High energy behavior of the quark elastic form factors within the Wilson integrals approach: perturbative and nonperturbative contributions

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Abstract

The Wilson integrals (world-line) approach is applied systematically to study the high-energy behavior of the light quarks form factors in the Sudakov regime. The one-loop order results for the quark-photon (color singlet form factor) and quark-gluon (color non-singlet form factor) vertices are presented. The role of non-perturbative contributions is discussed.

In the sector of the strong interactions of the Standard Model, the elastic form factors of quarks are the most elementary entities exhibiting the double-logarithmic behavior at large transferred momentum. These quantities enter the quark-photon and quark-gluon vertices in the calculations of various QCD processes at the partonic level, and are under active investigation nowadays [1, 2]. Very recently, the investigations of the form factors of quarks has been connected with the progress in experimental study of the constituent quarks and search for the perturbative and nonperturbative effects in their structure from low to high energies [3].

Here we apply the powerful Wilson integrals techniques [4, 5, 6] to perform the Sudakov resummation of soft gluon radiative corrections to the quark-vector boson vertices with large transferred momentum. The world-line formulation of a quantum theory is actively developed not only due to the wide range of applications in the perturbative QFT, but also from the point of view of the various string theories (for a recent review, see [7] and Refs. therein). The attractive feature of this approach is that it does not refer directly to the standard perturbative technique allowing one to avoid the diagrammatic calculations which are very involved in non-Abelian gauge theories. Therefore, it is equally suitable for the perturbative as well as for nonperturbative calculations [8]. In this report, we demonstrate how the Wilson integrals formalism can be applied to arbitrary gauge fields in n-dimensional space-time, in a particular case of the on-shell quark form factor.

The quark form factor can be defined via the amplitude of the elastic quark scattering (in the Sudakov kinematics) in an external gauge field—the quark on a mass shell comes from infinity, emits the hard vector boson at the origin, and goes away to infinity:

$$u_i(p)\,[\mathcal{M}_\mu]^a_{ij}\,v_j(p') = F\left[(p-p')^2;\xi\right]\bar{u}_i(p)t^a_{ij}\gamma_\mu v_j(p') \quad , \tag{1}$$

where p, p' are the momenta of the in-coming and out-going quarks, and ξ is the covariant gauge-fixing parameter. In this formulae, we write down the vector boson and quark color

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[2] The work is partially supported by RFBR (Grant Nos. 03-02-17291, 02-02-16194), Russian Federation President's Grant 1450-2003-2, and INTAS (Grant No. 00-00-366).

indices $a, (i,j)$ for generality, assuming that the external vector boson is a colored gluon [9]. The case of the photon is trivially restored by means of the replacement of t^a_{ij} by the unity matrix. The Sudakov kinematics is determined by the small masses of the quarks and large squared transferred momentum:

$$m^2 = p_1^2 = p_2^2 \ \ , \ \ (p_1p_2) = m^2 \cosh\chi \ , \ (p_1p_2) >> m^2 \tag{2}$$

Following the Refs. [5, 6], we express the contribution of the resumed soft gluon radiative corrections to the form factor as the vacuum average of two path-ordered exponentials

$$G^a_{ij} \ F(Q^2;\xi) = \left\langle 0 \middle| \mathcal{T}\left\{ W_{ii'} G^a_{i'j'} W_{j'j} \right\} \middle| 0 \right\rangle , \tag{3}$$

where

$$W_{ii'} = \mathcal{P} \exp \left[ig \ t^\alpha v^1_\mu \int_{-\infty}^0 d\sigma \ A^\alpha_\mu(v^1\sigma) \right] \Bigg|_{ii'} \tag{4}$$

and

$$W_{j'j} = \mathcal{P} \exp \left[ig \ t^\beta v^2_\mu \int_0^\infty d\sigma \ A^\beta_\mu(v^2\sigma) \right] \Bigg|_{j'j} . \tag{5}$$

The quark trajectories can be parameterized as:

$$\textbf{In}\text{-Trajectory:} \ \ x_\mu = v^1_\mu \ \tau \ \ , \ \ \tau \in [-\infty, 0] \ \ , \ \ v^1_\mu = p^1_\mu/m \ , \tag{6}$$

$$\textbf{Out}\text{-Trajectory:} \ \ y_\nu = v^2_\nu \ \sigma \ \ , \ \ \sigma \in [0, +\infty] \ \ , \ \ v^2_\nu = p^2_\nu/m \ . \tag{7}$$

For calculations to the $O(g^2)$ accuracy, it is convenient to present the gauge field propagator $D_{\mu\nu}(z)$:

$$\left\langle 0 \middle| \mathcal{T} A^\alpha_\mu(x) A^\beta_\nu(y) \middle| 0 \right\rangle = \mathcal{D}^{\alpha\beta}_{\mu\nu}(x-y) = \delta^{\alpha\beta} D_{\mu\nu}(x-y) \tag{8}$$

in the form [10]:

$$D_{\mu\nu}(z) = g_{\mu\nu}\partial_\rho\partial^\rho D_1(z^2) - \partial_\mu\partial_\nu D_2(z^2) \ . \tag{9}$$

First, we present the vacuum averaged Wilson integral Eq. (3) in n-dimensional space-time, for an arbitrary gauge field which can be of any origin, for instance, nonperturbative. The leading order g^2 terms stem from the expressions:

$$W^{(1)}_{LO} = -\frac{g^2}{2} \ t^a_{ij} \ C_F \ v^1_\mu v^1_{\mu'} \ \int_0^\infty d\sigma \int_0^\infty d\sigma' \ \theta(\sigma-\sigma') \ D_{\mu\mu'}\left[v^1(\sigma-\sigma')\right] \ , \tag{10}$$

$$W^{(2)}_{LO} = -\frac{g^2}{2} \ t^a_{ij} \ C_F \ v^2_\mu v^2_{\mu'} \ \int_{-\infty}^0 d\sigma \int_{-\infty}^0 d\sigma' \ \theta(\sigma-\sigma') \ D_{\mu\mu'}\left[v^2(\sigma-\sigma')\right] \ , \tag{11}$$

and

$$W^{(12)}_{LO} = -\frac{g^2}{2} \ G_F \ t^a_{ij} \ v^1_\mu v^2_\nu \ \int_0^\infty d\tau \int_0^\infty d\sigma \ D_{\mu\nu}(v^1\tau + v^2\sigma) \ . \tag{12}$$

The general expressions in n dimensions and arbitrary covariant gauge, with the gauge field two-point correlator expressed as Eq. (9):

$$W^{(1)} = W^{(2)} = -t^a_{ij} \ \frac{g^2}{2} C_F \ \left[(n-2) D_1(-\vec{b}^2) + 2\vec{b}^2 D'_1(-\vec{b}^2) + D_2(-\vec{b}^2) \right] \ , \tag{13}$$

$$W^{(12)}(\chi) = t^a_{ij}\; g^2 G_F \left[\chi \coth\chi \left((n-2)D_1(-\vec{b}^2) + 2\vec{b}^2 D_1'(-\vec{b}^2)\right) + D_2(-\vec{b}^2)\right] , \qquad (14)$$

up to $O(g^4)$ order terms. Here $C_F = (N_c^2 - 1)/2N_c$ is the quadratic Casimir operator in the fundamental representation, and G_F is the color factor which is $G_F = C_F$ for the external photon field (color singlet form factor) and $G_F = C_F - C_A/2$, $C_A = N_c$, for the gluon probe (color non-singlet form factor). The space-like transversal vector $\vec{b}$ is introduced in order to regulate the UV divergence.

Performing the standard renormalization procedure within the $\overline{MS}$ scheme, described in detail in Refs. [12, 11, 5], one finds for the perturbative gluon field (with λ^2 being the IR cutoff)

$$W^{(1)}_{LO}(\alpha_s, \mu^2/\lambda^2; \xi) = W^{(2)}_{LO}(\alpha_s, \mu^2/\lambda^2; \xi) = t^a_{ij} \frac{\alpha_s}{4\pi} C_F \left(1 - \frac{\xi}{2}\right) \ln \frac{\mu^2}{\lambda^2} , \qquad (15)$$

and

$$W^{(12)}_{LO}(\alpha_s, \chi, \mu^2/\lambda^2; \xi) = -t^a_{ij} \frac{\alpha_s}{2\pi} G_F \left[\chi \coth\chi - \frac{\xi}{2}\right] \ln \frac{\mu^2}{\lambda^2} . \qquad (16)$$

Here the UV-normalization point is taken $\mu^2 = 4\vec{b}^{-2}$. The total one-loop contribution to the form factor is given by the sum

$$F^{NS}_{LO}(Q^2; \xi) = W_0 + 2W^{(1)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}\right) + W^{(12)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}, \chi\right) + O(\alpha_s^2) . \qquad (17)$$

The high-energy asymptotic behavior of the form factor is determined by the (gauge-dependent) cusp anomalous dimension which stems from the renormalization of the Wilson integral (3) [5]:

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} + \delta(\alpha_s, \xi) \xi \frac{\partial}{\partial \xi}\right) \ln F(\chi) = -\frac{1}{2} \Gamma^{NS}_{cusp}\left[\alpha_s(\mu^2); \chi\right] . \qquad (18)$$

This anomalous dimension reads in our case:

$$\Gamma_{cusp}\left[\alpha_s(\mu^2); \chi\right] = \frac{\alpha_s}{\pi}\left[G_F \chi \coth\chi + \frac{C_F - G_F}{2}\xi - C_F\right] + O(\alpha_s^2) . \qquad (19)$$

It is easy to see that for the color singlet case, the gauge dependent term is cancelled, as it should be for the gauge invariant quantity.

Therefore, the leading (double-logarithmic) behavior of the quark form factor is found from the corresponding evolution equation, and is given by

$$F\left[Q^2\right] = \exp\left(-G_F \int_{\lambda^2}^{Q^2} \frac{d\mu}{2\mu}\; \ln \frac{Q^2}{\mu}\; \frac{\alpha_s(\mu)}{\pi} + \text{NLO terms}\right) F\left[\alpha_s(\lambda^2)\right] =$$

$$= \exp\left[-\frac{2G_F}{\beta_0} \ln Q^2 \ln\ln Q^2 + O(\ln Q^2) + \text{NLO terms}\right] F\left[\alpha_s(\lambda^2)\right] . \qquad (20)$$

In any case, the dependence from the gauge-fixing parameter ξ drops out of the leading logarithmic expression for Γ_{cusp}, and yields no influence on the main asymptotics. Thus we find that the cusp anomalous dimension for the non-singlet quark form factor has a negative sign—in contrast to the singlet case—what leads, in the large-Q^2 regime, to the enhancement, rather than suppression, of the contribution due to the resumed soft gluon radiative corrections.

The formulas (13, 14) can be explored to compute the leading contributions to the Wilson vacuum average for an arbitrary $SU(N_c)$ gauge field in any covariant gauge, in n-dimensional space-time. For instance, the nonperturbative instanton-induced corrections, as well as the IR-renormalon effects are evaluated within this approach in Refs. [10]. Within the Instanton Liquid Model (ILM) of the QCD vacuum [13], the the leading nonperturbative (instanton-induced) corrections B_I result in a finite renormalization of the subleading perturbative term, while the asymptotic "double-logarithmic" behaviour is controlled by the perturbative cusp anomalous dimension:

$$F\left[Q^2\right] = \exp\left[-\frac{2G_F}{\beta_0}\ln Q^2 \ln\ln Q^2 - \ln Q^2 \ \left(B_I - \frac{2G_F}{\beta_0}\right) + O(\ln\ln Q^2)\right] F_0\left[q_0^2;\lambda^2\right] \ , \tag{21}$$

where

$$B_I = \frac{\Gamma(\beta_0/2)[\Gamma(\beta_0/2-2)]^3}{2\,[\Gamma(\beta_0/2-3/2)]^4}\pi^2\bar{n}\bar{\rho}^4 \ln\frac{2\pi\sigma}{\lambda^2} \ . \tag{22}$$

Numerically, the magnitude of the instanton effects is determined by the vacuum diluteness parameter $f = \pi^2\bar{n}\bar{\rho}^4$—the so-called "packing fraction" $f \approx 0.12$ for the following ILM average density and instanton size:

$$\bar{n} \approx 1fm^{-4}, \quad \bar{\rho} \approx 1/3fm \ . \tag{23}$$

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