String-like meson spectra: phenomenology vs. QCD

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Abstract

The spectrum of masses and decay constants of non-strange meson resonances in the energy range 0–2.5 GeV is analyzed. We consider the possible corrections to the linear trajectories by matching two-point correlators to the Operator Product Expansion (OPE). From the ensuing sum rules we find that the allowed modifications to the linear Regge behavior must take a specific form. After fitting the lightest states in each channel the whole spectrum for meson masses and residues is obtained in a good agreement with phenomenology.

1 Introduction

Hadron phenomenology tells us that the masses squared of mesons with given quantum numbers lie approximately on equally-spaced linear trajectories[1] (see, for example, the modern reviews [1, 2]). This is a strong indication that QCD admits an effective string description, as this type of spectrum is characteristic e.g. of the bosonic[2] string [4]. Long ago it was noticed [3] that if one accepts this behavior, by including an infinite number of resonances it is possible to reproduce the parton-model logarithm present in two-point correlators.

This observation means that the exchange of an infinite number of vector mesons may be dual to the perturbative QCD continuum made of quarks and gluons. This quark-hadron duality was explicitly verified in QCD in 1+1 dimensions in the large-N_c limit [5,6]. At present, the QCD community understand by quark-hadron duality several, somewhat different, things. The most common assertion is that perturbation theory can be used to calculate a certain smeared hadronic cross section (so-called global duality) [7]. We use the term in a rather lax sense and understand by quark-hadron duality any matching of hadronic and partonic degrees of freedom.

In the context of large-N_c QCD, quark-hadron duality for mesons has received a lot of attention [8, 10–16].

In the present work we propose a systematic method [17] to take into account possible corrections to the linear trajectories in the V, A, S, P channels. Our ansatz for the meson mass spectra, once substituted into two-point correlators, is matched to the OPE, which gives a set of constraints on meson mass parameters. As we shall see, the OPE dictates a

[1] From now on, we shall term this behavior 'linear trajectory ansatz' (LTA)

[2] This string model is of course not consistent, a more realistic amplitude is the Lovelace-Shapiro amplitude but unfortunately it cannot be derived from any known string. See e.g. [32] for a discussion on this point

very particular class of corrections to the linear trajectory ansatz. These corrections are not anecdotal: they turn out to be essential to get good phenomenology and by taking them into account one does not need to introduce an artificial separation between low-lying and excited radial excitations in the different channels like in [11,12,15,16] Finally, the results are fitted to the experimental data [1,2,18].

2 Current-current correlators and QCD sum rules

Let us consider the two-point correlators of V, A, S, P quark currents in the large-N_c limit of QCD. On the one hand, by virtue of confinement they are saturated by an infinite set of narrow meson resonances, that is, they can be represented by the sum of related meson poles in Euclidean space

$$\Pi^J(Q^2) = \int d^4x \exp(iQx) \langle \bar{q}\Gamma q(x) \bar{q}\Gamma q(0)\rangle_{planar} = \sum_n \frac{Z_J(n)}{Q^2 + m_J^2(n)} + D_0^J + D_1^J Q^2, \quad (1)$$

$$J \equiv S, P, V, A; \qquad \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = const. \quad (2)$$

The last two terms both in the S, P and in the V, A channels represent a perturbative contribution, with D_0 and D_1 being contact terms required to eliminate divergences in the ultraviolet. On the other hand, their high-energy asymptotics is provided [9,19] by perturbation theory and the OPE. In the chiral limit

$$\Pi^V(Q^2) = \frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{\Lambda^2}{Q^2} - \frac{\alpha_s}{4\pi^3}\cdot\frac{\lambda^2}{Q^2} + \frac{\alpha_s}{12\pi}\cdot\frac{\langle (G^a_{\mu\nu})^2\rangle}{Q^4} - \frac{28}{9}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^6}, \quad (3)$$

$$\Pi^A(Q^2) = \frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{\Lambda^2}{Q^2} - \frac{\alpha_s}{4\pi^3}\cdot\frac{\lambda^2}{Q^2} + \frac{\alpha_s}{12\pi}\cdot\frac{\langle (G^a_{\mu\nu})^2\rangle}{Q^4} + \frac{44}{9}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^6}, \quad (4)$$

$$\Pi^S(Q^2) = -\frac{3}{8\pi^2}\left(1 + \frac{11\alpha_s}{3\pi}\right)Q^2\ln\frac{\Lambda^2}{Q^2} - \frac{3\alpha_s}{2\pi^3}\lambda^2\ln\frac{\Lambda^2}{Q^2} + \frac{\alpha_s}{8\pi}\cdot\frac{\langle (G^a_{\mu\nu})^2\rangle}{Q^2} - \frac{22}{3}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^4}, \quad (5)$$

$$\Pi^P(Q^2) = -\frac{3}{8\pi^2}\left(1 + \frac{11\alpha_s}{3\pi}\right)Q^2\ln\frac{\Lambda^2}{Q^2} - \frac{3\alpha_s}{2\pi^3}\lambda^2\ln\frac{\Lambda^2}{Q^2} + \frac{\alpha_s}{8\pi}\cdot\frac{\langle (G^a_{\mu\nu})^2\rangle}{Q^2} + \frac{14}{3}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^4}, \quad (6)$$

where we have defined

$$\Pi^{V,A}_{\mu\nu}(Q^2) \equiv \left(-\delta_{\mu\nu}Q^2 + Q_\mu Q_\nu\right)\Pi^{V,A}(Q^2). \quad (7)$$

The vacuum saturation for the 4-fermion condensate [9] in the large-N_c limit has been used. In Eqs. (3)-(6) the $\langle (G^a_{\mu\nu})^2\rangle$ and the $\langle\bar{q}q\rangle$ represent the gluon and the quark condensate, respectively. The normalization scale Λ denotes a momentum cutoff. The dimension 2 gluon condensate λ was introduced in [20]. This dimension-two condensate is gauge-dependent: for speculations concerning origin, measurement, and physical meaning of this condensate see [20,21]. We shall not include this term in our analysis.

As it can be seen from Eqs. (3)-(6) differences of correlators of opposite parity decrease very rapidly at large momenta

$$\Pi^V(Q^2) - \Pi^A(Q^2) = -8\pi\alpha_s \frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right), \quad (8)$$

$$\Pi^S(Q^2) - \Pi^P(Q^2) = -12\pi\alpha_s \frac{\langle\bar{q}q\rangle^2}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right). \quad (9)$$

This results in asymptotic restoration of chiral symmetry (CSR) at high energies, with the quark condensate $\langle\bar{q}q\rangle$ being an order parameter of the chiral symmetry breaking in QCD.

Expanding the right-hand side of Eq. (1) in inverse powers of Q^2 and comparing with Eqs. (3)-(6) one obtains three sum rules for each channel (we do not account for condensates of dimension eight and higher). If one deals with a finite number of resonances the procedure is straightforward. In the case of infinite number of resonances and given an ansatz for $m^2(n)$ one uses the Euler-Maclaurin summation formula or uses the ψ-function (both methods are equivalent). It should be noted here that, generally speaking, applying a summation procedure for a divergent sum and expanding in $1/Q^2$ are not commutative operations. A regulator is in general called for.

The CSR sum rules, obtained in this way, represent certain constraints on the parameters of meson mass spectra [22]. This information can be extracted and compared with phenomenology which is the subject of the next sections.

In this work we shall ignore the anomalous dimensions of the operators and the running of the coupling constant, and therefore we do not take into account the logarithmic corrections that they may introduce in the CSR sum rules. We shall come back to this issue in a separate paper.

3 Vector and axial-vector resonances

In the V, A case the residues $Z_{V,A}(n)$ in Eq. (1) have the structure

$$Z_{V,A}(n) = 2F^2_{V,A}(n),$$

where the quantities $F_{V,A}(n)$ represent the decay constants.

Let us now introduce the linear trajectory ansatz for the meson mass spectrum

$$m^2_{V,A}(n) = m^2_{V,A} + a_{V,A}n\,; \qquad F^2_{V,A}(n) = const\,; \qquad n = 0,1,2,\ldots\,. \quad (10)$$

In order to do the sums over all resonances we shall use the Euler-Maclaurin summation formula in Eq. (1)

$$\sum_{n=0}^{N} f(n) = \int_0^N f(x)\,dx + \frac{1}{2}[f(0) + f(N)] + \frac{B_1}{2!}[f'(N) - f'(0)] -$$

$$\frac{B_2}{4!}[f'''(N) - f'''(0)] + \ldots + (-1)^k \frac{B_{k+1}}{(2k+2)!}[f^{(2k+1)}(N) - f^{(2k+1)}(0)] + \ldots\,, \quad (11)$$

where $B_1 = 1/6\,, B_2 = 1/30\,, \ldots$ are Bernoulli numbers. We shall expand the final expression in powers of Q^{-2} and after comparing with the OPE's (3) and (4) one gets the set of CSR sum rules.

As we have already mentioned, it is not possible to extract further consequences from the individual V and A CSR sum rules. Subleading terms are ambiguous. Fortunately, the annoying logarithm is absent in differences of correlation functions. Namely

$$\Pi^V(Q^2) - \Pi^A(Q^2) = 2\sum_{n=0}^{\infty}\left(\frac{F_V^2(n)}{Q^2 + m_V^2(n)} - \frac{F_A^2(n)}{Q^2 + m_A^2(n)}\right) \tag{12}$$

This difference (and the equivalent for S, P) show a very fast CSR. The leading contribution is of $\mathcal{O}(1/Q^6)$: all perturbative and purely gluonic contributions cancel in the difference.

This fact was used among others by [12], where the following generalization of the Weinberg sum rules [25] was proposed

$$\sum_{n=0}^{\infty}\left(F_V^2(n)m_V^{2i}(n) - F_A^2(n)m_A^{2i}(n)\right) = C^{(i)}, \qquad i = 0, 1, \tag{13}$$

$$C^{(0)} = f_\pi^2, \qquad C^{(1)} = 0.$$

we emphasize that one sums over chiral pairs in the difference (13). This becomes important at the moment that one needs to cut-off the sums with a finite value of N. If the cut-off is placed in such a way that the 'chiral partner' of a given resonance, that is included, is left out, chiral symmetry will be explicitly broken by the regulator. It is thus vital to properly identify 'chiral partners'. This is not an issue in the V, A cases, but it will be so as we shall see in the S, P channels.

Our goal is to construct a class of Regge trajectories that does not lead to the unwanted terms. Namely, our ansatz must reproduce the parton-model logarithm and contain only inverse powers of large momentum squared Q^2 besides the logarithm. Recall that, using (11),

$$\sum_{n=0}^{N}\frac{F^2(n)}{Q^2 + m^2(n)} = \int_0^N \frac{F^2(x)\,dx}{Q^2 + m^2(x)} + \mathcal{O}\left(\frac{1}{Q^2}\right)\,. \tag{14}$$

Clearly the logarithm can be only produced by the integral. In order to generate the logarithm we require that

$$\int \frac{F^2(x)\,dx}{Q^2 + m^2(x)} = C\ln\left(\frac{Q^2 + m^2(x)}{\mu^2}\right) + \mathcal{O}\left(\frac{1}{Q^2}\right), \tag{15}$$

here C is a constant and μ is a normalization scale. Differentiating Eq. (15) with respect to x, one can see that the two requirements are satisfied only if

$$F^2(x) = t\,(x)\,\frac{dm^2(x)}{dx}\,, \quad t\,(x) = C + \Delta t\,(x)\ , \tag{16}$$

where $\Delta t\,(x)$ is a decreasing function to be defined. If we do not consider anomalous dimensions the function $\Delta t\,(x)$ does not have to contain any logarithms and other non-polynomial in Q^{-2} terms. Thus, the direct expansion of the integral

$$\int_0^{m(N)} \frac{\Delta t\,(x)\, d\left(m^2(x)\right)}{Q^2+m^2(x)} = \sum_{k>0} t_k \left(\frac{1}{Q^2}\right)^k, \qquad t_k = \int_0^{m(N)} \Delta t\,(x) \left(-m^2(x)\right)^{k-1} d\left(m^2(x)\right)$$

must exist at any order.

Convergence of the integrals is possible if the function $\Delta t\,(x)$ falls off as an exponential, or faster. Therefore, since one does not have yet any dynamical arguments let us take the simplest possibility, namely

$$t\,(x) = C + A_F e^{-B_F x}\,, \qquad B_F > 0\,. \tag{17}$$

Given an ansatz for $m^2(n)$, Eq. (16) together with (17) provides the condition of consistency with the OPE. It is clear that the LTA ansatz (10) is a simple particular case of Eq. (16). Any improvement of the ansatz (10) has to meet Eq. (16) as well. We remark that the corrections to linear trajectories, proposed in [13, 14], do not satisfy this requirement. Thus, these ansätze cannot be matched to the OPE.

The sum rules (13) can actually be generalized to values of i greater than 1

$$\sum_{n=0}^{\infty} \left(F_V^2(n) m_V^{2i}(n) - F_A^2(n) m_A^{2i}(n)\right) = C^{(i)}, \qquad i = 0, 1, \dots\,. \tag{18}$$

Now the $C^{(i)}$ contain a contribution from higher dimensional condensates. For the absolute convergence of Eq. (18) at a given i one has to have

$$F_V^2(n) - F_A^2(n) \sim \frac{1}{n^{1+\alpha}}, \qquad \alpha > i, \tag{19}$$

$$m_V^2(n) - m_A^2(n) \sim \frac{1}{n^{\beta}}, \qquad \beta > i. \tag{20}$$

Let us now discuss corrections to Eq. (10)

$$m_{V,A}^2(n) = m_{0;V,A}^2 + a\,n + \delta_{V,A}(n)\,. \tag{21}$$

Note that the convergence for any i requires $m_V = m_A$ and that the function $\delta_{V,A}(n)$ must decrease at least exponentially.

Collecting our observations, we propose the simplest ansatz satisfying all previous requirements, keeping of course in mind that this is just one of the possibilities

$$\begin{aligned} m_{V,A}^2(n) &= m^2 + an + A_m^{V,A} e^{-B_m n}, \\ F_{V,A}^2(n) &= \left(C + A_F^{V,A} e^{-B_F^{V,A} n}\right)\left(a - A_m^{V,A} B_m e^{-B_m n}\right), \\ C &= \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right), \qquad B_m > 0, \qquad B_F^{V,A} > 0. \end{aligned} \tag{22}$$

4 Scalar and pseudoscalar resonances

The S, P case differs from the V, A one and the discussion presented in the previous section needs some modifications. Due to the transversality (7), the V, A correlators required only one subtraction in Eq. (1). The S, P correlators require two subtractions. Accepting the definition

$$Z_{S,P}(n) \equiv 2F^2_{S,P}(n)m^2_{S,P}(n),$$

the analog of (15) in the S, P case takes the form

$$\int \frac{F^2(x)m^2(x)\,dx}{Q^2+m^2(x)} = -CQ^2 \ln\left(\frac{Q^2+m^2(x)}{\mu^2}\right) - GC\ln\left(\frac{Q^2+m^2(x)}{\mu^2}\right) + \dots\,, \tag{23}$$

where $C > 0$, $G < 0$ are constants. Let us make the following renormalization. The right-hand side of (23) can be rewritten as

$$\sum_n \frac{F^2(n)m^2(n)}{Q^2+m^2(n)} = \sum_n \frac{F^2(n)m^2(n)}{m^2(n)-G} - \sum_n \frac{F^2(n)m^2(n)\,(Q^2+G)}{(m^2(n)-G)\,(Q^2+m^2(n))}\,. \tag{24}$$

The first term in the right-hand side of (24) represents an infinite constant and can be eliminated. Then we have instead of Eq. (23)

$$\int \frac{F^2(x)m^2(x)\,(Q^2+G)\;dx}{(m^2(x)-G)\,(Q^2+m^2(x))} = CQ^2\ln\left(\frac{Q^2+m^2(x)}{\mu^2}\right) + GC\ln\left(\frac{Q^2+m^2(x)}{\mu^2}\right) + \dots\,. \tag{25}$$

Note incidentally, that the procedure above is tantamount to making a subtraction in the correlator at the point G. Repeating the discussion in the previous section, which has led to (16), one obtains in the S, P case

$$F^2_{S,P}(n) = \left(C + A^{S,P}_F e^{-B^{S,P}_F n}\right)\frac{dm^2_{S,P}(n)}{dn}\,. \tag{26}$$

Repeating the same arguments concerning the parameter B_m, as in the previous section, we propose the following parametrization of S, P masses

$$m^2_{S,P} = m^2 + an + A^{S,P}_m e^{-B_m n}. \tag{27}$$

5 Details of fits and results

In order to make the numerical fits we have to fix some parameters. They are: condensates $\langle\bar{q}q\rangle$ and $\langle\left(G^a_{\mu\nu}\right)^2\rangle$, constants f_π and α_s, pion residue $Z_\pi = 2\frac{\langle\bar{q}q\rangle^2}{f^2_\pi}$ (from current algebra), and general slope a (to reproduce the right asymptotics). The results are displayed in the tables at the end. The inputs general for all tables (if any) are: $a = (1120\,\text{MeV})^2$, $\langle\bar{q}q\rangle = -(240\,\text{MeV})^3$, $\frac{\alpha_s}{\pi}\langle\left(G^a_{\mu\nu}\right)^2\rangle = (360\,\text{MeV})^4$, $f_\pi = 87\,\text{MeV}$, $Z_\pi = 2\frac{\langle\bar{q}q\rangle^2}{f^2_\pi}$, $\alpha_s = 0.3$. The units are: $m(n)$, $F(n)$ — MeV; A_m — MeV^2; A_F, $B_{F,m}$ — MeV^0.

Reasonable fits were found for the case, when the ground S and P states do not belong to the corresponding trajectories. The price to pay is the appearance of two new parameters — $Z_S^{\rm out}$ and $m_S^{\rm out}$. The former can be fixed from the linear σ-model: $Z_S^{\rm out} = Z_\pi$. For closure of our equations set we introduce the new input in the CSR sum rules: $m_S(0)$. Thus, normalizing on the masses of the lowest V,A,P states, we are able to reproduce the whole spectrum for masses and residues of the all light V,A,S,P mesons due to the CSR sum rules. As it seen from Table 2, the agreement with phenomenology is very good (taking into account that we work in the large-N_C limit).

Table 1: Set I. An example of parameters for the mass spectra of our work. The corresponding experimental values [2,18] (if any) are displayed in brackets.

Case	Inputs	Predictions
For all $VASP$		$m_0 = 1240,\ B_m = 0.8$
V	$m(0) = 770\,(769.3 \pm 0.8)$	$A_m = -970^2,\ A_F = -0.004,\ B_F = 1.7 + i\pi$
A	$m(0) = 1130\,(1230 \pm 40)$	$A_m = -500^2,\ A_F = -2 \cdot 10^{-7},\ B_F = 0.1$
S	$m(0) = 1230$	$A_m = -165^2,\ A_F = -0.019,\ B_F = 0.6 + i\pi$ $m_S^{\rm out} = 770$
P	$m(0) = 1400\,(1300 \pm 100)$	$A_m = 650^2,\ A_F = 0.004,\ B_F = 0.48$

Table 2: Mass spectrum and residues for the parameter sets of previous Tables. The known experimental values [2,18] are displayed in brackets.

n	out	0	1	2	3
$m_V(n)$		770(769.3±0.8)	1540 (1465 ± 25)	1960	2280 (2149 ± 17)
$F_V(n)$		143(154±8)	152	139	135
$m_A(n)$		1130(1230±40)	1640 (1640 ± 40)	2000 (1971 ± 15)	2300 (2270 ± 50)
$F_A(n)$		142(123±25)	137	134	133
$m_S(n)$	770	1230	1670	2010	2300
$F_S(n)$	207	95	213	160	190
$m_P(n)$	0	1400	1730 (1801 ± 13)	2030 (2070 ± 35)	2310 (2360 ± 30)
$F_P(n)$	–	155	169	175	178

6 Summary

In the present work we have considered the problem of matching the vector, axial-vector, scalar, and pseudoscalar meson mass spectra $m^2(n)$ (n is the number of radial excitation) with universal slope to the Operator Product Expansion. The analysis was carried out for the light non-strange mesons in the large-N_c and chiral limits. The main result is that, given the masses of the ground states, the chiral symmetry restoration sum rules (following from this matching) allow to restore the whole spectrum for the V,A,S,P masses and residues. Let us summarize the important lessons which we gained from our analysis.

- The matching to the OPE can not be achieved by a simple linear parametrization of the mass spectrum.

- The convergence of the generalized Weinberg sum rules requires the universality of intercepts for masses of chiral partners. The tree-level limit implies universality of intercepts for all V,A,S,P spectra. Thus, there must exist deviations from the linear spectra (string picture) parametrizing the chiral symmetry breaking. These deviations must decrease at least exponentially.

- There are also deviations from the relation for the meson residues (decay constants) $F^2(n) \sim \frac{dm^2(n)}{dn}$. The analytic structure of OPE imposes the exponential decrease on these deviations (or faster).

We have proposed the simplest ansatz for the V,A,S,P meson masses and residues meeting the all general principles above. The numerical solutions of the CSR sum rules for these spectra were obtained and compared with the experimental data. Our fits agree well with the existing phenomenology.

Unfortunately, the underlying dynamics, which generates the non-linear contributions to the spectra of meson masses and residues, is not well known. We can say only that these deviations from the string picture seem to parametrize the chiral symmetry breaking in QCD [32] and, hence, must be proportional to the chiral condensate $\langle \bar{q}q \rangle$. Developing a theory of these non-linear contributions is an interesting task for future.

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