Quasimolecular and vortex states of nuclei through Harvey's prescription

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Abstract

In the last years developments in experimental technique improved the experimental resolution of the registration of elastically scattered complex particles at large angles. This enables the observation of fragmented momentum states as a signature of Bose- Einstein condensation in light nuclei. The condensate of α-particle "gas" is analogous to the recently discovered Bose condensates of bosonic atoms formed in magnetic traps. This situation is very similar to nuclear pairing where a limited number of Cooper pairs are involved and yet most nuclei manifest clear signatures of superfluidity. There could exist topological soliton-like excitations in nuclei with α cluster condensation. These excitations are vortices in superfluid, a property possessed by such Bose condensed nuclei. Given the exchange reaction mechanism, anomalous backward scattering may indicate the presence of vortex states in nuclei. These vortex states can be detected at backward angles. We show that the vortex-like states possess quantum numbers $J^P = N_\alpha^{(-1)^{N_\alpha}}$ of the self-conjugate N_α condensed states. We point out actual candidates, namely 3^- state in ^{12}C and 4^+ state in ^{16}O, although the topological nature as a vortex is obscure in such light nuclei.

1 Introduction

During the last years Anomalous large angular scattering (ALAS) became an instrument for the study of quasimolecular states. The observation of fragmented cluster states with the help of so-called Method of Inverse Geometry (IMG) gave a new impulse to the investigation of cluster states [1]. Here we would like to demonstrate how this method can be applied to the study of vortex states. The interest to the vortex states in nuclei appeared periodically in the scientific literature. In 1966 Zeldovich assumed the existence of rotating nuclear isomers [2]. He considered a nucleus as a superfluid liquid drop with quantum vortex going through the drop axis. The circulation of the velocity around a contour, surrounding a vortex is equal to $\hbar/m$, where m stands for boson mass. The total momentum of rotation is equal to $\hbar Z/2$, where Z is the nuclear charge. In 1973 Wong [3] noted that toroidal and spherical bubble nuclei in principle can exist. Then Fowler, Raha and Weiner considered stable vortex excitations in rotating nuclei [4]. They suggested hot spot model whose evolution is governed by nonlinear equation with a soliton solution.

Wilkinson [5] proposed a similar idea about alpha-neutron rings and chains. According to him valence neutrons in molecular type orbits between alpha clusters provide an additional binding energy to stabilize the chain structures. Later Bertsch, Broglia and Schrieffer again considered superfluidity rotation in nuclei [6]. They gave an estimate on the parameters of nuclear vortex. During the last years there was a discussion on nuclear vortex current and vorticity [7], as well as on the role played by toroidal dipole modes in nuclei [8]. At high energies nuclear transport theory predicts formation of unstable bubbles and rings in the central collision of heavy ions [9] of equal mass .

2 Results and discussion

For a better understanding of the possible vortex movement in nuclei we utilize the work of the Oxford group [10]. They used the Cranking Model for the description of clusters in nuclei. It was done through the diagonalization of the Hamiltonian in the rotating frame, which involves adding to the single particle Hamiltonian the term $-wj_x$, where w is the rotation frequency. Calculations in the framework of this model showed impressively pronounced cluster structure in light nuclei which becomes especially clear from the density contours for different configurations of alpha-particles. The same pictures can be obtained by calculation of pressure distribution inside vortices [11]. Recently Iwazaki noticed that the vortex excitations in alpha cluster condensed nuclei may exist only for certain alpha-particle configurations [12].

Recent experiments on the scattering of alpha-particles from light nuclei demonstrated the appearance of bands having the same angular momenta [13, 14]. They occur within the rotational bands which consist of even and odd states. The moment of inertia deduced from the band structure of the rotational states indicates that there are a few alpha particles orbiting around a core. On the other side, at some low density, few α-particles exist probably in a weakly self-bound gas-like state with, at most, pairwise two α-particle correlations of the ^{8}Be-type. Since α-particles are Bosons, these low density states form a Bose condensate [15, 16]. The condensate of α-particle "gas" is analogous to the recently discovered Bose condensates of bosonic atoms formed in magnetic traps [17]. This situation is very similar to nuclear pairing where a limited number of Cooper pairs are involved and yet most nuclei manifest clear signatures of superfluidity. More then 50 years ago Bogoliubov [18] derived his famous theory for the elementary excitations of a dilute Bose gas. This theory, originally applied to the homogenious systems, can be shown [19] to correspond to the linear limit of the time-dependent Ginzburg–Pitaevskii–Gross equation [20, 21, 22]. At high energy excitations of alpha-clusterized nuclei, when the shell structure becomes weak, one has to deal with Bose-Einstein condensate of alpha-particles which is governed by the following Gross-Pitaevsky equation [15]

$$-\frac{\hbar^2\nabla^2}{2m_\alpha}\psi_\alpha(r) + V_{ex}(r)\psi_\alpha(r) + N_\alpha \cdot \ g|\psi_\alpha(r)|^2\psi_\alpha(r) = \mu\psi_\alpha(r), \quad (1)$$

where m_α is the mass of α-particle, V_{ex} - an effective (oscillator) potential. N_α is the number of α-particles, g - the coupling constant and μ is the chemical potential. Neglecting the kinetic energy term in (1), one comes to the well known Thomas-Fermi solution $\rho_\alpha(r) \equiv N_\alpha \cdot |\psi_\alpha(r)|^2 \approx (\mu - V_{ex}(r))/g$. This solution shows that at high energies the fragmentation of cluster states takes place. The number of these fragmented states is proportional to the number of clusters N_α. If $g \geq 0$, $(\mu - V_{ex}) \gg g$ one can expand the local momentum $p_\alpha(r) \equiv \sqrt{2m_\alpha(\mu - V_{ex} - g\rho_\alpha)}$ in series of ρ_α. This standard semi-classical WKB procedure leads to the conclusion that discretization of momentum depends upon the number of α-particles

$$\int_{r_1}^{r_2} p(r)dr = (2N+1)\pi/2 + GN_\alpha, \tag{2}$$

where N is the radial quantum number. Relying on this result we can present the band structure of cluster states by the following formula for the spectrum

$$E = A + BL(L+1) + CN + DN^2 + GN_\alpha, \tag{3}$$

where the standard formula describing rotational molecular spectra was modified by adding a new term, which is proportional to the number of α-particles.

The Gross-Pitaevsky equation (1) has also a vortex solution [23], where the nonlinear radial equation for the vortexes is

$$\frac{1}{2m_\alpha r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) + \left(\mu - \frac{\gamma}{2m_\alpha r^2}\right)R(r) - gR(r)^3 = 0.$$

What could be the signature of the vortex structure in nuclei? What kind of experiments would shed light on this structure? In our opinion electron scattering with large momenta transfer can give us a clue to the vortex structure. Here are some candidates for scrutinized investigation, the states 3^- (9.64 MeV) in ^{12}C and 4^+ (16 MeV) in ^{16}O. On the other hand, experiments like ALAS are also appropriate for the study of vortex states. It is a well-known fact, there are lots of mechanisms yielding ALAS. For our purposes we need only one, namely the mechanism of elastic transfer.

As we have mentioned, the inverse geometry experiments demonstrated at large angles such phenomena as fragmentation of the cluster states, see Fig. 1. This picture corresponds to the spectrum (formula (3)), where the constant G is equal to 433 KeV, what corresponds to the correlation length between α-particles 2.4 fm. On the other hand, experiments like ALAS are also appropriate to the study of vortex states. Apart from the explanation of fragmentation of cluster states with the help of Bose condensate, we would like to combine the ideas of Cranking Model and inverse geometry method. Let us consider ALAS of α-particles on ^{28}Si in terms of inverse geometry method supposing the elastic exchange transfer mechanism. If the registration of elastic scattered nuclei takes place in the back hemisphere along oz-axis, we can present the conservation of energy before and after the elastic exchange process in the following way:

$$\underset{\text{before exch.}}{[(h - wj_z) + T]} = \underset{\text{after exch.}}{[h' + (T' - w'j_z)]},$$

where h and h' – Hamiltonians, T and T' – kinetic energies, w and w' – rotation frequencies along oz-axis, j_z – momentum projection on oz-axis whose eigenvalue is proportional to the number of alpha-particles. The prime denotes another particle in terms of elastic transfer process and the left-hand side corresponds to the energy before the exchange, while the right-hand side is the energy after the exchange.

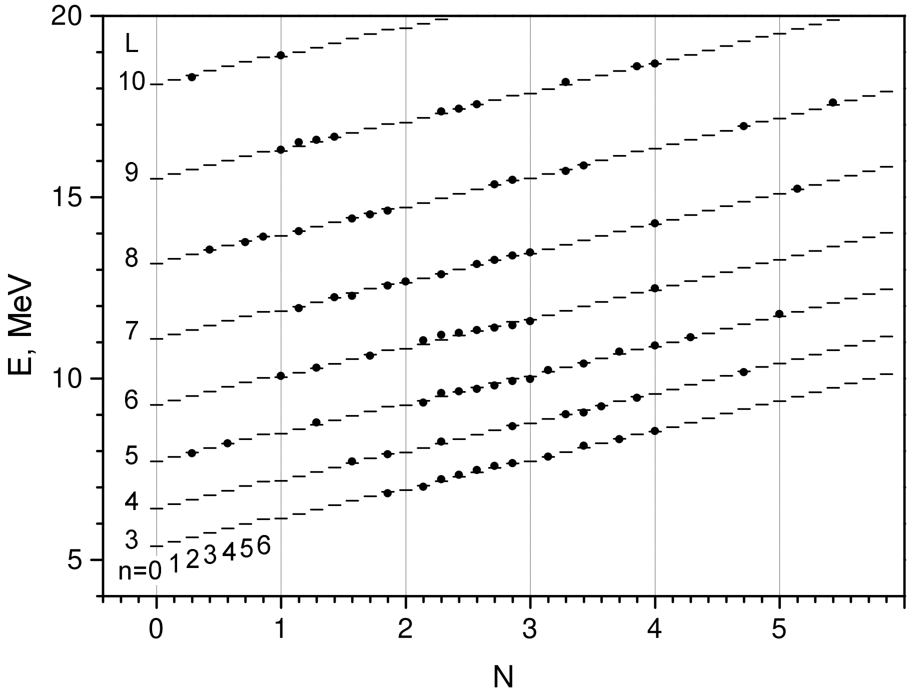

Figure 1: The fragmentation of cluster states observed in the α-scattering on ^{28}Si.

In terms of Bose condensate we can connect the wave function of the nucleus containing N_α alpha particles at rest to the wave function of the nucleus having velocity v. Momentum of the system in two reference frames is $P(v) = P + N_\alpha mv$. The idea of combining cranking model and inverse geometry method follows Harvey's prescription [24]. In this framework we estimated the energy of fragmented states. This is close to the values which are measured with the help of inverse geometry method, see Fig. 1.

Let us consider a chain of N_α alpha-clusters along Z-axis. The shell model predicts the following form of the wave functions: $[000]^4, [001]^4, [00, N_\alpha - 1]^4$. This situation is very close to the Bose-Einstein condensate of the dilute gas [25]. But the distance between alpha-clusters along the linear chain is determined by the effective alpha-alpha interaction. Applying the method of molecular orbitals to the alpha-particles of the chain we found that the number of orbitals corresponds to the number of alpha-particles.

3 Conclusion

In conclusion we would like to stress that ALAS in combination with inverse geometry is a promising method to study exotic states of nuclear matter. It is clear that certain vortices like vortex rings are impossible to observe experimentally, because these structures tend to disappear with a distance. Only some indirect ways like fragmentation of cluster states are appropriate for the observation of vortex states.

Work was supported in part by Deutsche Forschungsgemeinschaft (grant 436 RUS 113/24), Russian Foundation for Basic Research (grant 03-02-04021) and the Heisenberg–Landau Program (JINR).

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