Light particle trapping on the fermion induced domain wall

A. A. Andrianov [♭♯], V. A. Andrianov [♭], P. Giacconi [♯] and R. Soldati [♯]

[♭] V.A.Fock Department of Theoretical Physics, Sankt-Petersburg State University,
198504 Sankt-Petersburg, Russia
[♯] Dipartimento di Fisica, Universitá di Bologna and
Istituto Nazionale di Fisica Nucleare, Sezione di Bologna,
40126 Bologna, Italia

Abstract

A possibility for light fermions and Higgs bosons to be localized on the four-dimensional domain wall in five-dimensional world is examined. The mechanism of light particle trapping is accounted for by a strong self-interaction of five-dimensional pre-quarks. The fluctuation of the brane gives rise to a nearly sterile scalar particles, branons, which may be candidates for the dark matter.

1 Introduction

The possibility of location of the observable world on a four-dimensional surface – a domain wall or a 3-brane – in a space-time with dimension higher than four has been recently set forth as a theoretical concept [1]–[3] for solutions to the problems of the Planck mass scale, symmetry breaking scales and fermion mass hierarchy. Respectively, an experimental program has been posed for the forthcoming collider and non-collider physics research to discover new particles signaling on the existence of extra dimensions. The extensive literature on those subjects and their applications is now covered in few review articles [4]-[7]. Among other options the domain wall formation and the trapping of low-energy particles in its layer might be triggered [8]-[10] by a number of background fields living in the multi-dimensional bulk. Certainly, the dynamical origin of such background fields and the way of how they induce spontaneous symmetry breaking is to be explored.

In our talk we describe the non-compact *4 + 1*-dimensional fermion model with strong local four-fermion interaction that leads to the discrete symmetry breaking and to a domain wall pattern of the vacuum state [11]. As a consequence light massive Dirac particles and light scalar bosons, Higgs-like particles and "branons" [12], live in four dimensions whereas very heavy states may leave a brane, *i.e.* disappear from our world.

The main ingredients of the model are made of the five-dimensional fermion bi-spinors $\psi(X)$ coupled to a scalar field $\Phi(X)$. The extra-dimension coordinate is assumed to be space-like, $(X_\alpha) = (x_\mu, z)$, $\mu = (0, 1, 2, 3)$, and the subspace of x_μ eventually corresponds to the four-dimensional Minkowski space. The extra-dimension size is supposed to be large enough. The fermion wave function obeys then the Dirac equation

$$[\, i\gamma_\alpha \partial^\alpha - \Phi(X)\,]\psi(X) = 0 \ , \quad \gamma_\alpha = (\gamma_\mu, -i\gamma_5) \ , \quad \{\gamma_\alpha, \gamma_\beta\} = 2g_{\alpha\beta} \ ,$$

with γ_α being a set of the Dirac matrices.

The trapping of light fermions on a four-dimensional hyper-plane – the domain wall – located in the fifth dimension at $z = 0$ can be generated by a certain *topological* background configuration of the scalar field, for instance, by $\langle\Phi(X)\rangle_0 = M\tanh(Mz)$ as it induces a z-localized zero-mode in the four-dimensional fermion spectrum, i.e. an essentially four-dimensional massless fermion with a given chirality (left or right).

Meanwhile the real quarks and leptons of our world are mainly massive Dirac fermions. Therefore, for each light fermion on a brane one needs at least two five-dimensional proto-fermions $\psi_1(X), \psi_2(X)$, to obtain left- and right-handed parts of a four-dimensional Dirac bi-spinor as zero modes. In order to produce zero modes with different chiralities they have to couple to a background scalar field with opposite charges,

$$[\, i\,\not\partial - \tau_3\Phi(X)\,]\Psi(X) = 0\ , \quad \not\partial \equiv \widehat{\gamma}_\alpha\partial^\alpha\ , \quad \Psi(X) = \left(\begin{array}{c}\psi_1(X)\\ \psi_2(X)\end{array}\right)\ , \tag{1}$$

where $\widehat{\gamma}_\alpha \equiv \gamma_\alpha \otimes \mathbf{1}_2$ are Dirac matrices and $\tau_a \equiv \mathbf{1}_4 \otimes \sigma_a,\ a = 1,2,3$ are Pauli matrices acting on the bi-spinor components $\psi_i(X)$.

The next task is to supply these fermions with light masses. As the mass operator mixes left- and right-handed components of the four-dimensional fermion it is embedded in the Dirac operator (1) with the mixing matrix $\tau_1 m_f$ of the fields $\psi_1(X)$ and $\psi_2(X)$. Following the Standard Model fermion mass generation by means of Higgs scalars, one may introduce the second scalar field $H(x)$ replacing the bare mass $\tau_1 m_f$ by v.e.v. $\tau_1 < H(x) >$. Both scalar fields might be dynamical indeed and their self-interaction should justify the spontaneous symmetry breaking by certain classical configurations allocating light massive fermions on the domain wall.

2 Fermion model with self-interaction in 5D

After the preliminary motivation we formulate our model with the following Lagrange density

$$\mathcal{L}^{(5)}(\overline{\Psi},\Psi) = \overline{\Psi}\ i\,\not\partial\Psi + \frac{g_1}{4N\Lambda^3}\left(\overline{\Psi}\tau_3\Psi\right)^2 + \frac{g_2}{4N\Lambda^3}\left(\overline{\Psi}\tau_1\Psi\right)^2 ,$$
$$\Longrightarrow \overline{\Psi}(i\,\not\partial - \tau_3\Phi - \tau_1 H)\Psi - \frac{N\Lambda^3}{g_1}\,\Phi^2 - \frac{N\Lambda^3}{g_2}\,H^2\ .$$

where $\Psi(X)$ is an eight-component five-dimensional fermion field with the total number $N = N_f N_c$ of color and flavor states. The ultraviolet cut-off scale Λ bounds fermion momenta and g_1 and g_2 are suitable dimensionless effective couplings. In the second line the bosonization with the help of a pair of auxiliary scalar fields $\Phi(X)$ and $H(X)$ is performed to get the required trilinear coupling of fermions and scalars.

In this Lagrangian the discrete τ-symmetry: $\Psi \to \tau_1\Psi$, $\Phi \to -\Phi$ and $\Psi \to \tau_3\Psi$, $H \to -H$, does not allow the fermions to acquire a mass and prevents a breaking of translational invariance in the perturbation theory.

On the other hand, for sufficiently strong couplings, this system undergoes spontaneous breaking of the τ-symmetry. In order to describe it, the effective low-energy Lagrange density is derived with the help of FMR [13],

$$\begin{aligned}
\mathcal{L}_{\rm low}^{(5)} &= \overline{\Psi}_l(X)[\, i\,\partial\!\!\!/ - \tau_3\Phi(X) - \tau_1 H(X)\,]\Psi_l(X) \\
&+\frac{N\Lambda}{4\pi^3}\,\{\partial_\alpha\Phi(X)\partial^\alpha\Phi(X) + \partial_\alpha H(X)\partial^\alpha H(X) \\
&+2\Delta_1\Phi^2(X) + 2\Delta_2 H^2(X) - [\,\Phi^2(X) + H^2(X)\,]^2\Big\}\,. \qquad (2)
\end{aligned}$$

where the two mass scales Δ_i are introduced in order to parameterize the deviations from the critical point $g_i^{\rm cr} = 9\pi^3$, $\quad i = 1,2$:

$$\Delta_i(g_i) = \frac{2\Lambda^2}{9g_i}\,(g_i - g_i^{\rm cr}) \qquad \Delta_1(g_1) > \Delta_2(g_2)\ .$$

If $\Delta_1(g_1) > 0$, then the true minima appear at a non-vanishing vacuum expectation value of the scalar field $\Phi(X)$: namely,

$$(I) \qquad \Phi_I \equiv \langle\Phi(X)\rangle_0 = \pm\sqrt{\Delta_1(g_1)} \equiv \pm M\ , \quad H_I \equiv \langle H(X)\rangle_0 = 0\ .$$

This follows from the stationary point conditions,

$$2\left[M^2 - \Phi^2 - H^2\right]\Phi = \partial^\alpha\partial_\alpha\Phi\ , \quad 2\left[\Delta_2 - H^2 - \Phi^2\right]H = \partial^\alpha\partial_\alpha H\ ,$$

and from the positive definiteness of the second variation of the boson effective action. From the latter one we find the masses of composite scalars: $M_1 = 2M$ (the Nambu relation) and $M_2 = \sqrt{2\Delta_1 - 2\Delta_2}$. At the same time the generation of a dynamical fermion mass M occurs that breaks the τ_1-symmetry. Around this minimum the particle physics is entirely five-dimensional.

3 Domain walls

The existence of two minima in the potential gives rise to another set of vacuum solutions which connect smoothly the minima.

$$(J) \qquad \langle\Phi(X)\rangle_0 = \pm M\tanh(Mz)\ , \quad \langle H(X)\rangle_0 = 0\ ;$$

$$(K) \qquad \langle\Phi(X)\rangle_0 = \pm M\tanh(\beta z)\ , \quad \langle H(X)\rangle_0 = \pm\mu\,{\rm sech}(\beta z)\ .$$

where $\beta = \sqrt{M^2 - \mu^2} = \sqrt{2(\Delta_1 - \Delta_2)}$. The solution (K) exists only for $0 < 2\Delta_2 - M^2 \equiv \mu^2$. For such a range of parameters Δ_i this solution delivers a minimum (it can be found out of the second variation around it) whereas the solution (J) lies on a saddle point.

In both phases v.e.v. of the scalar field $\langle\Phi(X)\rangle$ has a kink shape and hence its coupling to fermions induces the trapping of the lightest, massless fermion state on the domain wall. However, only the solution (K) supplies this light fermion with a mass due to a non-zero v.e.v. of the field $\langle H(X)\rangle$. Let us focus on this phase.

At ultra-low energies much smaller than M, the physics on the vacuum (K) is essentially four-dimensional. It is described by the Dirac fermion with the mass

$$m_f \equiv \int_{-\infty}^{+\infty} dz\,\overline{\psi}_0(z)\,H_K(z)\,\psi_0(z) = \frac{\pi}{4}\,\mu.$$

As well one has two localized ultra-light modes in the spectrum of the second-variation operator for the scalar fields $\Phi(X)$ and $H(X)$. They produce two four-dimensional scalar bosons, a massless ϕ and a massive h. When assuming that $\mu \ll M$ one finds,

$$m_h = \mu\sqrt{2} + \mathcal{O}(\frac{\mu^2}{M}).$$

The ultra-low-energy effective four-dimensional Lagrangian density for the light states reads:

$$\mathcal{L}^{(4)} = \overline{\psi}(x)(i\gamma^\mu\partial_\mu - m_f)\psi(x) + \frac{1}{2}\Big(\partial_\mu\phi(x)\Big)^2 + \frac{1}{2}\Big(\partial_\mu h(x)\Big)^2 - \frac{1}{2}m_h^2 h^2(x)$$
$$-g_f\overline{\psi}(x)\psi(x)h(x) - \lambda_1\phi^4(x) - \lambda_2\phi^2(x)h^2(x) - \lambda_3 h^4(x) - \lambda_4\, h^3(x)\ ,$$

with the ultra-low energy effective couplings given by

$$g_f = \frac{\pi}{4}\sqrt{\frac{M\pi^3}{\Lambda N}}\ ,\ \lambda_1 = \frac{18M\pi^3}{35\Lambda N}\ ,\ \lambda_2 = \frac{2M\pi^3}{5\Lambda N}\ ,\ \lambda_3 = \frac{M\pi^3}{3\Lambda N}\ , \lambda_4 = \mu\pi\sqrt{\frac{M\pi}{\Lambda N}}\ .$$

Herein the heavy scalars and fermions with masses $\sim M$ have been decoupled.

Quite remarkably, the domain wall Lagrange density has a non-trivial large cut-off limit in the vicinity of the scaling point $\mu \ll M$, provided that the ratio $M/\Lambda < 1$ is fixed. The four-dimensional ultra-low-energy theory happens to be interacting.

We stress that *a priori* possible 3-point vertex $\phi^2(x)\, h(x)$ does not emerge and the coupling of light fermions to the massless scalar $\overline{\psi}(x)\psi(x)\phi(x)$ cannot appear in principle. Thereby the direct decay of the massive Higgs-like boson h into a pair of massless branons [12] is suppressed and the low-energy Standard Model matter turns out to be stable.

4 Manifest breaking of translational invariance

One can conceive that in reality the translational invariance in five dimensions is broken not only spontaneously but also manifestly due to the presence of a gravitational background, of other branes etc. In a full analogy with the Pseudo-Goldstone boson physics one can expect [11, 12] that the small manifest breaking of translational symmetry supplies the branons with a small mass.

In the model presented here the natural realization of the translational symmetry breaking can be implemented by the inhomogeneous scalar background fields coupling to the lowest-dimensional fermion currents. Let us restrict ourselves to the scenario of the type (K) and introduce two scalar defects with the help of the background fields $\mathrm{F}_\Phi(z)$ and $\mathrm{F}_H(z)$ which are supposed to be quite small. These scalar defect fields catalyze the translational symmetry breaking and the domain wall formation by means of their interactions with the fermion currents,

$$\mathcal{L}^{(5)}_{\mathrm{F}} = -\ \mathrm{F}_\Phi(z)\,\overline{\Psi}(X)\tau_3\Psi(X)\ -\ \mathrm{F}_H(z)\,\overline{\Psi}(X)\tau_1\Psi(X)\ .$$

In units comparable with the low energy effective action (2),

$$\mathrm{F}_\Phi(z) \equiv \frac{g_1\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_\Phi(z)\ , \qquad \mathrm{F}_H(z) \equiv \frac{g_2\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_H(z)\ ,$$

Let us introduce a defect of topological type:

$$\mu\, \mathrm{f}_\Phi(z) = M\,\gamma \tanh(\bar\beta z); \quad \mathrm{f}_H(z) = \kappa\, \mathrm{sech}(\bar\beta z)\ ,$$

where γ, κ are dimensionless parameters. For this particular ansatz the solution of the modified equations for stationary configurations of scalar fields can be found analytically [11] in a similar form,

$$\langle \Phi(X) \rangle_0 = M\, a \tanh(\bar\beta z)\ , \quad \langle H(X) \rangle_0 = \mu\,(1+\xi)\, \mathrm{sech}(\bar\beta z)\ .$$

where for $\mu/M \ll 1, \kappa \ll 1, \xi \ll 1$ one obtains approximately $a \approx 1$, $\bar\beta \approx \beta$ and $\kappa \approx 2\xi + (\gamma/2)$. Further on we use ξ as an input parameter instead of κ.

The branon mass happens to be triggered entirely by the topological defect,

$$(m_\phi)^2 \approx \gamma \mu^2\ .$$

One can see a strong polarization effect induced by a topological defect: the local minimum is guaranteed only for asymptotics at infinities which are coherent in their signs, *i.e.*, for positive γ.

The Higgs mass encodes both topological and non-topological vacuum perturbations,

$$(m_h)^2 \approx \mu^2 \left(2 + 6\xi + \frac{1}{2}\gamma\right)\ .$$

As $\gamma > 0$ the topological defect makes the Higgs particle heavier. However the sign of ξ is not fixed by the requirement to provide a local minimum. Therefore the Higgs to fermion ($\sim$ top-quark) mass mass ratio can be substantially reduced with an appropriate choice of a non-topological part of the defect $\sim \xi$. In particular, the Higgs masses may be well adjusted to a phenomenologically acceptable value ~ 135 GeV for a reasonably small value of a defect $\xi \sim 0.4$. The induced coupling constants $\lambda_1, \lambda_2, \lambda_3$ appear to be insensitive to a very small background defect whereas the fermion mass and the constant λ_4 are subject to rescaling $m_f \approx m_f^{(0)}(1+\xi)$, $\lambda_4 \approx \lambda_4^{(0)}(1+\xi)$.

Thus a very small topological defect does not change the main dynamics of domain wall trapping of light fermions and scalars. In particular, the Standard Model matter remains stable and the "branons", being now massive, yet do not decay directly into a pair of fermion and anti-fermion. It makes then difficult to register them in the collider experiments. The branon matter is more elusive for experiments because it is not produced by fermions at the leading order in the Yukawa coupling and does not appear as a decay product of the Higgs bosons. Still the fermion annihilation may result in missing energy events due to radiative creation of branon pair mediated by Higgs bosons. The corresponding Feynman integral is well convergent and therefore, at very low momenta, the amplitude of this process can be estimated to be of order $g_f^2 \cdot \lambda_2 \cdot m_f^2/\mu^2$. We notice that if they are produced by very light fermions, *e.g.* e^+e^-, the very last ratio is negligibly small putting such signals into the experimental background. Nonetheless, for top-quarks (in the fusion production) there might be a possibility to discover branon pair signals for sufficiently light branons.

Anyhow, the nearly sterile, massive branons seem to be good candidates for saturation of the dark matter of our Universe[12].

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