# ALPHA-PARTICLE STATES AND $\alpha$-CONDENSATE IN NUCLEAR MATTER AT THE NORMAL DENSITY

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**Abstract**

A theory of alpha-particle states of nuclei based on the microscopic Hamiltonian of the Elliott SU(3) model is constructed. The Hamiltonian well describes a rather wide set of low-lying states. We show that it is also true for a lot of highly excited states. The problem of $\alpha$-condensed nuclear matter of normal density is discussed in the light of the theory.

## 1 Introduction.

At present the idea of the existence of the alpha-particle (AP) condensate in $N = Z$ nuclear matter grows in popularity [1]. The basic line of theoretical investigations is the search for the $\alpha$-condensed states near $k\alpha$-particle threshold ($k = Z/2$) at a very low density. A few candidates for such states in each light $N = Z$-even nucleus are predicted in this way. At the same time the main test of the $\alpha$-condensed nuclear states is the strong coupling of such states to the $\alpha$+nucleus channel (open or closed) populated in the elastic $\alpha$-scattering from nuclei, the radiative capture, $\alpha$-transfer reactions etc. Thus it is reasonable to treat the problem of the $\alpha$-condensate in connection with the alpha-particle (AP) states. A wide experimental evidence concerning these states has been obtained up to now [2, 3] for even-even (2s-1d)-shell nuclei both with $N = Z$ and $N = Z + 2$ nucleon numbers. Except for the near-magic regions of the nuclei, more than one hundred levels in the spectrum of each of the studied nuclei ($^{22}Ne$, $^{32}S$, $^{36}Ar$, etc.) was observed.

In this contribution a microscopic theory of $\alpha$-particle states of nuclei is presented. The theory allows one to make certain conclusions concerning the $\alpha$-condensate in the nuclear matter.

## 2 Alpha-particle spectra in the extended Elliott model

First of all let us discuss some pronounced features of the $\alpha$-particle spectra of $16 \leq A \leq 44$ nuclei known from spectroscopic tables, refs. [2, 3] and some other papers. The spectra are characterized by the following peculiarities:

1. There is a large number of AP states for a nonmagic nucleus. To explain the level density using a two-body model (for $^{32}S$ as an example) one needs a nuclear potential with the normal depth and the width of about 15 fm. So the resonances being strongly coupled to AP channel are different from the two-body type ones.

2. Although the level density for any angular momentum L is high, for the nuclei from the middle of (2s-1d)-shell it is about one order of magnitude less than total. In one and

the same energy region only a small part of nuclear levels turns out to be visible and the most part of them is not populated. The selectivity of $\alpha$-particle scattering (and other mentioned reactions) is strong.
3. The A-dependence of the level density is strong and irregular. Spectra of nuclei irrelevant to magic or magic core+alpha types are much more dense than those falling in this category.

As it is seen from the $\alpha$-transfer reaction data the spectroscopic factors of alpha-particles are large for the ground and some low-lying states of $N = Z$-even nuclei. So such states can also be considered as the alpha-particle ones.

The above mentioned properties place rigid constraints on a theoretical model. The desired model should contain a description of two-body asymptotics, the spectra of the resonance states, and the coupling between them. For the sake of brevity we restrict ourselves to a short review of the methods involved. Let us consider a (2s-1d)-shell N=Z-even nucleus. An AP asymptotic state is the antisymmetrized product of the cluster $\Psi_\alpha$, core $\Psi_{(A-4)}$ and relative motion $\phi_E^L(\rho)$ wave functions (WFs):

$$\Psi_{as} = \hat{A}(\Psi_\alpha \Psi_{(A-4)} \phi_E^L(\rho)), \tag{1}$$

where $\hat{A}$ is the antisymmetrizer. The ground state of an (A-4) target nucleus is well described by the Elliott model [4, 5] and so its WF is characterized by the spin $S = 0$, the isospin $T = 0$, the angular momentum and the total spin $L = J = 0$, the Young frame $[f_c] = [4^{(A-4)/4}]$, and the Elliott symbol of SU(3)-symmetry $(\lambda\mu)$. The global Wigner SU(4) symmetry of the state is determined by the product of the Young frames of the AP [4] and the core $[f_c]$ and consequently takes the only value $[f] = [4] \times [f_c] = [4^{A/4}]$. Other global quantum numbers $(S, T, L)$ satisfy the ordinary selection rules. Since the WF of the $\alpha$-particle is SU(3)-scalar, the global SU(3) Elliott symbol of the state is determined by the product

$$(\lambda\mu) \subset (\lambda_c\mu_c) \times (n0), \tag{2}$$

where the last representation is related to the WF of the $\alpha+$ core relative motion. The value $n$ is determined due to the possibility to expand the two-body continuum WF of the relative motion $\phi_E^L(\rho)$ onto a series of harmonic oscillator WFs $\phi_{nL}(\rho)$, where $n$ is the principal quantum number (QN) of the oscillator functions. As a result, a WF of the relative motion is characterized by the infinite number of the Elliott symbols $(n0)$. In this way the components of the asymptotic WFs are written in the Elliott model also.

The basic hypothesis of the model is the existence of AP resonances which are described by the same QNs $[f], (\lambda\mu), L, S, T$ as the main components of WFs of the asymptotic channels, the strong coupling of the discussed resonances and channels characterized by one and the same set of the QNs, and the weak coupling of the channels to other nuclear states. Naturally all partial QNs of the asymptotic states are not conserved in the colliding region because the antisymmetrizer is a pitiless terminator of partial quantities at short distances.

Phenomenological argument in favor of the hypothesis is that for a magic core $(\lambda_c\mu_c) =$ (00) and so $(\lambda\mu)$ takes the only value $(n0)$ according to exp. (2), while if the hypothesis were not valid, the list of $(\lambda\mu)$ would be too wide even for this core. Thus if one would exclude $(\lambda\mu)$-conservation requirement there would be too many states populated by the discussed $\alpha$-particle reactions both for the magic core case and for another situation. Similar consequences appear if one would get rid of the Young frame conservation.

The resonances under discussion are to be the eigenstates of a Hamiltonian. To conserve the Elliott SU(3) symbols this Hamiltonian must be constructed from the number of the commuting invariant operators of SU(3) group and the related subgroups:

$$H = H_{osc.} + f_1(\hat{L}^2) + f_2(QQ) + f_3((Q \otimes Q)Q) + f_4(Q(\hat{L} \otimes \hat{L})), \tag{3}$$

where $H_{osc.}$ is the oscillator Hamiltonian, $\hat{L}$ is the operator of angular momentum, $Q$ is the quadrupol operator (see Ref.[5]). The approach is the extension of the Elliott SU(3) model [4, 5] of alpha-particle states extending it by inclusion of the $Q^3$, $Q\hat{L}^2$ terms into the model Hamiltonian. It may be written through the Casimir operators, thus it is a typical Hamiltonian of broken symmetry.

It is clear that the eigenvalues of the Hamiltonian are easily expressed through the QNs $(\lambda\mu)$, $L$ and $\Omega$ - the eigenvalue of the Bargman operator $\hat{\Omega}$ [5]. On the other hand, the proposed model incorporates the Hamiltonian and the WFs written through the nucleon variables. Consequently it is a microscopic model.

Despite the coincidences of the QNs the WFs of the $\alpha$-particle resonances at first glance do not manifest a cluster structure which is defined by analogy with eq. (1). Indeed these WFs are typical SU(3) shell-model ones. What is the connection? To make it visible a multicluster generalization of two-body Wildermuth-Kannelopulos identity [6] is constructed. It is possible to write the translationally-invariant shell model WF in the form:

$$\Psi_A \equiv \hat{A}(\prod_{i=1}^{k} \Psi_i \times \hat{N}^{-1/2}\Psi_{rel.}\Delta). \tag{4}$$

Here $\Psi_A$ is the A-nucleon WF of the multicluster system; $\Psi_i$ is the WF of a cluster i, $\Psi_{rel.}$ is the WF of the relative motion of k clusters in the SU(3) scheme, $\hat{N}$ is the (multicluster) overlap kernel of the resonating group model. The letter $\Delta$ denotes the global (characterizing the whole system) QNs. If one uses the alpha-particles as the clusters (constituents), then $\Delta \equiv \{[f] = [4^{A/4}], (\lambda\mu), L, S = 0, T = 0, J = L, \delta\ \}$, where $\delta$ denotes complementary QNs. The discussed above AP resonances are characterized by the same QNs. The Hamiltonian constructed from the Casimir operators results in the same spectrum both in the full space of antisymmetrized A-nucleon WFs and in the space of the WFs of the relative motion of AP $\Psi_{rel.}$ (all exchange matrix elements are equal to zero), so APs can be considered as structureless constituents. The only difference between the spectra is that some states contained in the space of the WFs of the relative motion turn out to be forbidden by the Pauli principle in the full space and some other states there turn out to be multiple (that is why the QN $\delta$ is introduced).

So the list of alpha-particle states is limited by, first, the Pauli principle restrictions and, second, the prescription of the basic hypothesis of the model requiring the coincidence of their QNs with the asymptotic ones.

An $N = Z$-even nuclear system behaves as a system of $k$ stable $\alpha$-particles. Consequently in the proposed model an arbitrary $\alpha$-particle resonance is a multicluster $\alpha$-

Table 1: Classification of the $\alpha$-particle levels of $^{20}Ne$ nucleus.

| $(\lambda\mu)$ | $n=8$ | $n=9$ | $n=10$ | $L^{\pi}$ | $n=8$ | $n=9$ | $n=10$ |
|---|---|---|---|---|---|---|---|
| $(80)$ | 1 | 0 | 0 | $0^+$ | 1 | 0 | 3 |
| $(90)$ | 0 | 1 | 0 | $1^-$ | 0 | 1 | 0 |
| $(10,0)$ | 0 | 0 | 3 | $2^+$ | 1 | 0 | 3 |
| | | | | $3^-$ | 0 | 1 | 0 |
| | | | | $4^+$ | 1 | 0 | 3 |
| | | | | $5^-$ | 0 | 1 | 0 |
| | | | | $6^+$ | 1 | 0 | 3 |
| | | | | $7^-$ | 0 | 1 | 0 |
| | | | | $8^+$ | 1 | 0 | 3 |
| | | | | $9^-$ | 0 | 1 | 0 |
| | | | | $10^+$ | 0 | 0 | 3 |

particle state. Low-lying states of $1p$-, $(2s-1d)$-shell nuclei are well described by the Elliott model. Therefore these states possessing the QNs $[f]=[4^k], S=0, T=0$ are also the states of such a type. Another choice of the partition of WF of a system is not valid, because in this case the antisymmetrizer turns out to be the origin of channel coupling. There is one exception. The multicluster partition remains valid under the operation of the antisimmetrizer, if a heavier magic constituent ( $^{16}O$, $^{40}Ca$) is involved. However qualitative properties of $^{20}Ne$ spectrum testify that multi-alpha model of the states is preferable, because some levels are absent in the spectrum if $^{16}O+\alpha$ model is explored.

Is the present model capable to explain the above mentioned peculiarities of the $\alpha$-partical spectra? To elucidate that let us analyze these spectra in $^{20}Ne$ and $^{22}Ne$ nuclei. The existence of one additional neutron pair does not break the validity of the relation of the type (4) for the resonance states and therefore such a system can be considered as 5-$\alpha$-particle + 2n-cluster one. According to exp. (2) the number of the resonances increases rapidly from $^{20}Ne$ nucleus to $^{22}Ne$ due to the fact that for the latter one the Elliott symbol of a target takes the form $(\lambda_c\mu_c)=(40)$.

The list of multiplicities of the allowed Elliott symbols and the spins for $^{20}Ne$ and $^{22}Ne$ nuclei is presented in tabls. 1 and 2. As it is seen from the tables, a rapid production of the $\alpha$-particle states from $^{20}Ne$ to $^{22}Ne$ takes place. Naturally the set of such states for a nucleus from the middle of $(2s-1d)$-shell ($^{32}S$ for example) is still larger. Thus the mentioned near-magic effect turns out to be explained.

On the other hand, it is clear that the total number of states of $^{22}Ne$ nucleus in the present $2\hbar\omega \sim 25$ MeV range is essentially greater than the number of the states listed in tabl. 2. So the explanation of property 2 is also obtained.

Finally the model turns out to be capable to describe the qualitative properties of the $\alpha$-particle states in a very natural way.

Possibilities of a quantitative description of the discussed states may be illustrated by the example of the spectrum of the $^{16}O$ nucleus. Here the linear combination of the Casimir operators in Hamiltonian (3) is used for the fit. In addition to square terms $L^4$ and $Q^4$, the even-odd splitting term are used. The value $\hbar\omega$ is fixed. The procedure of the

Table 2: Classification of the $\alpha$-particle levels of $^{22}Ne$ nucleus.

| $(\lambda\mu)$ | $n=8$ | $n=9$ | $n=10$ | $L^{\pi}$ | $n=8$ | $n=9$ | $n=10$ |
|---|---|---|---|---|---|---|---|
| (82) | 1 | 0 | 0 | $0^+$ | 3 | 0 | 99 |
| (63) | 1 | 0 | 0 | $1^-$ | 0 | 10 | 0 |
| (44) | 2 | 0 | 0 | $2^+$ | 7 | 0 | 164 |
| (11, 1) | 0 | 1 | 0 | $3^-$ | 0 | 19 | 0 |
| (92) | 0 | 2 | 0 | $4^+$ | 10 | 0 | 239 |
| (73) | 0 | 3 | 0 | $5^-$ | 0 | 23 | 0 |
| (54) | 0 | 4 | 0 | $6^+$ | 8 | 0 | 239 |
| (14, 0) | 0 | 0 | 3 | $7^-$ | 0 | 19 | 0 |
| (12, 1) | 0 | 0 | 3 | $8^+$ | 5 | 0 | 194 |
| (10, 2) | 0 | 0 | 21 | $9^-$ | 0 | 12 | 0 |
| (83) | 0 | 0 | 30 | $10^+$ | 0 | 0 | 119 |
| (64) | 0 | 0 | 45 | | | | |

choice of the data from the spectroscopic tables was the following. Ordinary Elliot rules [5] were used for the subthreshold part of the spectrum. For the analysis of the higher part of the spectrum, the experimentally detected levels for which the complete set of information about $\alpha$-decay properties is available besides the information about the final state of a daughter nucleus were involved. An additional requirement of the high value of the $\alpha$-spectroscopic factor ($W \geq 0.1$) of WF of a level was imposed. The results of the fit are presented in fig. 1.

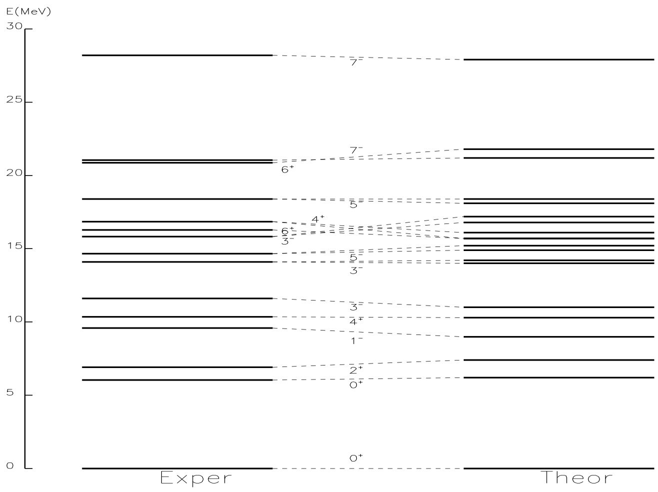


Figure 1: Experimental and theoretical spectra of $\alpha$-particle states of $^{16}O$ nucleus

Doubling of the experimental level is an example of degeneracy of the solutions of the microscopic Schrödinger equation with Hamiltonian (3). The QN $\delta$ is involved especially for denoting this degeneracy. The reverse doubling of the very close, in comparison with their widths, theoretical levels is connected, probably, with the situations in which the resolution of the experiment is not high enough.

It is seen that Hamiltonian (3) reproduces the experimental data rather well. There is the only case ($3^+$ in 16 MeV vicinity) for which the deviation of the theoretical value is greater than 1 MeV. The model demonstrates the $\alpha$-particle nature of many other states of $^{16}O$ nucleus for which the data are not complete ($\alpha$-particle widths or decay modes at all are not known). These states are not presented in fig. 1 to prevent its overloading.

So the results obtained in the model turn out to be well justified by the experiment.

# 3 Alpha-particle condensate in nuclei.

The states of nuclei generated by the SU(3) Hamiltonian and populated in the discussed above reactions manifest in these processes the properties of nuclear system, condensed to multi-alpha-particle matter.

The picture of the process looks as follows: $\alpha$-particles penetrate each other, and the interaction and nucleon exchange result in a disintegration (a loss of individuality) of the $\alpha$-particles. Instead of this all the properties of multi-alpha-particle system are conserved and the memory about asymptotic $\alpha$-particle channel turns out to be absolute (by analogy with elastic time-dependent Hartry-Fock picture). The nucleon dynamics determined by the Hamiltonian is the origin of stability of $\alpha$-particle states. And what is more, taking into account the fact that numerous ground and low-lying states of light nuclei are multi-alpha-particle ones one can conclude that the discussed condensation takes place at the normal nuclear density. Internal structure of $\alpha$-particles, exchange effects, nucleon degrees of freedom turn out to be visible for other entrance or exit channel only.

So the nucleon system described by Hamiltonian (3) and "$\alpha$-particle" boundary conditions behaves as $\alpha$-particle matter. All nucleons turn out to be condensed to $\alpha$-particles. Peculiar kind of the $\alpha$-particle shell model is valid for the WF $\Psi_{rel.}$.

However the above mentioned selection rules appearing due to the Pauli principle put the limits on the number of $\alpha$-particles placed on one $\alpha$-particle level. Indeed, if one considers step by step occupation of $\alpha$-particle levels in the ground states of $N = Z$-even nuclei the level with principal QN of the alpha-particle center of mass motion $n_\alpha = 0$ is completed by one; next ($n_\alpha = 4$), by three, next ($n_\alpha = 8$) is completed by six $\alpha$-particles etc. Occupation of a level with an intermediate value of $n_\alpha$ is allowed only if the lower levels are not completely occupied i.e. in case when excited states are considered. So the Pauli principle prevents condensation of a great number of $\alpha$-particles on one level. Properties of alpha-condensation are essentially different from the ordinary Bose-Einstein condensation.

# 4 Conclusion.

1. A microscopic SU(3) theory of the $\alpha$-particle states of light nuclei is developed. It is rather general since it allows one to investigate arbitrary characteristics of the discussed states such as spectroscopic factors (reduced widths), B(E2)-probabilities, inelastic scattering channels etc.
2. The approach seems to be very natural since a lot of qualitative properties of $\alpha$-particle systems appear automatically in it. Indeed the properties of alpha-particle spectra (level densities and their variation from nucleus to nucleus, shapes of rotational bands etc.) are well reproduced in the model.
3. Quantitative characteristics of the $\alpha$-particle spectra are also well reproduced. The ground, low-lying and highly-excited states of the nuclei under discussion are well described by the model.
4. The point of the theory is the precisely solvable model of cluster stability. The Hamiltonian of the model generates the properties of A-nucleon system completely reduced to the properties of $k\alpha$-particle one, i.e. a system behaves as the $\alpha$-condensed matter.
5. The properties of multi-alpha-particle states obtained in the model are the typical properties of $\alpha$-condensed matter and this condensation takes place in the states of the normal density on frequent occasions.
6. The Pauli principle which is inherent in an A-nucleon system prevents condensation of a large number of $\alpha$-particles on one level. Therefore $\alpha$-condensation is a more complicated subject than the ordinary Bose-Einstein.

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