Weak decays of the B_c meson

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Abstract

Semileptonic and nonleptonic decays of the B_c meson are studied in the framework of the relativistic quark model. The heavy quark expansion in inverse powers of the heavy (b, c) quark masses is used to simplify calculations while the light final quarks in the B and D mesons are treated relativistically. The decay form factors are explicitly expressed through the overlap integrals of the meson wave functions in the whole accessible kinematic range. The obtained results are compared with the predictions of other approaches.

1 Introduction

The investigation of weak decays of mesons composed of heavy quark and antiquark gives a very important insight in the heavy quark dynamics. The properties of the B_c meson are of special interest, since it is the only heavy meson consisting of two heavy quarks with different flavors. This difference of quark flavors forbids annihilation into gluons. As a result, the excited B_c meson states lying below the BD production threshold undergo pionic or radiative transitions to the pseudoscalar ground state which is considerably more stable than corresponding charmonium or bottomonium states and decays only weakly. The Collider Detector at Fermilab (CDF) Collaboration [1] reported the discovery of the B_c ground state in $p\bar{p}$ collisions. More experimental data are expected to come in near future from the Tevatron and Large Hadron Collider (LHC).

The characteristic feature of the B_c meson is that both quarks forming it are heavy and thus their weak decays give comparable contributions to the total decay rate. Therefore it is necessary to consider both the b quark decays $b \to c, u$ with the $\bar{c}$ quark being a spectator and $\bar{c}$ quark decays $\bar{c} \to \bar{s}, \bar{d}$ with b quark being a spectator. The former transitions lead to semileptonic decays to charmonium and D mesons while the latter lead to decays to B_s and B mesons. The estimates of the B_c decay rates indicate that the c quark decays give the dominant contribution ($\sim 70\%$) while the b quark decays and weak annihilation contribute about 20% and 10%, respectively (for a recent review see e.g. [2] and references therein). However, from the experimental point of view the B_c decays to charmonium are easier to identify. Indeed, CDF observed B_c mesons [1] analyzing their semileptonic decays $B_c \to J/\psi l\nu$.

The important difference between the B_c semileptonic decays induced by $b \to c, u$ and $c \to s, d$ transitions lies in the substantial difference of their kinematic ranges. In the case of B_c decays to charmonium and $D^{(*)}$ mesons the kinematic range (the square of momentum transfer to the lepton pair varies from 0 to $q^2_{\rm max} \approx 10$ GeV^2 for decays to J/ψ and $q^2_{\rm max} \approx 18$ GeV^2 for decays to D mesons) is considerably broader than for decays to $B_s^{(*)}$ and $B^{(*)}$ mesons ($q^2_{\rm max} \approx 0.8$ GeV^2 for decays to B_s and $q^2_{\rm max} \approx 1$ GeV^2 for decays to B mesons). As a result in the B_c meson rest frame the maximum recoil momentum of the final charmonium and D mesons is of the same order of magnitude as their masses, while the maximum recoil momentum of the $B_s^{(*)}$ and $B^{(*)}$ mesons is considerably smaller than the meson masses.

In this talk we consider semileptonic B_c decays in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. This model has been successfully applied to calculations of mass spectra, radiative and weak decays of heavy quarkonia and heavy-light mesons [3, 4, 5, 6, 7, 8]. In our recent paper [9] we applied this model to investigation of properties of the B_c meson and heavy quarkonia. The relativistic wave functions obtained there are used for the calculation of the transition matrix elements. The consistent theoretical description of B_c decays requires a reliable determination of q^2 dependence of the decay amplitudes in the whole kinematic range. In the most of previous calculations the corresponding decay form factors were determined only at one kinematic point either $q^2 = 0$ or $q^2 = q^2_{\rm max}$ and then extrapolated to the allowed kinematic range using some phenomenological ansatz (mainly (di)pole or Gaussian). Our aim is to explicitly determine the q^2 dependence of form factors in the whole kinematic range in order to avoid extrapolations thus reducing uncertainties.

2 Relativistic quark model

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [10] of the Schrödinger type [11]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are the center of mass energies on mass shell given by

$$E_1 = \frac{M^2-m_2^2+m_1^2}{2M}, \quad E_2 = \frac{M^2-m_1^2+m_2^2}{2M}. \tag{3}$$

Here $M = E_1 + E_2$ is the meson mass, $m_{1,2}$ are the quark masses, and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Constructing the quasipotential of the quark-antiquark interaction, we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by [3]

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k}),$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{6}$$

where κ is the Pauli interaction constant characterizing the long-range anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_V(r) = (1-\varepsilon)Ar + B, \qquad V_S(r) = \varepsilon Ar, \tag{7}$$

reproducing

$$V_{\rm conf}(r) = V_S(r) + V_V(r) = Ar + B, \tag{8}$$

where ε is the mixing coefficient.

The expression for the quasipotential of the heavy quarkonia, expanded in v^2/c^2 without and with retardation corrections to the confining potential, can be found in Refs. [3] and [9, 4], respectively. The structure of the spin-dependent interaction is in agreement with the parameterization of Eichten and Feinberg [12]. The quasipotential for the interaction of a heavy quark with a light antiquark without employing the expansion in inverse powers of the light quark mass is given in Ref. [5]. All the parameters of our model like quark masses, parameters of the linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ are fixed from the analysis of heavy quarkonium masses [3] and radiative decays [6]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.16$ GeV have usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion for the semileptonic $B \to D$ decays [7] and charmonium radiative decays [6]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [3]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa = -1$. It has been known for a long time that the correct reproduction of the spin-dependent quark-antiquark interaction requires either assuming the scalar confinement or equivalently introducing the Pauli interaction with $\kappa = -1$ [13, 3, 4] in the vector confinement.

3 Matrix elements of the electroweak current

In order to calculate the exclusive semileptonic decay rate of the B_c meson, it is necessary to determine the corresponding matrix element of the weak current between meson states.

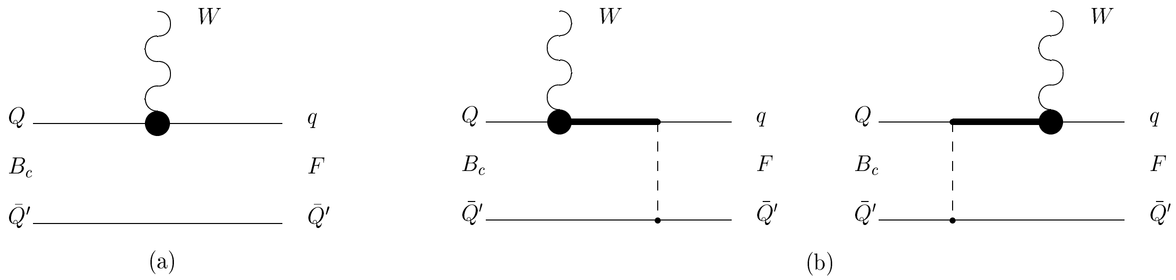

Figure 1: (a) Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (9). (b) Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}$ in (5). Bold lines denote the negative-energy part of the quark propagator.

In the quasipotential approach, the matrix element of the weak current $J^W_\mu = \bar{q}\gamma_\mu(1-\gamma_5)Q$, associated with $b \to q$ ($Q = b$ and $q = c, u$) or $c \to q$ ($Q = c$ and $q = s, d$) transitions, between a B_c meson with mass M_{B_c} and momentum p_{B_c} and a final meson F ($F = \psi, D, B_s, B$) with mass M_F and momentum p_F takes the form [14]

$$\langle F(p_F)|J^W_\mu|B_c(p_{B_c})\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}_{F\,\mathbf{p}_F}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_c\,\mathbf{p}_{B_c}}(\mathbf{q}), \tag{9}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{M\,\mathbf{p}_M}$ are the meson ($M = B_c, F$) wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_M$.

The contributions to Γ come from Figs. 1a, 1b. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the quark-antiquark interaction. In the leading order of the v^2/c^2 expansion for heavy quarkonia and in the heavy quark limit $m_{b,c} \to \infty$ for heavy-light mesons only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes already at the subleading order. The vertex functions look like

$$\Gamma^{(1)}_\mu(\mathbf{p},\mathbf{q}) = \bar{u}_q(p_q)\gamma_\mu(1-\gamma^5)u_Q(q_Q)(2\pi)^3\delta(\mathbf{p}_{Q'}-\mathbf{q}_{Q'}), \tag{10}$$

and

$$\begin{aligned}\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q}) \;=\; & \bar{u}_q(p_q)\bar{u}_{Q'}(p_{Q'})\Big\{\gamma_{1\mu}(1-\gamma_1^5)\frac{\Lambda_Q^{(-)}(k)}{\epsilon_Q(k)+\epsilon_Q(p_q)}\gamma_1^0\mathcal{V}(\mathbf{p}_{Q'}-\mathbf{q}_{Q'}) \\ & +\mathcal{V}(\mathbf{p}_{Q'}-\mathbf{q}_{Q'})\frac{\Lambda_q^{(-)}(k')}{\epsilon_q(k')+\epsilon_q(q_Q)}\gamma_1^0\gamma_{1\mu}(1-\gamma_1^5)\Big\}u_Q(q_Q)u_{Q'}(q_{Q'}),\end{aligned} \tag{11}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1a and 1b, $\mathbf{k} = \mathbf{p}_q - \boldsymbol{\Delta}$; $\mathbf{k}' = \mathbf{q}_Q + \boldsymbol{\Delta}$; $\boldsymbol{\Delta} = \mathbf{p}_F - \mathbf{p}_{B_c}$;

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

Here [14]

$$p_{q,Q'} = \epsilon_{q,Q'}(p)\frac{p_F}{M_F} \pm \sum_{i=1}^{3} n^{(i)}(p_F)p^i,$$
$$q_{Q,Q'} = \epsilon_{Q,Q'}(q)\frac{p_{B_c}}{M_{B_c}} \pm \sum_{i=1}^{3} n^{(i)}(p_{B_c})q^i,$$

and $n^{(i)}$ are three four-vectors given by

$$n^{(i)\mu}(p) = \left\{\frac{p^i}{M},\ \delta_{ij} + \frac{p^i p^j}{M(E+M)}\right\}, \quad E = \sqrt{\mathbf{p}^2 + M^2}.$$

The general structure of the current matrix element (9) is rather complicated, because it is necessary to integrate both with respect to d^3p and d^3q. The δ-function in the expression (10) for the vertex function $\Gamma^{(1)}$ permits to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has the usual structure of an overlap integral of meson wave functions and can be calculated exactly (without employing any expansion) in the whole kinematic range, if the wave functions of the initial and final meson are known. The situation with the contribution $\Gamma^{(2)}$ is different. Here, instead of a δ-function, we have a complicated structure, containing the potential of the $q\bar{q}$-interaction in meson. Thus in the general case we cannot get rid of one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (9). Therefore, it is necessary to use some additional considerations in order to simplify calculations. The main idea is to expand the vertex function $\Gamma^{(2)}$, given by (11), in such a way that it will be possible to use the quasipotential equation (1) in order to perform one of the integrations in the current matrix element (9).

$B_c \to \psi, \eta_c$ transitions. The natural expansion parameters for B_c decays to charmonium are the active heavy b and c quark masses as well as the spectator c quark mass. We carry such an expansion up to the second order in the ratios of the relative quark momentum $\mathbf{p}$ and binding energy to the heavy quark masses $m_{b,c}$. It is important to take into account the fact that in the case of weak B_c decays caused by $b \to c, u$ quark transition the kinematically allowed range is large ($|\mathbf{\Delta}_{\max}| = (M_{B_c}^2 - M_F^2)/(2M_{B_c}) \sim 2.4$ GeV for decays to charmonium and ~ 2.8 GeV for decays to D mesons). This means that the recoil momentum $\mathbf{\Delta}$ of a final meson is large in comparison to the relative momentum $\mathbf{p}$ of quarks inside a meson (~ 0.5 GeV), being of the same order as the heavy quark mass almost in the whole kinematic range. Thus we do not use expansions in powers of $|\mathbf{\Delta}|/m_{b,c}$ or $|\mathbf{\Delta}|/M_F$, but approximate in the expression (11) for $\Gamma^{(2)}$ the heavy quark energies $\epsilon_{b,c}(p+\Delta) \equiv \sqrt{m_{b,c}^2 + (\mathbf{p}+\mathbf{\Delta})^2}$ by $\epsilon_{b,c}(\Delta) \equiv \sqrt{m_{b,c}^2 + \mathbf{\Delta}^2}$, which become independent of the quark relative momentum $\mathbf{p}$. Making these replacements and expansions we see that it is possible to integrate the current matrix element (9) either with respect to d^3p or d^3q using the quasipotential equation (1). Performing integrations and taking the sum of the contributions of $\Gamma^{(1)}$ and $\Gamma^{(2)}$ we get the expression for the current matrix element, which contains ordinary overlap integrals of meson wave functions and is valid in the whole kinematic range. Thus this matrix element can be easily calculated using numerical wave functions found in our meson mass spectrum analysis [9, 4].

***$B_c \to D^{(*)}$* transitions.** In this case the heavy b quark undergoes the weak transition to the light u quark. The constituent u quark mass is of the same order of magnitude as the relative momentum and binding energy, thus we cannot apply the expansion in inverse powers of its mass. Nevertheless, taking into account the fact that the recoil momentum of the final meson in this decay is large almost in the whole kinematic range (as it was discussed above), we can neglect the relative momentum $\mathbf{p}$ of quarks inside a meson with respect to the large recoil momentum $\mathbf{\Delta}$. Thus in the region of large recoil ($|\mathbf{\Delta}| \gg |\mathbf{p}|$) we can use the same expressions of the $\Gamma^{(2)}$ contribution to the current matrix element both for the $B_c \to D^{(*)}$ and $B_c \to \psi, \eta_c$ transitions. Moreover, the smallness of the $\Gamma^{(2)}$ contribution, which is proportional to the small binding energy, and its weak dependence on momentum transfer allows one to extrapolate these formulas to the whole kinematic range. As numerical estimates show, such extrapolation introduces only small uncertainties.

***$B_c \to B_s(B)^{(*)}$* transitions.** The heavy c quark undergoes the weak transition to the light s or d quark. The constituent s, d quark masses are of the same order of magnitude as the relative momentum and binding energy, thus we cannot apply the expansion in inverse powers of their masses. The heavy quark expansion in $1/m_{c,b}$ significantly simplifies the structure of the $\Gamma^{(2)}$ contribution to the decay matrix element, but the momentum $\mathbf{p}$ dependence of the light quark energies $\epsilon_q(p)$ still prevents to perform one of the integrations. It is important to note that the kinematically allowed range for B_c decays to B_s and B meson is not large ($|\mathbf{\Delta}_{\max}| = (M_{B_c}^2 - M_F^2)/(2M_{B_c}) \sim 0.8$ GeV for decays to B_s and ~ 0.9 GeV for decays to B mesons). This means that the recoil momentum $\mathbf{\Delta}$ of a final meson is of the same order as the relative momentum $\mathbf{p}$ of quarks inside a heavy-light meson (~ 0.5 GeV) in the whole kinematic range. Taking also into account that the final B_s and B mesons are weakly bound [5], we can replace the light quark energies by the center of mass energies on mass shell $\epsilon_q(p) \to E_q = (M_F^2 - m_b^2 + m_q^2)/(2M_F)$. We used such a substitution in our analysis of heavy-light meson mass spectra [5] which allowed us to treat the light quark relativistically without an unjustified expansion in inverse powers of its mass. Making these replacements and expansions we see that it is possible to integrate the current matrix element (9) either with respect to d^3p or d^3q using the quasipotential equation (1). Performing integrations and taking the sum of the contributions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ we get the expression for the current matrix element, which contains ordinary overlap integrals of meson wave functions and is valid in the whole kinematic range.

4 B_c decay form factors and weak decay rates

The matrix elements of the weak current J^W for B_c decays to pseudoscalar mesons P can be parametrized by two invariant form factors:

$$\langle P(p_F)|\bar{q}\gamma^\mu b|B_c(p_{B_c})\rangle = f_+(q^2)\left[p_{B_c}^\mu + p_F^\mu - \frac{M_{B_c}^2 - M_P^2}{q^2}\, q^\mu\right] + f_0(q^2)\frac{M_{B_c}^2 - M_P^2}{q^2}\, q^\mu, \quad (12)$$

where $q = p_{B_c} - p_F$; M_{B_c} is the B_c meson mass and M_P is the pseudoscalar meson mass.

The corresponding matrix elements for B_c decays to vector mesons V are parametrized

Table 1: Form factors of weak B_c decays.

Transition	$f_+(q^2)$	$f_0(q^2)$	$V(q^2)$	$A_1(q^2)$	$A_2(q^2)$	$A_0(q^2)$
$B_c \to \eta_c, J/\psi$						
$q^2 = q^2_{\max}$	1.07	0.92	1.34	0.88	1.33	1.06
$q^2 = 0$	0.47	0.47	0.49	0.50	0.73	0.40
$B_c \to \eta_c', \psi'$						
$q^2 = q^2_{\max}$	0.08	0.05	-0.16	0.03	0.10	0.08
$q^2 = 0$	0.27	0.27	0.24	0.18	0.14	0.23
$B_c \to D, D^*$						
$q^2 = q^2_{\max}$	1.20	0.64	2.60	0.62	1.78	0.97
$q^2 = 0$	0.14	0.14	0.18	0.17	0.19	0.14
$B_c \to B_s(B_s^*)$						
$q^2 = q^2_{\max}$	0.99	0.86	6.25	0.76	2.62	0.91
$q^2 = 0$	0.50	0.50	3.44	0.49	2.19	0.35
$B_c \to B(B^*)$						
$q^2 = q^2_{\max}$	0.96	0.80	8.91	0.72	2.83	1.06
$q^2 = 0$	0.39	0.39	3.94	0.42	2.89	0.20

by four form factors

$$\langle V(p_F)|\bar{q}\gamma^\mu b|B(p_{B_c})\rangle = \frac{2iV(q^2)}{M_{B_c}+M_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B_c\rho}p_{F\sigma}, \tag{13}$$

$$\begin{aligned}\langle V(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_{B_c})\rangle &= 2M_V A_0(q^2)\frac{\epsilon^*\cdot q}{q^2}\, q^\mu + (M_{B_c}+M_V)A_1(q^2)\left(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}\, q^\mu\right)\\ &\quad - A_2(q^2)\frac{\epsilon^*\cdot q}{M_{B_c}+M_V}\left[p^\mu_{B_c} + p^\mu_F - \frac{M^2_{B_c}-M^2_V}{q^2}\, q^\mu\right],\end{aligned} \tag{14}$$

where M_V and ϵ_μ are the mass and polarization vector of the final vector meson. The following relations hold for the form factors at the maximum recoil point of the final meson ($q^2 = 0$)

$$f_+(0) = f_0(0),$$

$$A_0(0) = \frac{M_{B_c}+M_V}{2M_V}A_1(0) - \frac{M_{B_c}-M_V}{2M_V}A_2(0).$$

In the limit of vanishing lepton mass, the form factors f_0 and A_0 do not contribute to the semileptonic decay rates. However, they contribute to nonleptonic decay rates in the factorization approximation.

Now we apply the method for the calculation of the decay matrix elements described in the previous section. The explicit expressions for the form factors can be found in Refs. [17, 18]. Their calculated values are given in Table 1.

The differential semileptonic decay rates can be expressed in terms of the form factors as follows.

Table 2: Semileptonic decay rates Γ (in 10^{-15} GeV) of B_c decays.

Decay	our	[19]	[20]	[21]	[22]	[23]	[24]	[25]	[26]	[27]
$B_c \to \eta_c e\nu$	5.9	14	11	11.1	14.2	2.1(6.9)	8.6	6.8	4.3	8.31
$B_c \to \eta_c' e\nu$	0.46		0.60		0.73	0.3				0.605
$B_c \to J/\psi e\nu$	17.7	33	28	30.2	34.4	21.6(48.3)	17.5	19.4	16.8	20.3
$B_c \to \psi' e\nu$	0.44		1.94		1.45	1.7				0.186
$B_c \to De\nu$	0.019	0.26	0.059	0.049	0.094	0.005(0.03)			0.001	0.0853
$B_c \to D^* e\nu$	0.11	0.49	0.27	0.192	0.269	0.12(0.5)			0.06	0.204
$B_c \to B_s e\nu$	12	29	59	14.3	26.6	11.1(12.9)	15	12.3	11.75	26.8
$B_c \to B_s^* e\nu$	25	37	65	50.4	44.0	33.5(37.0)	34	19.0	32.56	34.6
$B_c \to Be\nu$	0.6	2.1	4.9	1.14	2.30	0.9(1.0)			0.59	1.90
$B_c \to B^* e\nu$	1.7	2.3	8.5	3.53	3.32	2.8(3.2)			2.44	2.34

(a) $B_c \to Pe\nu$ decays

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B_c \to Pe\nu) = \frac{G_F^2 \Delta^3 |V_{qQ}|^2}{24\pi^3} |f_+(q^2)|^2. \tag{15}$$

(b) $B_c \to Ve\nu$ decays

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B_c \to Ve\nu) = \frac{G_F^2 \Delta |V_{qQ}|^2}{96\pi^3} \frac{q^2}{M_{B_c}^2} \left(|H_+(q^2)|^2 + |H_-(q^2)|^2 + |H_0(q^2)|^2\right), \tag{16}$$

where G_F is the Fermi constant, V_{qQ} is the CKM matrix element,

$$\Delta \equiv |\mathbf{\Delta}| = \sqrt{\frac{(M_{B_c}^2 + M_{P,V}^2 - q^2)^2}{4M_{B_c}^2} - M_{P,V}^2}.$$

The helicity amplitudes are given by

$$H_\pm(q^2) = \frac{2M_{B_c}\Delta}{M_{B_c} + M_V} \left[V(q^2) \mp \frac{(M_{B_c} + M_V)^2}{2M_{B_c}\Delta} A_1(q^2) \right], \tag{17}$$

$$H_0(q^2) = \frac{1}{2M_V\sqrt{q^2}} \left[(M_{B_c} + M_V)(M_{B_c}^2 - M_V^2 - q^2) A_1(q^2) - \frac{4M_{B_c}^2 \Delta^2}{M_{B_c} + M_V} A_2(q^2) \right]. \tag{18}$$

The results of our calculations of the semileptonic and nonleptonic (in the factorization approximation) decay rates of B_c meson are given in Tables 2, 3 in comparison with predictions of other approaches based on quark models [19, 21, 22, 24, 25, 27], QCD sum rules [20] and on the application of heavy quark symmetry relations [23, 26] to the quark model. Our predictions for the CKM favored semileptonic B_c decays to charmonium ground states are almost 2 times smaller than those of QCD sum rules [20] and quark models [19, 21, 22], but agree with quark model results [24, 25, 26, 27]. Note that the ratios of the $B_c \to J/\psi e\nu$ to $B_c \to \eta_c e\nu$ decay rates have close values in all approaches except [23]. In the case of semileptonic decays to radially excited charmonium states our prediction for the decay to the pseudoscalar η_c' state is consistent with others, while the

Table 3: Nonleptonic B_c decay rates Γ (in 10^{-15} GeV).

Decay	our	[20]	[21]	[22]	[24]	[23]	[27]
$B_c^+ \to \eta_c \pi^+$	$0.93a_1^2$	$1.8a_1^2$	$1.59a_1^2$	$2.07a_1^2$	$1.47a_1^2$	$0.28a_1^2$	$1.49a_1^2$
$B_c^+ \to \eta_c \rho^+$	$2.3a_1^2$	$4.5a_1^2$	$3.74a_1^2$	$5.48a_1^2$	$3.35a_1^2$	$0.75a_1^2$	$3.93a_1^2$
$B_c^+ \to J/\psi \pi^+$	$0.67a_1^2$	$1.43a_1^2$	$1.22a_1^2$	$1.97a_1^2$	$0.82a_1^2$	$1.48a_1^2$	$1.01a_1^2$
$B_c^+ \to J/\psi \rho^+$	$1.8a_1^2$	$4.37a_1^2$	$3.48a_1^2$	$5.95a_1^2$	$2.32a_1^2$	$4.14a_1^2$	$3.25a_1^2$
$B_c^+ \to \eta_c K^+$	$0.073a_1^2$	$0.15a_1^2$	$0.119a_1^2$	$0.161a_1^2$	$0.15a_1^2$	$0.023a_1^2$	$0.115a_1^2$
$B_c^+ \to \eta_c K^{*+}$	$0.12a_1^2$	$0.22a_1^2$	$0.200a_1^2$	$0.286a_1^2$	$0.24a_1^2$	$0.041a_1^2$	$0.198a_1^2$
$B_c^+ \to J/\psi K^+$	$0.052a_1^2$	$0.12a_1^2$	$0.090\ a_1^2$	$0.152a_1^2$	$0.079a_1^2$	$0.076a_1^2$	$0.0764a_1^2$
$B_c^+ \to J/\psi K^{*+}$	$0.11a_1^2$	$0.25a_1^2$	$0.197a_1^2$	$0.324a_1^2$	$0.18a_1^2$	$0.23a_1^2$	$0.174a_1^2$
$B_c^+ \to \eta_c' \pi^+$	$0.19a_1^2$			$0.268a_1^2$		$0.074a_1^2$	$0.248a_1^2$
$B_c^+ \to \eta_c' \rho^+$	$0.40a_1^2$			$0.622a_1^2$		$0.16a_1^2$	$0.587a_1^2$
$B_c^+ \to \psi' \pi^+$	$0.12a_1^2$			$0.252a_1^2$		$0.22a_1^2$	$0.0708a_1^2$
$B_c^+ \to \psi' \rho^+$	$0.20a_1^2$			$0.710\ a_1^2$		$0.54a_1^2$	$0.183a_1^2$
$B_c^+ \to \eta_c' K^+$	$0.014a_1^2$			$0.020a_1^2$		$0.0055a_1^2$	$0.0184a_1^2$
$B_c^+ \to \eta_c' K^{*+}$	$0.021a_1^2$			$0.031a_1^2$		$0.008a_1^2$	$0.0283a_1^2$
$B_c^+ \to \psi' K^+$	$0.009a_1^2$			$0.018a_1^2$		$0.01a_1^2$	$0.00499a_1^2$
$B_c^+ \to \psi' K^{*+}$	$0.011a_1^2$			$0.038\ a_1^2$		$0.03a_1^2$	$0.00909a_1^2$
$B_c^+ \to B_s \pi^+$	$25a_1^2$	$167a_1^2$	$15.8a_1^2$	$58.4a_1^2$	$34.8a_1^2$	$30.6a_1^2$	$65.1a_1^2$
$B_c^+ \to B_s \rho^+$	$14a_1^2$	$72.5a_1^2$	$39.2a_1^2$	$44.8a_1^2$	$23.6a_1^2$	$13.6a_1^2$	$42.7a_1^2$
$B_c^+ \to B_s^* \pi^+$	$16a_1^2$	$66.3a_1^2$	$12.5a_1^2$	$51.6a_1^2$	$19.8a_1^2$	$35.6a_1^2$	$25.3a_1^2$
$B_c^+ \to B_s^* \rho^+$	$110a_1^2$	$204a_1^2$	$171a_1^2$	$150a_1^2$	$123a_1^2$	$110.1a_1^2$	$139.6a_1^2$
$B_c^+ \to B_s K^+$	$2.1a_1^2$	$10.7a_1^2$	$1.70a_1^2$	$4.20a_1^2$		$2.15a_1^2$	$4.69a_1^2$
$B_c^+ \to B_s K^{*+}$	$0.03a_1^2$		$1.06a_1^2$			$0.043a_1^2$	$0.296a_1^2$
$B_c^+ \to B_s^* K^+$	$1.1a_1^2$	$3.8a_1^2$	$1.34a_1^2$	$2.96a_1^2$		$1.6a_1^2$	$1.34a_1^2$
$B_c^+ \to B^0 \pi^+$	$1.0a_1^2$	$10.6a_1^2$	$1.03a_1^2$	$3.30a_1^2$	$1.50a_1^2$	$1.97a_1^2$	$3.64a_1^2$
$B_c^+ \to B^0 \rho^+$	$1.3a_1^2$	$9.7a_1^2$	$2.81a_1^2$	$5.97a_1^2$	$1.93a_1^2$	$1.54a_1^2$	$4.03a_1^2$
$B_c^+ \to B^{*0} \pi^+$	$0.26a_1^2$	$9.5a_1^2$	$0.77a_1^2$	$2.90a_1^2$	$0.78a_1^2$	$2.4a_1^2$	$1.22a_1^2$
$B_c^+ \to B^{*0} \rho^+$	$6.8a_1^2$	$26.1a_1^2$	$9.01a_1^2$	$11.9a_1^2$	$6.78a_1^2$	$8.6a_1^2$	$8.16a_1^2$
$B_c^+ \to B^0 K^+$	$0.09a_1^2$	$0.70a_1^2$	$0.105a_1^2$	$0.255a_1^2$		$0.14a_1^2$	$0.272a_1^2$
$B_c^+ \to B^0 K^{*+}$	$0.04a_1^2$	$0.15a_1^2$	$0.125a_1^2$	$0.180a_1^2$		$0.032a_1^2$	$0.0965a_1^2$
$B_c^+ \to B^{*0} K^+$	$0.04a_1^2$	$0.56a_1^2$	$0.064a_1^2$	$0.195a_1^2$		$0.12a_1^2$	$0.0742a_1^2$
$B_c^+ \to B^{*0} K^{*+}$	$0.33a_1^2$	$0.59a_1^2$	$0.665a_1^2$	$0.374a_1^2$		$0.34a_1^2$	$0.378a_1^2$
$B_c^+ \to B^+ \bar{K}^0$	$34a_2^2$	$286a_2^2$	$39.1a_2^2$	$96.5a_2^2$	$24.0a_2^2$		$103.4a_2^2$
$B_c^+ \to B^+ \bar{K}^{*0}$	$13a_2^2$	$64a_2^2$	$46.8a_2^2$	$68.2a_2^2$	$13.8a_2^2$		$36.6a_2^2$
$B_c^+ \to B^{*+} \bar{K}^0$	$15a_2^2$	$231a_2^2$	$24.0a_2^2$	$73.3a_2^2$	$8.9a_2^2$		$28.9a_2^2$
$B_c^+ \to B^{*+} \bar{K}^{*0}$	$120a_2^2$	$242a_2^2$	$247a_2^2$	$141a_2^2$	$82.3a_2^2$		$143.6a_2^2$
$B_c^+ \to B^+ \pi^0$	$0.5a_2^2$	$5.3a_2^2$	$0.51a_2^2$	$1.65a_2^2$	$1.03a_2^2$		
$B_c^+ \to B^+ \rho^0$	$0.7a_2^2$	$4.4a_2^2$	$1.40a_2^2$	$2.98a_2^2$	$1.28a_2^2$		
$B_c^+ \to B^{*+} \pi^0$	$0.13a_2^2$	$4.8a_2^2$	$0.38a_2^2$	$1.45a_2^2$	$0.53a_2^2$		
$B_c^+ \to B^{*+} \rho^0$	$3.4a_2^2$	$13.1a_2^2$	$4.50a_2^2$	$5.96a_2^2$	$4.56a_2^2$		

$a_{1,2}$ are the Wilson coefficients in the operator product expansion.

Table 4: Branching fractions (in %) of exclusive B_c decays calculated for the fixed values of the B_c lifetime $\tau_{B_c} = 0.46$ ps, $a_1 = 1.14$ for b-quark decays and $a_1 = 1.20$, $a_2 = -0.317$ for c-quark decays.

Decay	Br	Decay	Br	Decay	Br
b-quark decays					
$B_c \to \eta_c e\nu$	0.42	$B_c^+ \to \eta_c \pi^+$	0.085	$B_c^+ \to \eta' \pi^+$	0.017
$B_c \to \eta_c' e\nu$	0.032	$B_c^+ \to \eta_c \rho^+$	0.21	$B_c^+ \to \eta_c' \rho^+$	0.036
$B_c \to J/\psi e\nu$	1.23	$B_c^+ \to J/\psi \pi^+$	0.061	$B_c^+ \to \psi' \pi^+$	0.011
$B_c \to \psi' e\nu$	0.031	$B_c^+ \to J/\psi \rho^+$	0.16	$B_c^+ \to \psi' \rho^+$	0.018
$B_c \to D e\nu$	0.013	$B_c^+ \to \eta_c K^+$	0.007	$B_c^+ \to \eta_c' K^+$	0.001
$B_c \to D^* e\nu$	0.037	$B_c^+ \to \eta_c K^{*+}$	0.011	$B_c^+ \to \eta_c' K^{*+}$	0.002
		$B_c^+ \to J/\psi K^+$	0.005	$B_c^+ \to \psi' K^+$	0.001
		$B_c^+ \to J/\psi K^{*+}$	0.010	$B_c^+ \to \psi' K^{*+}$	0.001
c-quark decays					
$B_c \to B_s e\nu$	0.84	$B_c^+ \to B_s K^{*+}$	0.003	$B_c^+ \to B^{*0} K^{*+}$	0.033
$B_c \to B_s^* e\nu$	1.75	$B_c^+ \to B_s^* K^+$	0.11	$B_c^+ \to B^+ \bar{K}^0$	0.24
$B_c \to B e\nu$	0.042	$B_c^+ \to B^0 \pi^+$	0.10	$B_c^+ \to B^+ \bar{K}^{*0}$	0.09
$B_c \to B^* e\nu$	0.12	$B_c^+ \to B^0 \rho^+$	0.13	$B_c^+ \to B^{*+} \bar{K}^0$	0.11
$B_c^+ \to B_s \pi^+$	2.52	$B_c^+ \to B^{*0} \pi^+$	0.026	$B_c^+ \to B^{*+} \bar{K}^{*0}$	0.84
$B_c^+ \to B_s \rho^+$	1.41	$B_c^+ \to B^{*0} \rho^+$	0.68	$B_c^+ \to B^+ \pi^0$	0.004
$B_c^+ \to B_s^* \pi^+$	1.61	$B_c^+ \to B^0 K^+$	0.009	$B_c^+ \to B^+ \rho^0$	0.005
$B_c^+ \to B_s^* \rho^+$	11.1	$B_c^+ \to B^0 K^{*+}$	0.004	$B_c^+ \to B^{*+} \pi^0$	0.001
$B_c^+ \to B_s K^+$	0.21	$B_c^+ \to B^{*0} K^+$	0.004	$B_c^+ \to B^{*+} \rho^0$	0.024

one for the decay to ψ' is considerably smaller (with the exception of Ref. [27]). For the CKM suppressed semileptonic decays of B_c to D mesons our results are in agreement with those of Ref. [23]. Our predictions for the CKM favored semileptonic B_c decays to $B_s^{(*)}$ are smaller than those of QCD sum rules [20] and quark models [19, 21, 22, 27], but agree with quark model results [23, 24, 25, 26]. For the CKM suppressed semileptonic decays of B_c to $B^{(*)}$ mesons our results are in agreement with the ones based on the application of heavy quark symmetry relations [23, 26] to the quark model.

As one sees from Tables 2, 3 the theoretical predictions for B_c weak decay rates differ substantially. Thus experimental measurements of corresponding decay rates can discriminate between various approaches.

5 Conclusions

In this talk we considered weak semileptonic and nonleptonic B_c decays in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. The weak decay form factors were calculated explicitly in the whole kinematic range using the heavy quark expansion for the initial active heavy quark $Q = b, c$ and spectator antiquark $\bar{Q}' = \bar{c}, \bar{b}$. The final light quark was treated completely relativistically without applying the unjustified expansion in inverse powers of its mass. The leading order contribution of the heavy quark expansion was treated exactly, while in calculating the subleading order contribution additional simplifying replacements were used. It was shown that such substitutions introduce only minor errors which are of the same order as the higher order terms in the heavy quark expansion. Thus the decay form factors were evaluated up to the subleading order of the heavy quark expansion. The overall subleading contributions are small and weakly depend on the momentum transfer q^2.

We calculated semileptonic and nonleptonic (in the factorization approximation) B_c decay rates. Our predictions for the branching fractions are summarized in Table 4, where we use the central experimental value of the B_c meson lifetime [16]. From this table we see that weak B_c semileptonic decays to the ground and first radially excited states of charmonium and to D mesons, associated with $\bar{b} \to \bar{c}, \bar{u}$ quark transition, yield $\sim 1.7\%$ and corresponding energetic nonleptonic decays (to charmonium and $K^{(*)}$ or π, ρ mesons) contribute $\sim 0.6\%$. The semileptonic decays to B_s and B mesons, associated with $c \to s, d$ quark transition, give in total $\sim 2.0\%$ of the B_c decay rate, while the energetic nonleptonic decays provide the dominant contribution $\sim 19.3\%$. All these decays (to B_s, B, charmonium and D mesons) add up to $\sim 23.6\%$ of the B_c total decay rate.

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