Spontaneous emission of two three-level atoms in the finite-Q cavity

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Abstract

On the basis of the master equation for the density matrix dynamics of the collective spontaneous radiation of two Λ - type three-level atoms interacting with two modes of quantum electromagnetic field in the damped cavity was considered. The behaviour of the photon numbers in modes was investigated in the case when both atoms are in excited state at the initial time.

1 Introduction

In resent years the much interest was attracted by spontaneous radiation of the atomic system in the cavity. This interest has been provoked by experiments on Rydberg atoms in high-Q cavities [1]. The dynamics of the atomic system in the cavity may be considered on the basis of the Jaynes-Cummings-like models, composed of a two-level or three-level atoms interacting with one or two modes of electromagnetic field in lossless cavity. These models have received considerable interest over resent years [2]. This interest was due to a fact that these models are exactly solvable in RWA and yield the nontrivial effects due to the quantum nature of the electromagnetic field. These include the collapse and revival of the atomic inversion and mean photon number, production of squeezed states of electromagnetic field and other properties.

A natural extension of the Jaynes-Cummings-like models would be to take into account the loss of the photon from the cavity. The dynamics of a single two-level atom with one- and two-photon transitions which couples with one mode of the electromagnetic field in the finite-Q cavity was considered in [3]. Different properties of collective radiation of two two-level atoms in a damped cavity were investigated in [4, 5]. In papers [6, 7] the authors considered behaviour of the three-level atom interacting with one and two different modes of the radiation field in the damped cavity. It is of interest to investigate the collective dynamics of two three-level atoms interacting with two different modes of the radiation field in a damped cavity. We shall restrict our attention to the case of spontaneous radiation. In our previous paper [8]we investigated the special case of one-atom initial excitation of two atom system. In this work we shall examine a more interesting case when at the initial time both atoms are at the upper level 3.

2 Model Hamiltonian and kinetic equations

Let us consider the system consisting of two identical three-level atoms with Λ- type transitions interacting resonantly with two modes of the quantum radiation fields in a finite-Q cavity. Following refs.[3] let us write down the master equation for the density matrix of the considered system in the form

$$\frac{\partial \rho}{\partial t} = -i/\hbar\,[H,\rho] - \sum_{i=1}^{2} k_i \left(a_i^+ a_i \rho - 2\,a_i \rho\, a_i^+ + \rho\, a_i^+ a_i\right), \tag{1}$$

where

$$H = H_A + H_F + H_{AF},$$

$$H_A = \sum_{j=1}^{3} \varepsilon_j R_{jj}$$

is the hamiltonian of two free three-level atoms,

$$H_F = \sum_{i=1}^{2} \hbar\omega_i a_i^+ a_i$$

is the hamiltonian of the free two-mode quantum electromagnetic field,

$$H_{AF} = \sum_{i=1}^{2}\sum_{l=1}^{2} \hbar g_i \left(a_i R_{3i}^{(l)} + a^+ R_{i3}^{(l)}\right)$$

is the hamiltonian of the atom-field interaction in the dipole and RWA aproximation. Here ε_j is the energy of the j-th level of the three-level atom, $R_{jj}^{(l)}$ is the population operator of the j-th level in the l-three-level atom, $R_{3i}^{(l)}$ are operators describing the transitions between levels i and 3 $(i = 1, 2)$, ω_i are the frequencies of two modes of the radiation field interacting resonantly with atoms, a_i^+ (a_i) is the operator of creation (annihilation) of a photon with frequencies ω_i obeying the relations $\varepsilon_3 - \varepsilon_1 = \hbar\omega_1$, $\varepsilon_3 - \varepsilon_2 = \hbar\omega_2$, g_i are the atom-field coupling constants, and $2k_1$ and $2k_2$ are the rates of loss of photons of two modes from the cavity.

Using the interaction representation

$$W(t) = e^{i/\hbar\, H_{AF} t}\,\rho(t)\,e^{-i/\hbar\, H_{AF} t},$$

$$\tilde{O}(t) = e^{i/\hbar\, H_{AF} t}\,O\,e^{-i/\hbar\, H_{AF} t},$$

(where O is an arbitrary operators of the atom or field subsystems) one can write the equation for the density matrix (1) in the form

$$\frac{\partial W}{\partial t} = -\sum_{i=1}^{2} k_i \left(\tilde{a}_i^+ \tilde{a}_i W - 2\,\tilde{a}_i W\,\tilde{a}_i^+ + W\,\tilde{a}_i^+ \tilde{a}_i\right), \tag{2}$$

Let us choose the initial conditions for the system under consideration. We shall restrict our consideration to the spontaneous radiation, therefore the electromagnetic field is in a vacuum state in both modes at the initial time. We shall examine below the case of both atoms at the upper level 3 at the initial time. The set of the eigenfunctions (dressed states) and eigenvalues of the Hamiltonian H_{AF} which is necessary to describe the the evolution of the system in a lossless cavity, have the following form

$$|\Psi_i >= C_i \left\{ |3,3;0,0> + A_i^{(1)}|1,1;2,0> + A_i^{(2)}||2,2;0,2> + A_i^{(3)}(|3,1;1,0> + |1,3;1,2>) + \right.$$

$$\left. + A_i^{(4)}(|3,2;0,1> + |3,2;0,1>) + A_i^{(5)}(|1,2;1,1> + |2,1;1,1>) \right\},$$

where

$$A_i^{(1)} = \frac{3\sqrt{2}}{\varepsilon_i^2 - \alpha^2 - 3}, \qquad A^{(2)})_i = \frac{\sqrt{2}(\varepsilon_i^2 - \alpha^2 - 6)}{\varepsilon_i^2 - \alpha^2 - 3}, \qquad A_i^{(3)} = \frac{3\varepsilon_i}{2(\varepsilon_i^2 - \alpha^2 - 3)},$$

$$A_i^{(4)} = \frac{\varepsilon_i(\varepsilon_i^2 - \alpha^2 - 6)}{2\alpha^2(\varepsilon_i^2 - \alpha^2 - 3)}, \qquad A_i^{(5)} = \frac{\varepsilon - i^2 = 2\alpha^2 - 6}{2\alpha(\varepsilon_i^2 - \alpha^2 - 3)}, \qquad (i = 1,2,3,4)$$

and ε_i are the roots of the algebraic equation $\varepsilon^4 - 7(\alpha^2 + 1)\varepsilon^2 + 6\alpha^4 + 28\alpha^2 + 6 = 0$ and $E_i = \hbar\varepsilon_i$;

$$|\Psi_1^0 >= \frac{1}{\sqrt{2(1+\alpha^2}} \{|3,1;1,0> - |1,3;1,0> + \alpha(|3,2;0,1 - |2,3;0,1>)\},$$

$$E_1 = 0;$$

$$|\Psi_2^0 >= \frac{1}{\sqrt{2}} \left\{ |3,3;0,0> - \frac{1}{\sqrt{2}}(|1,1;2,0> + (|2,2;0,2>) \right\},$$

$$E_1 = 0; \tag{3}$$

$$|\Psi_3^0 >= \alpha \frac{2}{\alpha^4 + 4\alpha^2 + 1} \left\{ \frac{\alpha}{\sqrt{2}}|1,1;2,0> + \frac{1}{\alpha\sqrt{2}}|2,2;0,2> - (|1,2;1,1 + |2,1;1,1>) \right\},$$

$$E_3 = 0;$$

$$|\Psi_4^0 >= \alpha^2 \frac{2}{\sqrt{3\alpha^4 + 2\alpha^2 + 3}} \left\{ |3,3;0,0> - \frac{\alpha^2 - 1}{\sqrt{2}\alpha^2}|2,2;0,2> - \frac{1}{\alpha}(|1,2;1,1 + |2,1;1,1>) \right\},$$

$$E_4 = 0;$$

$$|\Psi_5^0 >= \alpha^2 \frac{2}{\sqrt{3\alpha^4 + 2\alpha^2 + 3}} \left\{ |3,3;0,0> - \frac{\alpha^2 - 1}{\sqrt{2}\alpha^2}|2,2;0,2> - \frac{1}{\alpha}(|1,2;1,1 + |2,1;1,1>) \right\},$$

$$E_4 = 0;$$

Here $|x, y; n_1, n_2>$ are the eigenstates of free Hamiltonian $H_A + H_F$:

$$|x, y; n_1, n_2 >= |x>_{A_1} \; |y>_{A_2} \; |n_1>_{F_1} \; |n_2>_{F_2},$$

where $x, y = 1,2,3$ are the numbers of the atomic levels; $n_1, n_2 = 0,1$ are the photon numbers in the modes and $\alpha = g_2/g_1$.

Solution of equation (2) in the basis (3) for arbitrary values of $\alpha = g_2/g_1$ is a very difficult problem. Therefore, in present paper the shall consider the specific case of three-level atom with equal coupling constants $g_1 = g_2 = g$, i.e. $\alpha = 1$ and $k_1 = k_2 = k$.

In this case the set of dressed states required to describe the system dynamics in lossless cavity have became

$$
\begin{aligned}
|\Psi_1>=\frac{1}{5}\Big\{&|3,3;0,0>+\frac{1}{2}(|1,1;2,0>+|2,2;0,2>)+\frac{1}{2}(|1,2;1,1>+|2,1;1,1>)\\
&+\frac{5}{8}(|3,1;1,0>+|1,3;1,0>+|3,2;0,1>+|2,3;0,1>)\Big\}, \qquad \varepsilon=+\sqrt{10};\\
|\Psi_2>=\frac{1}{5}\Big\{&|3,3;0,0>+\frac{1}{2}(|1,1;2,0>+|2,2;0,2>)+\frac{1}{2}(|1,2;1,1>+|2,1;1,1>)\\
&-\frac{5}{8}(|3,1;1,0>+|1,3;1,0>+|3,2;0,1>+|2,3;0,1>)\Big\}, \qquad \varepsilon=-\sqrt{10};\\
|\Psi_3>=\frac{1}{2}\Big\{&|1,1;2,0>-|2,2;0,2>+\frac{1}{\sqrt{2}}(|3,1;1,0>+|1,3;1,0>)-\\
&-(|3,2;0,1>+|2,3;0,1>)\}, \quad \varepsilon=+\sqrt{2};\\
|\Psi_4>=\frac{1}{2}\Big\{&|1,1;2,0>-|2,2;0,2>-\frac{1}{\sqrt{2}}(|3,1;1,0>+|1,3;1,0>)+\\
&+(|3,2;0,1>+|2,3;0,1>)\}, \quad \varepsilon=-\sqrt{2};\\
|\Psi_1^0>=\frac{1}{10}\Big\{&|3,3;0,0>+\frac{1}{2}(|1,1;2,0>+|2,2;0,2>)-\\
&-2(|1,2;1,1>+|2,1;1,1>)\}, \qquad \varepsilon=0;\\
|\Psi_2^0>=\frac{1}{2}\{&|3,1;1,0>-|1,3;1,0>+|3,2;0,1>-|2,3;0,1>\}, \qquad \varepsilon=0;\\
|\Psi_3^0>=\frac{1}{2}\Big\{&|3,3;0,0>-\frac{1}{2}(|1,1;2,0>+|2,2;0,2>)\Big\}, \qquad \varepsilon=0.
\end{aligned} \tag{4}
$$

For finite-Q cavity one must also take into account the set of the following dressed-states

$$
\begin{aligned}
|\varphi_{1,\pm}>&=\frac{1}{2}\{|3,1;0,0>-|1,3;0,0>\pm(|2,1;0,1>-|1,2;0,1>)\}, \qquad \varepsilon=\pm1;\\
|\varphi_{2,\pm}>&=\frac{1}{2}\{|3,2;0,0>-|2,3;0,0>\pm(|1,2;1,0>-|2,1;1,0>)\}, \qquad \varepsilon=\pm1; \qquad (5)\\
|\zeta_{1,\pm}>&=\frac{1}{\sqrt{3}}\Big\{|1,1;1,0>\pm\frac{\sqrt{3}}{2}(|3,1;0,0>+|1,3;0,0>)+\frac{1}{2}(|1,2;0,1>+|2,1;0,1>)\Big\},\\
&\qquad \varepsilon=\pm\sqrt{3};\\
|\zeta_{2,\pm}>&=\frac{1}{\sqrt{3}}\Big\{|2,2;0,1>\pm\frac{\sqrt{3}}{2}(|3,2;0,0>+|2,3;0,0>)+\frac{1}{2}(|1,2;1,0>+|2,1;1,0>)\Big\},\\
&\qquad \varepsilon=\pm\sqrt{3};
\end{aligned}
$$

$$|\Xi_1>=|1,1;0,0>; \quad |\Xi_2>=|2,2;0,0>; \quad |\Xi_3>=|1,2;0,0>; \quad |\Xi_1>=|2,1;0,0>,$$

$$\varepsilon = 0;$$

Using the eigenstates (4) - (5) we can obtain from the master equation (2) the kinetic equation for matrix element of W in the dressed-state representation. Let $k_1, k_2 << g_1, g_2$. Then we can employ the so-called secular approximation [3] and deduce the closed set of kinetic equations. We shall restrict our attention to calculation intensities of radiation in both modes (to obtain the mean photon numbers and atomic populations one must use the full set of dressed states (4)-(5))) . The set of kinetic equations for matrix elements of W that are necessary for this problem are

$$
\begin{aligned}
&< \Psi_i|\dot{W}(t)|\Psi_j >= -\frac{11}{5}k < \Psi_i|\dot{W}(t)|\Psi_j > \quad (i,j=1,2),\\
&< \Psi_i|\dot{W}(t)|\Psi_j >= -3k < \Psi_i|\dot{W}(t)|\Psi_i > \quad (i,j=3,4),\\
&< \Psi_1^0|\dot{W}(t)|\Psi_1^0 >= -\frac{18}{5}k < \Psi_1^0|\dot{W}(t)|\Psi_1 > +\frac{2}{\sqrt{5}}k < \Psi_3^0|\dot{W}(t)|\Psi_1^0 >;\\
&< \Psi_3^0|\dot{W}(t)|\Psi_3^0 >= -2k < \Psi_1^0|\dot{W}(t)|\Psi_1 > +\frac{2}{\sqrt{5}}k < \Psi_3^0|\dot{W}(t)|\Psi_1^0 >;\\
&< \Psi_3^0|\dot{W}(t)|\Psi_1^0 >= -\frac{14}{5}k < \Psi_3^0|\dot{W}(t)|\Psi_1 > +\frac{1}{\sqrt{5}}k(< \Psi_3^0|\dot{W}(t)|\Psi_1^0 > + < \Psi_3^0|\dot{W}(t)|\Psi_1^0 >);\\
&< \Psi_2^0|\dot{W}(t)|\Psi_2^0 >= -2k < \Psi_2^0|\dot{W}(t)|\Psi_2^0 >,\\
&< \Psi_2^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_2^0|\dot{W}(t)|\Psi_i > \quad (i=1,2), \qquad (6)\\
&< \Psi_i|\dot{W}(t)|\Psi_j >= -\frac{13}{10}k < \Psi_i|\dot{W}(t)|\Psi_j > \qquad (i=1,2; j=3,4),\\
&< \Psi_1^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_1^0|\dot{W}(t)|\Psi_i >, \qquad (i=1,2);\\
&< \Psi_1^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_1^0|\dot{W}(t)|\Psi_i >, \qquad (i=3,4);\\
&< \Psi_3^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_3|\dot{W}(t)|\Psi_i > +\frac{2}{\sqrt{5}} < \Psi_1^0|\dot{W}(t)|\Psi_i >), \qquad (i=1,2);\\
&< \Psi_1^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_3|\dot{W}(t)|\Psi_i > +\frac{2}{\sqrt{5}} < \Psi_3^0|\dot{W}(t)|\Psi_i >), \qquad (i=1,2);\\
&< \Psi_3^0|\dot{W}(t)|\Psi_i >= -2k < \Psi_3|\dot{W}(t)|\Psi_i > +\frac{2}{\sqrt{5}} < \Psi_1^0|\dot{W}(t)|\Psi_i >), \qquad (i=3,4);
\end{aligned}
$$

3 Solutions of the kinetic equations and behaviour of observables

For spontaneous radiation (when at t =0 the electromagnetic field is in the vacuum state and both atoms are prepared on the upper excited level 3) we have :

$$W(0) = |3,3;0,0><3,3;0,0| =$$

$$= \frac{1}{4}\left\{ \frac{4}{5}\sum_{1}^{2} |\Psi_i><\Psi_i| + \frac{2}{5}|\Psi_1^0><\Psi_1^0| + 2|\Psi_3^0><\Psi_3^0| + \right.$$

$$+2\sqrt{2}(|\Psi_1><\Psi_1^0| + |\Psi_2><\Psi_1^0| + |\Psi_1^0><\Psi_1| + |\Psi_1^0><\Psi_2|) +$$

$$+\frac{2\sqrt{2}}{5}(|\Psi_1><\Psi_3^0| + |\Psi_2><\Psi_3^0| + |\Psi_3^0><\Psi_1| + |\Psi_3^0><\Psi_2|) +$$

$$\left. +\frac{2}{\sqrt{5}}(|\Psi_1^0><\Psi_3^0| + |\Psi_3^0><\Psi_1^0| \right\}.$$

Then the initial values of the non-zero matrix elements of the density matrix $W(0)$ are

$$<\Psi_i|W(0)|\Psi_j>= \frac{1}{5} \qquad (i,j=1,2); <\Psi_i|W(0)|\Psi_i>=0 \qquad (i=3,4);$$

$$<\Psi_1^0|W(0)|\Psi_1^0>= \frac{1}{10}; \qquad <\Psi_3^0|W(0)|\Psi_3^0>= \frac{1}{2}; \qquad <\Psi_1^0|W(0)|\Psi_3^0>= \frac{1}{2\sqrt{5}}; \tag{7}$$

$$<\Psi_i|W(0)|\Psi_1^0>= \frac{\sqrt{2}}{2}; \qquad <\Psi_i|W(0)|\Psi_3^0>= \frac{\sqrt{2}}{10}.$$

The non-zero solutions of equations (6) with initial conditions (7) are

$$<\Psi_i|\dot{W}(t)|\Psi_j>= \frac{1}{5}\exp(-\frac{11}{5}kt), \quad (i,j=1,2),$$

$$<\Psi_1^0|\dot{W}(t)|\Psi_1^0>= \frac{1}{10}\exp(-\frac{8}{5}kt);$$

$$<\Psi_3^0|\dot{W}(t)|\Psi_3^0>= \frac{1}{2}\exp(-\frac{8}{5}kt);$$

$$<\Psi_1^0|\dot{W}(t)|\Psi_3^0> \approx \frac{1}{4}\exp(-\frac{8}{5}kt);$$

$$<\Psi_1^0|\dot{W}(t)|\Psi_1^0>= \frac{1}{10}\exp(-\frac{8}{5}kt); \tag{8}$$

$$<\Psi_1^0|\dot{W}(t)|\Psi_i>= \frac{\sqrt{2}}{20}\left\{(5+\sqrt{5})\exp\left[-\frac{2}{5}(5-\sqrt{5})kt\right] + (5-\sqrt{5})\exp\left[-\frac{2}{5}(5+\sqrt{5})kt\right]\right\};$$

$$<\Psi_3^0|\dot{W}(t)|\Psi_i>= \frac{\sqrt{2}}{20}\left\{(5+\sqrt{5})\exp\left[-\frac{2}{5}(5-\sqrt{5})kt\right] - (5-\sqrt{5})\exp\left[-\frac{2}{5}(5+\sqrt{5})kt\right]\right\}$$

$$(i=1,2).$$

Using solutions (8) we can find the average for any dynamical value. In particular, for intensities of electromagnetic fields in the in the secular approximation in both modes one can obtain

$$I_1(t) = I_2(t) =< \dot{N}_1(t) >=< \dot{N}_2(t) >\approx \frac{\sqrt{2}}{25}\left\{(5+\sqrt{5})^2 \exp\left[-\frac{2}{5}(5-\sqrt{5})kt\right]\right. \qquad 9)$$

$$\left. -(5-\sqrt{5})^2 \exp\left[-\frac{2}{5}(5+\sqrt{5})kt\right]\sin(\sqrt{10}t) - \frac{4\sqrt{10}}{25}\exp(-\frac{8}{5}kt)\sin(2\sqrt{(10)t})\right\}.$$

It is interesting to compare the above results with the expressions for intensities of excited single three-level atom radiation which are given in the case $\alpha = 1$ by [7]

$$I_1^{(single)}(t) = I_2^{(single)}(t) = \frac{1}{2}\sqrt{2}\exp(-kt)\sin(2\sqrt{2}t). \qquad (10)$$

Comparing (9) and (10) for high-Q cavity and $kt << 1$ we have the relations for the maximum values of the radiation rates

$$\frac{(I_1)_{max}}{(2I_1^{(single)}(t))_{max}} \approx \frac{4}{\sqrt{5}}.$$

This implies the occurence the collective radiation of two atoms and the collective interaction between them.

Thus, we have presented a treatmeant of collective spontaneous emission from a system of two three-levell atoms with one-photon transitions interacting with two modes of quantum elecromagnetic field in a cavity with finite Q. The case of two initially excited atoms has been considered. The time evolution of radiation rates was obtained. The considered system was shown to have the collective properties. The analysys of time behaviour of mean photon numbers and atomic populations for considered models and for different values of coupling constants will be a subject of a subsequent paper.

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