M. Sher, Ann.Phys.(N.Y.) 148 (1983) 95

[11] Y. Okada, M. Yamaguchi, T. Yanagida Phys.Lett. B262 (1991) 54
J. Ellis, G. Ridolfi, F. Zwirner, Phys.Lett. B257 (1991) 83
H. Haber, R. Hempfling, Phys.Rev.Lett. 66 (1991) 1815
R. Barbieri, M. Frigeni, F. Caravaglios, Phys.Lett. B258 (1991) 167

[12] M. Carena, J.R. Ellis, A. Pilaftsis, C.E.M. Wagner, Nucl.Phys. B586 (2000) 92

[13] S. Coleman, E. Weinberg, Phys.Rev. D7 (1973) 1888

[14] M. Carena, J.R. Espinosa, M. Quiros, C.E.M. Wagner, Phys.Lett. B355 (1995) 209