Coherent many-photon dynamics of two-level atom in non-ideal cavity

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Abstract

Dynamics of detuned two-level Jaynes-Cummings model with multiphoton transitions in the finite Q-cavity was considered. The time evolution of the mean photon number and atomic populations was investigated for various values of the model parameters. The type of field statistics and photon bunching/antibunching effects were examined.

The Jaynes-Cummings model provides many interesting quantum features of photon-atom interactions [1] and was experimentally realized on Rydberg atoms in high-Q cavity. In its original form this model describes the interaction of an undamped two-level atom with non-decaying quantized field mode under the rotating-wave approximation. However, the model may be generalized or extended in several ways. For example, it is easily generalized to the case with multiphoton transition through the virtual levels. Another possibility is to take into account photon leakage. Dissipative dynamics of the Jaynes-Cummings model in the finite-Q cavity using the Markov master equation approach was studied by several authors for the case of one-photon and two-photon transitions in two-level and three-level atoms (see references in [2, 3]). It is of interest to generalize these results to the case of arbitrary many-photon transitions in a two-level atom taking into account the detuning between the atom and the field frequencies.

1 Hamiltonian and kinetic equations

Our system comprises one two-level atom of resonance frequency ω_0 interacting non-resonantly with a single mode of the quantized radiation field of frequency ω by means of m-photon transitions

$$H = H_A + H_F + H_{AF}, \tag{1}$$

where

$$H_A = \hbar\omega_0 R^z$$

is the free two-level atom Hamiltonian;

$$H_F = \hbar\omega a^+ a$$

is the free electromagnetic field Hamiltonian and

$$H_{AF} = \hbar g(a^m R^+ + (a^+)^m R^-)$$

is the interaction Hamiltonian. Here $a(a^+)$ are the cavity-field annihilation (creation) operators, $R^{\pm}$ are the operators describing the transitions from ground to excited level in two-level atom, R^z is the inversion operator and g is the coupling constant. We denote the field-atom frequency difference

$$\Delta = m\omega - \omega_0$$

and we refer to Δ as the detuning parameter.

The master equation for the density matrix ρ of the combined system of the atom and the field in the finite-Q cavity is given by

$$\frac{\partial \rho}{\partial t} = -i/\hbar\,[H,\rho] - k\left(a^+a\rho - 2\,a\rho\,a^+ + \rho\,a^+a\right), \tag{2}$$

where $2k$ is the rate of photons leakage from the cavity.

Using the representation

$$W = e^{\frac{i}{\hbar}Ht}\rho(t)e^{-\frac{i}{\hbar}Ht}$$

one can obtain from (1) the equation for the density matrix in the form

$$\frac{\partial W}{\partial t} = -k\,\left(a^+aW - 2\,aW\,a^+ + W\;a^+a\right), \tag{3}$$

To solve the equation (3) one can use the dressed-state representation, i.e. representation consisting of the complete set of eigenstates of H. For the case of initially excited two-level atom and lossless cavity which are given by

$$|\Psi_n^{\pm}\rangle = \sqrt{\frac{1\mp\delta(n)}{2}}|2,n\rangle \pm \sqrt{\frac{1\pm\delta(n)}{2}}|1,n+m\rangle, \tag{4}$$
$$E_n^{\pm} = \hbar\omega\left(n+\frac{m}{2}\right) \pm \frac{1}{2}\hbar\Omega(n),$$

where $\Omega(n) = \sqrt{\Delta^2 + 4g^2(n+m)!/n!}$, $\delta(n) = \Delta/\Omega(n)$. Here $|\alpha;n>$ are the eigenstates of the Hamiltonian $H_A + H_F$:

$$|\alpha;n> = |\alpha>_A\;|n>_F,$$

where $\alpha = 1,2$; $n = 0,1,2,\ldots$.

In the case of finite-Q cavity one can add to (4) the states taking into account the possibility of the decrease of the photon number in cavity

$$|\Psi^{leak}> = |1;l>, (l = 0,1,2...m-1)$$

and corresponding the energy $E = \hbar\omega l - \frac{1}{2}\hbar\omega_0$.

In the secular approximation ($k << g$) the equations for the diagonal elements of W are found to be

$$\begin{aligned}\langle\Psi_n^+|\dot W|\Psi_n^+\rangle &= -k[2n+m(1+\delta(n))]\langle\Psi_n^+|W|\Psi_n^+\rangle + \\ &+ 2k\left[(\gamma_n^+)^2\langle\Psi_{n+1}^+|W|\Psi_{n+1}^+\rangle + (\gamma_n^-)^2\langle\Psi_{n+1}^-|W|\Psi_{n+1}^-\rangle\right], \quad (5)\\ \langle\Psi_n^-|\dot W|\Psi_n^-\rangle &= -k[2n+m(1-\delta(n))]\langle\Psi_n^-|W|\Psi_n^-\rangle + \\ &+ 2k\left[(\Gamma_n^-)^2\langle\Psi_{n+1}^+|W|\Psi_{n+1}^+\rangle + (\Gamma_n^+)^2\langle\Psi_{n+1}^-|W|\Psi_{n+1}^-\rangle\right]. \quad (6)\end{aligned}$$

$$\langle\Psi_l^{leak}|\dot{W}|\Psi_l^{leak}\rangle = -2k(l\langle\Psi_l^{leak}|W|\Psi_l^{leak}\rangle + (l+1)\langle\Psi_{l+1}^{leak}|W|\Psi_{l+1}^{leak}\rangle), \tag{7}$$
$$l = 0, 1, 2 ... m-2.$$

$$\begin{aligned}\langle\Psi_{m-1}^{leak}|\dot{W}|\Psi_{m-1}^{leak}\rangle &= -k\left(2(m-1)\langle\Psi_{m-1}^{leak}|W|\Psi_{m-1}^{leak}\rangle - \right.\\ &- \left. m\left[(1+\delta(0)\langle\Psi_0^+|W|\Psi_0^+\rangle + (1-\delta(0))\langle\Psi_0^-|W|\Psi_0^-\rangle\right]\right), \end{aligned}\tag{8}$$

where

$$\gamma_n^+ = \frac{1}{2}(\sqrt{(n+1)(1-\delta(n))(1-\delta(n+1))} + \sqrt{(n+m+1)(1+\delta(n))(1+\delta(n+1))}),$$

$$\gamma_n^- = \frac{1}{2}(\sqrt{(n+1)(1-\delta(n))(1+\delta(n+1))} - \sqrt{(n+m+1)(1+\delta(n))(1-\delta(n+1))}),$$

$$\Gamma_n^- = \frac{1}{2}(\sqrt{(n+1)(1+\delta(n))(1-\delta(n+1))} - \sqrt{(n+m+1)(1-\delta(n))(1+\delta(n+1))}),$$

$$\Gamma_n^+ = \frac{1}{2}(\sqrt{(n+1)(1+\delta(n))(1+\delta(n+1))} + \sqrt{(n+m+1)(1-\delta(n))(1-\delta(n+1))}).$$

The non-zero off-diagonal elements W are given by

$$\langle\Psi_n^{\pm}|\dot{W}(t)|\Psi_n^{\mp}\rangle = -k(2n+m)\langle\Psi_n^{\pm}|W|\Psi_n^{\mp}\rangle. \tag{9}$$

The solutions of the equations (9) are

$$\langle\Psi_n^{\pm}|W(t)|\Psi_n^{\mp}\rangle = \langle\Psi_n^{\pm}|W(0)|\Psi_n^{\mp}\rangle \exp(-k(2n+m)t).$$

Solutions of the Eqs (5)-(8) for arbitrary initial states of the atom and field may be obtained only numerically.

2 Coherent-field initial state

In this paper we consider only the coherent initial states of the field. For this field and and for the excited atom for the density matrix we have

$$\rho(0) = \rho_A(0)\rho_F(0), \tag{10}$$

$$W_F(0) = \sum_{n=0}^{\infty} p_n|n><n|, \tag{11}$$

where $p_n = exp(-\bar{n})\frac{\bar{n}^n}{n!}$. In the dressed state representation the initial density matrix becomes

$$\begin{aligned}W(0) &= \frac{1-\delta(n)}{2}|\Psi_n^+\rangle\langle\Psi_n^+| + \frac{1+\delta(n)}{2}|\Psi_n^-\rangle\langle\Psi_n^-| + \\ &+ \frac{\sqrt{1-\delta^2(n)}}{2}(|\Psi_n^+\rangle\langle\Psi_n^-| + |\Psi_n^-\rangle\langle\Psi_n^+|). \end{aligned}\tag{12}$$

For mean photon number and atomic populations in considered case we have

$$
\begin{aligned}
< N(t) > &= \sum_{n=0}^{\infty} \Big[(n+0.5m(1+\delta(n)))\langle\Psi_n^+|W|\Psi_n^+\rangle + \\
&+ (n+0.5m(1-\delta(n)))\langle\Psi_n^-|W|\Psi_n^-\rangle - \\
&- m\sqrt{1-\delta^2(n)}cos(\Omega(n)t)\langle\Psi_n^+|W|\Psi_n^-\rangle\Big] + \sum_{l=0}^{m-1} l\langle\Psi_l^{leak}|W|\Psi_l^{leak}\rangle,
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
< R_1(t) > &= \sum_{n=0}^{\infty} \Big[0.5(1+\delta(n))\langle\Psi_n^+|W|\Psi_n^+\rangle + 0.5(1-\delta(n))\langle\Psi_n^-|W|\Psi_n^-\rangle - \\
&- \sqrt{1-\delta^2(n)cos(\Omega(n)t)}\langle\Psi_n^+|W|\Psi_n^-\rangle\Big] + \sum_{l=0}^{m-1} \langle\Psi_l^{leak}|W|\Psi_l^{leak}\rangle,
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
< R_2(t) > &= \sum_{n=0}^{\infty} \Big[0.5(1-\delta(n))\langle\Psi_n^+|W|\Psi_n^+\rangle + 0.5(1+\delta(n))\langle\Psi_n^-|W|\Psi_n^-\rangle + \\
&+ \sqrt{1-\delta^2(n)}cos(\Omega(n)t)\langle\Psi_n^+|W|\Psi_n^-\rangle\Big].
\end{aligned} \tag{15}
$$

The results of numerical calculations for the mean photon number and time behavior of atomic populations obtained from equations (22)-(24) are presented in the Figures (1)-(5).

For a single-photon transitions one can observe characteristic collapses and beginning of Rabi oscillations for the mean atomic level populations and mean photon numbers in the field mode. In the case of two-photon transition collapses and revivals are strongly periodical. For three- and four-photon processes there is no such a clear revival regime.

In the presence of a relatively small detuning ($\sim 10g$) for a single-photon transition collapse region is wider than that in the resonant case. For the two-photon processes small detuning results in destroying the collapses and revivals after several Rabi cycles. Evolution picture for the average observables does not show clear Rabi cycles.

Under the condition of large detuning ($\sim 100g$) the mean populations for the ground and excited states oscillate near 0 and 1 respectively for any m-photon transitions ($m = 1, 2, 3...$). Atomic and field frequencies are strongly different, so the probability of an atom to emit a photon is very small. The mean photon number is clearly seen to relax more rapidly.

A finite Q-cavity influences considerably on the mean photon number dynamics . Field dynamics is a superposition of a relaxation processes (as a result of the photon leakage) and Rabi cycles. Oscillations of atomic population reach a steady-state value, which goes to 1 for ground state and to 0 for excited state with the decrease of the cavity quality.

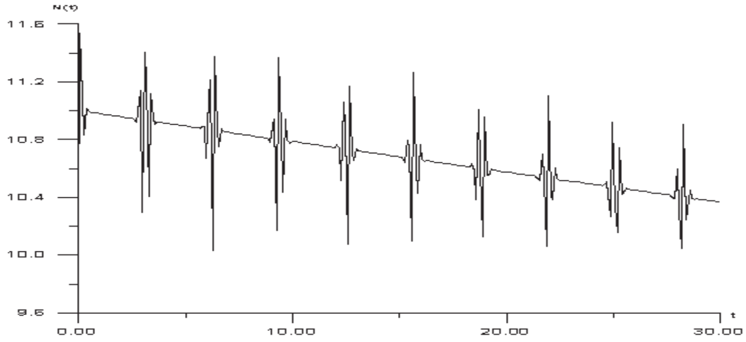

Figure 1: Mean photon number for $< N >= 10, m = 2, \delta = 0, g = 1$ and $k = 0.001$ (in g units)

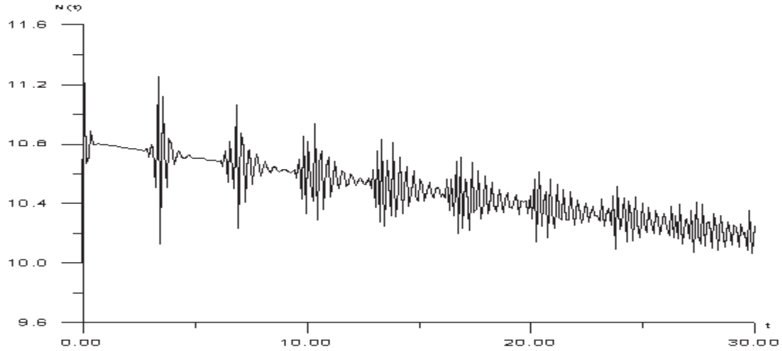

Figure 2: Mean photon number for $< N >= 10, m = 2, \delta = 10, g = 1$ and $k = 0.001$ (in g units)

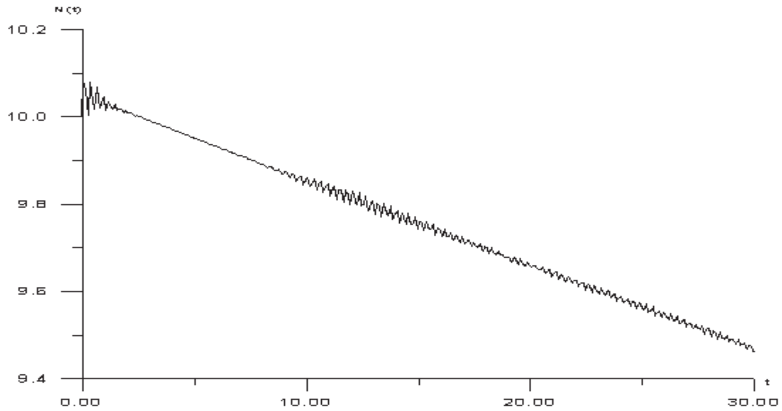

Figure 3: Mean photon number for $< N >= 10, m = 2, \delta = 100, g = 1$ and $k = 0.001$ (in g units)

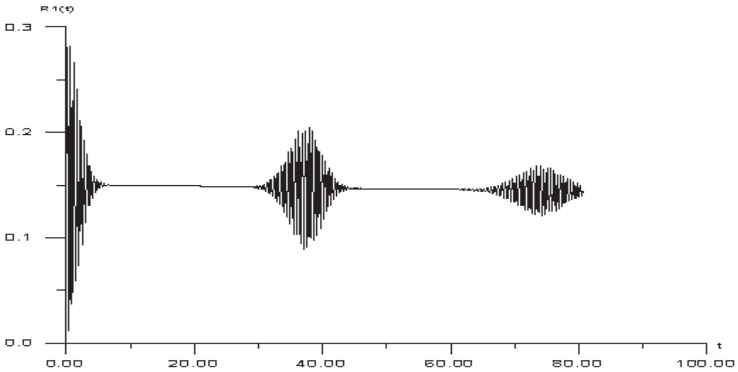

Figure 4: Mean atomic population for $< N >= 10, m = 1, \delta = 10, g = 1$ and $k = 0.001$ (in g units)

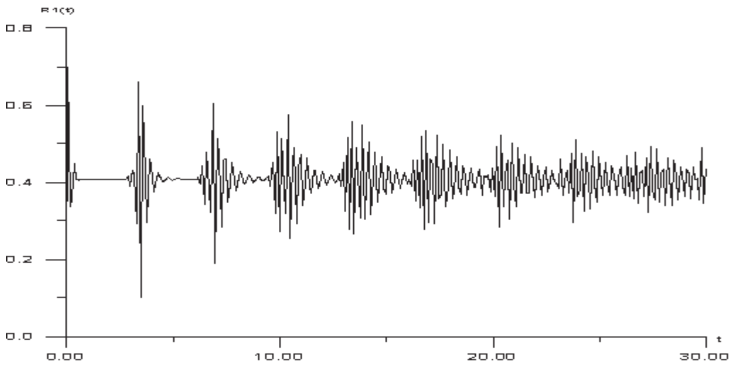

Figure 5: Mean atomic population for $< N >= 10, m = 2, \delta = 10, g = 1$ and $k = 0.001$ (in g units)

References

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