ESFRAD. FORTRAN code for calculation of QED corrections to polarized ep-scattering by the electron structure function method

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Abstract

The main features of the electron structure function method for calculations of the higher order QED radiative effects to polarized deep-inelastic *ep*-scattering are presented. A new FORTRAN code ESFRAD based on this method was developed. A detailed quantitative comparison between the results of ESFRAD and other methods implemented in the codes POLRAD and RADGEN for calculation of the higher order radiative corrections is performed.

1 Introduction

The observed cross section in the electron–proton deep inelastic scattering

$$e^-(k_1) + p(p_1) \to e^-(k_2) + X(p_x) \tag{1}$$

($k_1^2 = k_2^2 = m^2$) is affected by electromagnetic effects caused by real and virtual photon emission and e^+e^- – pair production, which cannot be removed with any experimental cuts. Therefore contributions of these effects called radiative corrections (RC), to different observables has to be calculated theoretically and then subtracted from experimental data (see [1] for details).

In the present report the model–independent RC are considered. Such RC include contributions due to radiation of real but not observed photons and electron–positron pairs as well as loop corrections in the lepton part of interaction. The model–dependent RC arise from electromagnetic effects in the hadron part of interaction. Such RC are described by the box–type and proton vertex diagrams and real photon radiation by the proton. Their calculation requires additional assumptions about the hadron structure and, therefore, they have additional theoretical uncertainties. The model–dependent RC are the subject of separate theoretical studies (see, e.g, Ref. [2, 3, 4, 5] for the elastic *ep*-scattering case).

The structure of model–independent RC in ultrarelativistic approximation has a simple form. The contribution of virtual and real particles to the observed cross section in the n-th order of perturbation theory can be presented as the following sum,

$$\frac{d\sigma_n^{RC}}{dQ^2dy} = \left(\frac{\alpha}{\pi}\right)^n \sum_{i=0}^{n} C_{ni}(Q^2,y)L^i + \mathcal{O}(\frac{m^2}{Q^2}), \tag{2}$$

where $Q^2 = -(k_1 - k_2)^2$, $y = (k_1 p_1 - k_2 p_1)/k_1 p_1$, $L = \log Q^2/m^2$ and the coefficients $C_{ni}(Q^2, y)$ are independent on the electron mass. Each of the leading terms $C_{ni}(Q^2, y)L^i$ describes along with elastic (i.e., without no emitted photons) and inelastic events with the number of hard collinear photons up to n. Since at modern accelerator energies the quantity L is about 10 - 20, the main contribution to RC (2) gives so-called leading order (LO) terms which are proportional to $(\alpha L/\pi)^n$ and all the real photons as well as e^+e^--pairs in this case are collinear of the parental electrons. Note that the contribution $\sim (\alpha/\pi)^n L^{n-1}$ is usually called next-to-leading order (NLO).

At present only the lowest order radiated corrections ($n = 1$) have been calculated completely for inelastic [6] and elastic lepton-hadron scattering [7] using the covariant approach developed in [8]. The formalism of [6] was used later to develop a FORTRAN code POLRAD [9] and a Monte Carlo generator RADGEN [10] for simulation of radiative events in deep–inelastic ep–scattering.

Unfortunately, evaluation of the higher order effects with $n \geq 2$ were estimated not so precise as lowest ones because of the extremely complicated integration over the unobserved real photon phase space. Firstly, the soft photon approximation was used by by D. Yennie, S. Frauchi and H. Suura [11] to describe the multi–photon radiation. Then the calculation of RC within the electron structure function (ESF) method was proposed by Fadin and Kuraev [12] who applied the Drell–Yan representation for the electron–positron annihilation cross section. According to this method, the radiatively corrected observed cross section can be written as a convolution of two ESF with its hard part. Within the leading accuracy the hard part coincides with the Born cross section.

Further development of the ESF method was done in [13]. It consists in modification of the hard part of the cross section taking into account the possibility of one non–collinear hard photon radiation. Such modification allows to go beyond the leading approximation and to compute partly NLO contributions into RC of the order $(\alpha/\pi)^n L^{(n-1)}$

$$\frac{d\sigma^{RC}_{LL}}{dQ^2 dy} = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi} L\right)^n C_{nn}(Q^2, y) + \left(\frac{\alpha}{\pi}\right)^n L^{(n-1)} C_{nn-1}(Q^2, y), \tag{3}$$

while the NLO contribution of the order α/π is calculated exactly. There exists another source of the NLO terms like the second one in the right–hand side of Eq.(3), which could not be evaluated in [13]. Such terms arise due to the improvement of ESFs by including their non-leading pieces. The respective calculations are in progress now.

The second order correction ($\sim$ $(\alpha L/\pi)^2$) for unpolarized and polarized inelastic lepton-hadron scattering was evaluated numerically in [14] and [15], respectively, just the ESF method. In this report we analyze the results of the new FORTRAN code ESFRAD [16] based on approach developed in [13] for both unpolarized and polarized deep-inelastic scattering and compare it with the codes POLRAD and RADGEN that treat the higher–order QED effects differently.

2 Master Formula

A master formula for the observed cross section within the ESF method is derived according to Fig. 1 that should be considered as a diagram for the cross section (not for the

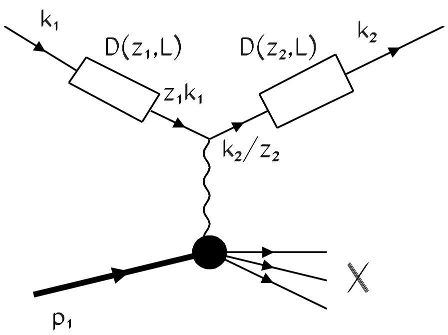


Figure 1: Diagram for the corrected cross section within the ESF method

amplitude),

$$\frac{d^2\sigma^{obs}(k_1,k_2)}{dQ^2dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} dz_2 D(z_1,L)\frac{1}{z_2^2}D(z_2,L)\frac{d\sigma^{hard}(\tilde{k}_1,\tilde{k}_2)}{d\tilde{Q}^2d\tilde{y}}, \tag{4}$$

where the physical meaning of integration variables z_1 and z_2 can be understood in Fig.1. The hard cross section in the integrand depends on so–called shifted variables which are defined as follows

$$\tilde{k}_1 = z_1k_1,\ \tilde{k}_2 = \frac{k_2}{z_2},\ \tilde{Q}^2 = \frac{z_1}{z_2}Q^2,\ \tilde{y} = 1 - \frac{1-y}{z_1z_2}. \tag{5}$$

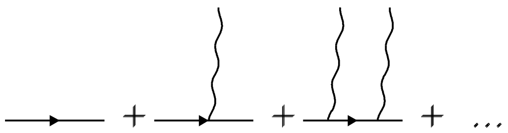
The lowest limits of integration can be found as a solution of the standard π-production threshold inequality for DIS process: $(p_1+\tilde{k}_1-\tilde{k}_2)^2 > (M+m_\pi)^2$ that leads to

$$z_{2m} = \frac{1-y+xyz_1}{z_1-z_{th}},\ z_{1m} = \frac{1-y+z_{th}}{1-xy},\ z_{th} = \frac{m_\pi(m_\pi+2M)}{2p_1k_1} \tag{6}$$

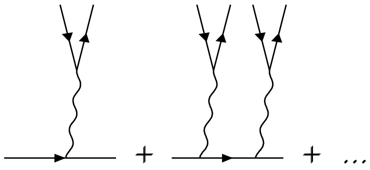
The electron structure function $D(z_1,L)$ gives the energy fraction (z_1) distribution of electrons with virtuality up to Q^2 which is created by the initial electron, whereas $D(z_2,L)$ is in fact the fragmentation function of the heavy intermediate electron into real one with the energy fraction z_2. Function $D(z,L)$ absorbs all the leading contributions connected with real and virtual photon emission and pair productions

$$D(z,L) = D^\gamma(z,L) + D_N^{e^+e^-}(z,L) + D_S^{e^+e^-}(z,L),\ \int\limits_0^1 dz[D^\gamma(z,L) + D_N^{e^+e^-}(z,L)] = 1. \tag{7}$$

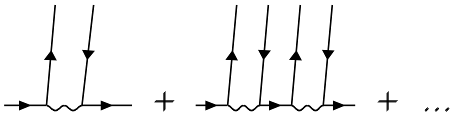
Roughly speaking, the contribution to the function D^γ comes from all the graphs with radiation of real and virtual photons, which is represented here in symbolic form as follows



A nonsinglet part of $D^{e^+e^-}$ is described in the same graphs but with attached e^+e^--pairs to the photon lines.



The graphs that have at least one intermediate photon which propagates along the parental electron line contribute to the singlet part of $D^{e^+e^-}$.



For details about the D-function see, for example, [17].

The hard cross section in the master formula consists of the sum of the Born cross section and NLO part of lowest order RC. It can be written as

$$\frac{d\sigma^{hard}(k_1,k_2)}{dQ^2dy} = \frac{1}{[1-\alpha/\pi\Pi(Q^2)]^2}\left[\frac{d\sigma_0(k_1,k_2)}{dQ^2dy} + \left(\frac{\alpha}{\pi}\right)C_{10}(Q^2,y)\right], \tag{8}$$

where $\Pi(Q^2)$ is an intermediate photon self energy and the coefficient $C_{10}(Q^2,y)$ enters Eq.(2).

The explicit expressions for the electron structure functions, used in ESFRAD as well as the more detailed description of the hard cross section can be found in [13, 16].

3 Numerical Results

For the numerical analysis performed within experiments on fixed proton targets at JLab (E_{beam} = 4 GeV) and HERMES (E_{beam} = 27.5 GeV), NMC parametrization for the unpolarized hadron structure function F_2 is used. The polarized hadron structure functions g_1 and g_2 are expressed by the asymmetries

$$A_1 = a + x^b(1-\exp(cx)), \tag{9}$$

with $a = 1.90202\times10^{-2}$, $b = -1.16312\times10^{-3}$, $c = -1.8451$ and

$$A_2 = \frac{0.53Mx}{\sqrt{Q^2}} \tag{10}$$

in a standard way.

The y-dependences of the relative RC to unpolarized part of the cross section

$$\delta_u = \frac{d\sigma^{obs}/dQ^2dy - d\sigma^0/dQ^2dy}{d\sigma^0/dQ^2dy}, \tag{11}$$

calculated in different FORTRAN codes and methods are presented on Fig. 2. It should be noted that the lowest order RC are the same for POLRAD, RADGEN and ESFRAD. But the higher order corrections in these codes are estimated in different way:

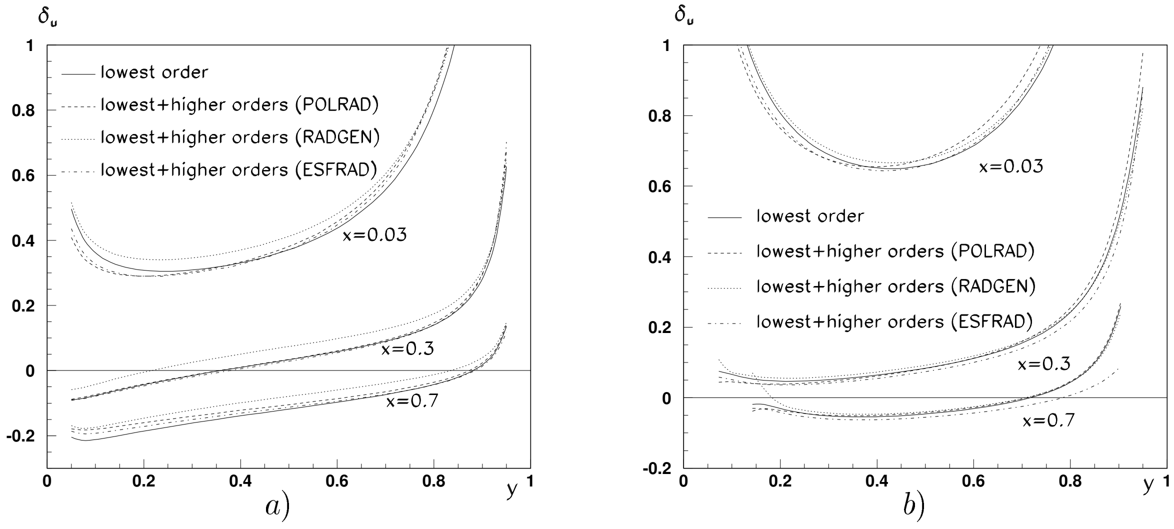


Figure 2: Relative radiative corrections to the unpolarized cross section defined by formulae (11) for a) HERMES (E_{beam} = 27.57 GeV) and b) JLab (E_{beam} = 4 GeV) kinematic conditions

- In RADGEN the soft photon approximation is used to account multi–photon events.
- In POLRAD only the second order leading correction is included.
- In ESFRAD together with the leading contribution the next-to-leading one is partly incorporated in accordance with Eq.(3).

It can be seen from Fig. 2(a) that at HERMES kinematics POLRAD and ESFRAD give very similar results for unpolarized part of the cross section while the behavior of the higher order correction generated by RADGEN coincides with POLRAD and ESFRAD only at large values of the variable y. At the same time at JLab kinematics (see Fig. 2(b)) the higher order effects in the considered different approaches show more distinctions. In this case the results of POLRAD and ESFRAD agree only at low y and there is noticeable disagreement between them when y increases and approaches its maximum value. On the other hand, RADGEN disagrees with POLRAD and ESFRAD at low y and coincides with POLRAD at large y. Note that for any given kinematical point (x, y) the value of RC for JLab is shifted toward positive values compared to the corresponding RC for HERMES.

Radiative corrections to the polarized asymmetries (9), (10) are presented in Fig. 3 and Fig. 4, respectively. One can see that the higher order effects give a very small contribution to the total correction in the entire kinematic region. It means that the effective higher order correction has a rather factorized form and disappears in the ratio of the polarization–dependent and unpolarized parts of the cross section. It should be noted that the main difference in the behavior of RC to A_1 and A_2: when x is growing RC to A_1 increases while RC to A_2 decreases. As well as for unpolarized case the value of RC to asymmetries for JLab is larger than the corresponding RC for HERMES.

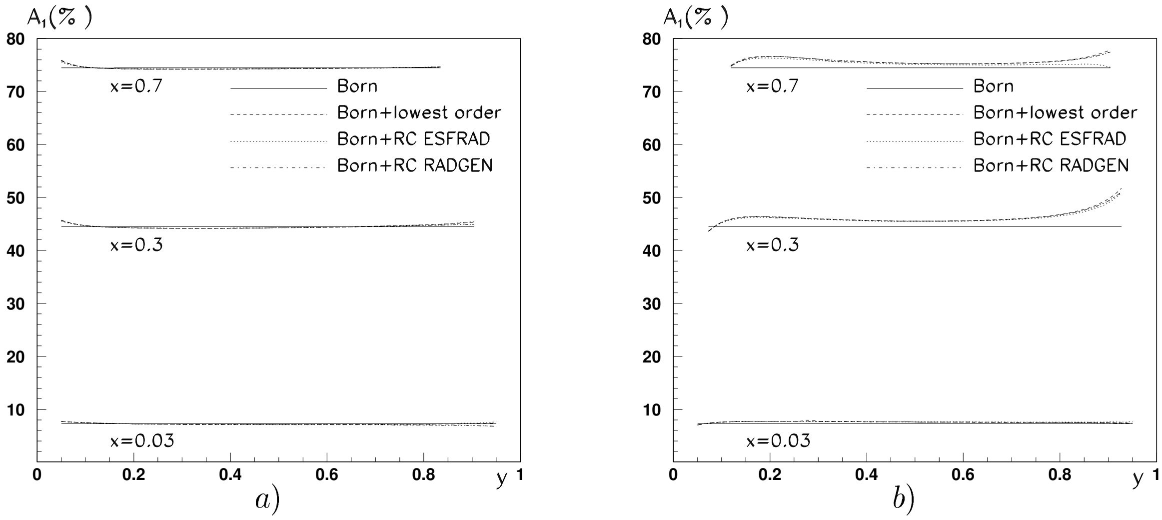


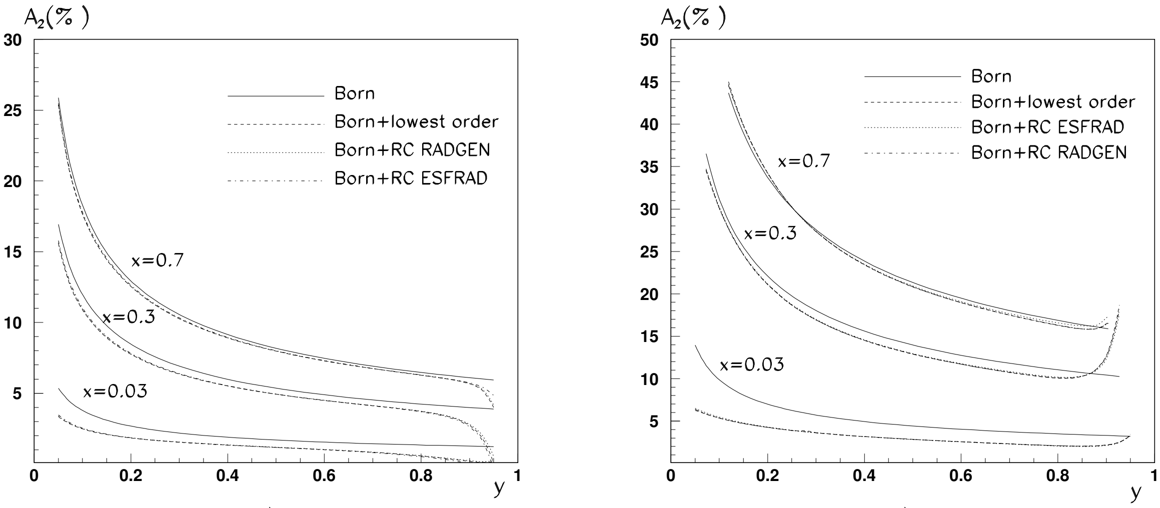
Figure 3: Radiative corrections to A_1 defined by expression (9) for a) HERMES ($E_{beam} =$ 27.57 GeV) and b) JLab ($E_{beam} = 4$ GeV) kinematic conditions

4 Conclusion

In this report the electron structure function method for calculation of the higher order radiative corrections is overviewed briefly and a comparative analysis of the FORTRAN code ESFRAD developed on the basis of ESF approach, with POLRAD and RADGEN is performed. All these codes account exactly for the first–order correction but the higher order effects are incorporated into them in different ways.

Our numerical analysis of the higher order RC in deep-inelastic events shown that

- RC to the unpolarized cross section and the spin asymmetries at JLab kinematics conditions ($E_{beam} = 4$ GeV) are larger than at HERMES ones ($E_{beam} = 27.5$ GeV).
- The higher order corrections generated by ESFRAD and POLRAD have the similar behavior at HERMES but differ noticeably at JLab conditions.
- At large values of y where RC is very important all three codes give the same result at HERMES kinematics but at JLab the higher order contribution of ESFRAD decreases considerably the total RC as compared with POLRAD and RADGEN.
- The higher order corrections contribute very little to polarization asymmetries that means that effectively they have rather factorized form and practically are canceled in the ratio of unpolarized and polarized parts of the cross section in the entire kinematic region allowed by HERMES and JLab conditions.
- With the increasing x the total RC to asymmetry A_1 increases, while RC to asymmetry A_2 decreases at both HERMES and JLab experimental conditions.



a) *b)*

Figure 4: Radiative corrections to A_2 defined by expression (10) for a) HERMES ($E_{beam} = 27.57$ GeV) and b) JLab ($E_{beam} = 4$ GeV) kinematic conditions

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