Time variation of fundamental constants: experiment and theory

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Abstract

We present in some detail the phenomenon of slow time variation of fundamental constants of nature. The main observational constraints on the variation of the gravitational constant, on the fine structure constant and on the proton-to-electron mass ratio are reviewed. We also elaborate on some theoretical schemes which naturally lead to such variations.

1 Introduction

All the laws of nature establishing relations between various dynamical characteristics involve parameters (or factors) which are assumed to be independent of the time or the space-time location and are thus regarded as *constants of nature*. For example, the gravitational interaction contains the gravitational constant, G. In fact, the statement about the constancy of the "constants" is just a hypothesis, though quite an important one. It is a part of the Copernican principle (see a thorough discussion of this issue in Ref. [1]) and is crucial for comparing and reproducing experiments. However, it is not senseless to question its validity. In particular, one may conceive that the constant G is in fact a smooth and slow varying function $G(t, \vec{r})$. This may be understood as a way to modify the standard physics framework or, equivalently, a way to introduce new elements of the physics beyond the Standard Model.

In this contribution by "variation of constants" we *will not mean* the change of couplings with increasing energy transfer in particle processes, that is, the running of couplings in the renormalization group context. Instead, here we will be concerned with the variation of couplings — like the fine structure constant α, the gravitational constant G, or the gauge coupling $\alpha_{\rm g}$ of the electroweak theory, amongst others — and mass parameters — like for example the proton-to-electron mass ratio $\mu = m_{\rm p}/m_{\rm e}$ — with time during the cosmological evolution in the low-energy limit. Of course, such variations on Solar System or geological time scales are indeed very small. There are at least two reasons for considering this effect. Firstly, recent observational data [2, 3, 4] can be interpreted as an indication of the time variation of α and μ. Secondly, there is a number of theoretical models which contain built-in mechanisms of such variations.

The issue of the level of "fundamentality" of a given coupling or parameter is not simple. The status of a coupling depends on the theory considered, namely whether it is effective or "fundamental" (microscopic). Some of the "fundamental" couplings of a large scale (effective) theory remain to be fundamental in the underlying microscopic theory, others do not. A pragmatic approach is to fix a theoretical framework characterized by a set of known parameters and then pose questions like why the parameters have the values they do really have and whether they are constants. Thus, questioning the constancy of fundamental parameters is essentially trying to understand a more fundamental theory behind. A discussion of this issue of fundamentality of couplings can be found in Ref. [5]. Moreover, it is important to realize that the only meaningful question whatsoever is, in fact, to ask about the variation of a *dimensionless* parameter, like α, or μ, or

$$\alpha_{\rm G} \equiv \frac{Gm_{\rm p}^2}{\hbar c} = \frac{m_{\rm p}^2}{M_{\rm Pl}^2} \approx 5.9 \cdot 10^{-39},$$

since only in this case the measurement of the variation is independent of the choice of the system of units and of the choice of standard rulers and clocks.

The issue of variation of the physical constants was first addressed by P.A.M. Dirac [6] who formulated it within the framework of his Large Number Hypothesis. We quote him literally: "Very large and very small dimensionless universal constants cannot be pure mathematical numbers and should rather be considered as variable parameters characterizing the state of the Universe". He basically considered the couplings α, $\alpha_{\rm G}$, $\alpha_{\rm W} \equiv (G_{\rm F} m_{\rm p}^2 c)/\hbar^3$ and their combinations

$$\delta \equiv \frac{H_0 \hbar}{m_{\rm p} c^2} \sim 2h \cdot 10^{-42} \quad \text{and} \quad \epsilon \equiv \frac{G\rho_0}{H_0^2} \sim 5h^{-2} \cdot 10^{-4},$$

where $H_0 \simeq 70\ \mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{Mpc}^{-1}$ is the present day Hubble constant and ρ_0 is the actual density of the Universe, and asked which of them vary with time. He also noticed that the relative magnitude of electrostatic and gravitational forces between a proton and an electron

$$\frac{\alpha_{\rm G}}{\mu\alpha} = \frac{G m_{\rm p} m_{\rm e}}{e^2} \sim 3.7 \cdot 10^{-40}$$

and the inverse number of times an electron has orbited around a proton during the age of the Universe, (the age of the Universe in atomic time)

$$\frac{H_0 e^2}{m_{\rm e} c^3} = 4\pi\alpha\mu\delta \sim 2.4 \cdot 10^{-40}$$

are of the same order. This prompted Dirac to assume that $\delta \propto H_0$ and $\alpha_{\rm G}$ vary with time as $\propto 1/t$.

We would like to note that the anthropic considerations allow only a certain interval of admissible values for some of the constants, though do not tell whether they are varying or not. For example, for the fine structure constant such interval is limited by

$$\frac{1}{170} < \alpha < \frac{1}{80},$$

where the lower bound comes from the requirement that the Grand Unification takes place at the scale $M_{\rm GUT} < M_{\rm Pl}$, and the upper one from the requirement that the proton lifetime $\tau_{\rm p}$ exceeds the age of the Universe $t_{\rm U}$. With the present value of α

$$\tau_{\rm p} \sim \frac{1}{\alpha^2}\frac{\hbar}{m_{\rm p}c^2}e^{\frac{1}{\alpha}} \sim 10^{32}\ {\rm yr}, \quad t_{\rm U} = \frac{c}{H_0} \sim 10^{17}\ {\rm s}.$$

The plan of the paper is the following. In Sect. 2 we review experimental bounds on the time variation of the fine structure constant α, of the gravitational constant G, and of the proton-to-electron mass ratio μ. Sect. 3 is devoted to discuss some of the theoretical approaches which can describe such phenomena. We first discuss the general features of these models, stressing an intrinsic relation between the time variation of couplings and the violation of the local equivalence principle. In the rest of the section we review the Bekenstein-Sandvik-Barrow-Magueijo models, the string inspired models with a runaway dilaton and three classes of models with extra dimensions. A discussion of the comparison of the theoretical predictions of these models with the observational data is also presented there. Finally, in Sect. 4 we elaborate our conclusions.

2 Observational results

For more than six decades physicists and astronomers have been interested in devising new methods and techniques to detect and measure any hypothetical time variation of fundamental constants or, at least, to obtain upper bounds on their variation. A comprehensive review of the observational results and techniques used for such observations can be found in Ref. [1], here we briefly summarize the main results.

2.1 Variation of α

The recent interest in the variation of fundamental constants is partially motivated by the results of the astrophysical observations of Ref. [2, 3], who claim to have detected a temporal variation of the fine structure constant. To estimate the value of α in the Early Universe they analyzed the fine doublet splitting from distant quasar absorption systems using the alcali doublet (AD) method first applied in [7]. The idea of the method is the following. As it is well known, in the leading approximation the fine splitting $\Delta\lambda = \lambda_1 - \lambda_2$ of two spectral lines of wavelength λ_1 and λ_2 is given by

$$\frac{\Delta\lambda}{\lambda} = A(Z\alpha)^2 + \mathcal{O}(\alpha)^4, \qquad (1)$$

where $\lambda = (\lambda_1 + \lambda_2)/2$ is the mean wavelength of the doublet, Z is the atomic number and A is a constant. The remarkable feature is that A does not involve other fundamental parameters, like the Planck constant or nucleon and electron masses. Furthermore, it is assumed that the Universe is the Friedmann-Robertson-Walker universe with the metric given by

$$ds^2 = -dt^2 + a(t)^2 \sum_{i,j=1}^{3} \gamma_{ij} dx^i dx^j, \qquad (2)$$

where t is the cosmological time. Let us denote the present value of t by t_0. The redshift z corresponding to the moment t is defined by the ratio of the wavelength observed now and the wavelength at that moment, and is related to the ratio of the values of scale factor as $1+z = \lambda^{\rm obs}/\lambda^{\rm em} = a_0/a(t)$, where $a_0 \equiv a(t_0)$. The corresponding look-back time is $t_z = t_0 - t$. For example, for a matter dominated Universe with the mass density $\Omega_{\rm M} = 1$ we have $a(t) \propto t^{2/3}$ and

$$t_z = t_{\rm U}\left[1 - \frac{1}{(1+z)^{3/2}}\right] \approx H_0^{-1} z + \mathcal{O}(z^2).$$

where the age of the Universe is $t_{\rm U} = 2/(3H_0) \approx 14$ Gyr. Furthermore, it is supposed that quasars have a cosmological origin and, hence, that their spectra were formed at an epoch corresponding to redshift z. The absorption resonance lines of the ions CIV, MgII, SiIV, and others are observed in clouds at the redshift z and the corresponding wavelengths of the doublets are measured. Hence, $\lambda_i^{\rm obs}(z) = \lambda_i^{\rm em}(\alpha(z))(1+z), \quad i = 1,2$, where we have indicated explicitely the possible dependence of the fine structure constant on z. Then the observed fine doublet splitting ratio of the system at the redshift z is equal to

$$\left(\frac{\Delta\lambda}{\lambda}\right)_z \equiv \frac{\lambda_2^{\rm obs}(z) - \lambda_1^{\rm obs}(z)}{\lambda^{\rm obs}(z)} = \frac{\lambda_2^{\rm em}(\alpha(z)) - \lambda_1^{\rm em}(\alpha(z))}{\lambda^{\rm em}(\alpha(z))} \approx A(Z\alpha)^2.$$

Denoting the same ratio measured at the laboratory now as $(\Delta\lambda/\lambda)_0$ one gets

$$\frac{\left(\frac{\Delta\lambda}{\lambda}\right)_z}{\left(\frac{\Delta\lambda}{\lambda}\right)_0} - 1 \approx 2\frac{\alpha(z) - \alpha(0)}{\alpha(0)}.$$

and, consequently, the time variation of the fine structure constant in terms of the doublet splitting ratios measured from observations of a distant absorption system and the same system in the laboratory in the leading approximation is given by:

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(z) - \alpha(0)}{\alpha(0)} \approx \frac{1}{2}\left[\frac{\left(\frac{\Delta\lambda}{\lambda}\right)_z}{\left(\frac{\Delta\lambda}{\lambda}\right)_0} - 1\right]. \tag{3}$$

This method has been extended to the more precise many-multiplet method which is most often used in the observations nowadays. Three sets of data has been analyzed so far:

- A sample of 30 low-z quasar absorption Mg/Fe systems, which already gave indications of a non-negligible time variation of α [8].
- Another sample of 49 additional absorption systems. This sample allowed to measure a non-zero $\Delta\alpha/\alpha$ at the 4.1σ confidence level for redshifts $0.5 < z < 3.5$ [9].
- A big sample of 128 new absorption systems. This sample gave the most precise determination of $\Delta\alpha/\alpha = (-0.57 \pm 0.10) \times 10^{-5}$ at 5.7σ, averaged over $0.2 < z < 3.7$ — see Fig. 1, and Ref. [9].

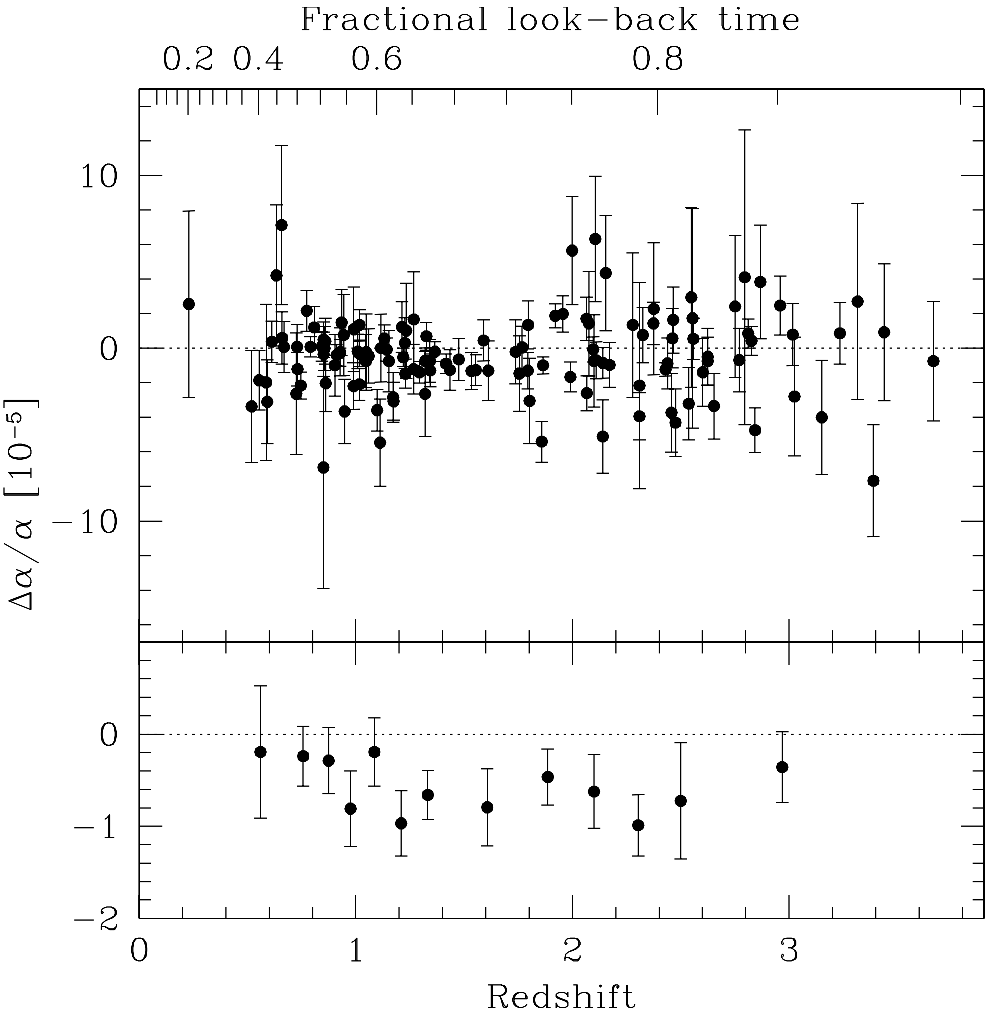

Figure 1: $\Delta\alpha/\alpha$ for 128 absorption systems with 1σ errors, from Ref. [9].

Thus, these results provide convincing evidence that α was *smaller* in the past. It is worth noting that all this amount of data supports the idea that this is not just an upper bound but a direct measurement of the rate of change of α. Possible systematic effects are very difficult to estimate [10]. However, a careful analysis of systematic errors [11] confirms that there are no systematic effects which can easily mimic a negative value of $\Delta\alpha/\alpha$ comparable to that found, and that the known systematic effects could lead only to more significant deviations of $\Delta\alpha/\alpha$ from zero. However, whether this detection is genuine or it is affected by systematic errors is still a matter of debate [12].

Other observations provide only upper bounds on the variation of α and do not exclude that its variation could be zero. One of the most stringent bounds is given by the Oklo natural nuclear reactor. It is a prehistorical natural fission reactor that operated about $t_z \approx 2 \times 10^9$ years ago ($z \approx 0.15$) during $(2.3 \pm 0.7) \times 10^5$ yr in the Oklo uranium mine in Gabon, West Africa. This phenomenon, first discovered by the French Comissariat à l'Énergie Atomique in 1972, consists in an anormally low relative concentration of the ^{149}Sm isotope with respect to ^{150}Sm. The isotopic ratio turns out to be $\eta_{\rm O} \approx 0.02$ whereas the normal one is $\eta_{\rm N} \approx 0.9$. A possible explanation is a different value of the capture resonance energy at the time of the reaction. From this the following bound on the variation of α can be derived [13]:

$$-0.9 \times 10^{-7} < \frac{\Delta\alpha}{\alpha} < 1.2 \times 10^{-7} \text{ at } z \approx 0.15$$

This estimate, however, involves a number of assumptions about, for instance, the temperature of neutrons, the original abundances of the different isotopes, and about the dependence of nuclear energy on α, that are difficult to control. Similar values are obtained from the so-called "rhenium constraint" which relies on the fact that the β-decay energy is sensitive to the variation of α. The analysis of the Re/Os ratio in $t_z = 4.6$ Gyr old ($z \approx 0.45$) iron-rich meteorites gives [14]:

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 3 \times 10^{-70}.$$

The reader should be aware that there is no direct contradiction between the local (low-z) constraints and the high-z data since they correspond to very different cosmological times. We also would like to mention that the local data do not give a direct measurement of $\Delta\alpha/\alpha$, but just upper bounds, and that in the local data many (but otherwise plausible) assumptions and parameters difficult to control are involved.

Additional high-z constraints come also from observations of the cosmic microwave background radiation (CMB) composed of photons emitted at the time of recombination of H and He when the Universe became transparent for the radiation at $z \sim 10^3$ or $t \approx 300$ Myr. A variation of α modifies the power spectrum of density fluctuations, and consequently the height of the first acoustic peak of the CMB anisotropy spectrum. Observations of COBE, BOOMERANG and DASI [15] yield:

$$\frac{\Delta\alpha}{\alpha} = -0.005^{+0.07}_{-0.04} \text{ at the } 68\% \text{ C.L.}$$

Big Bang Nucleosynthesis (BBN) also provides bounds on the variation of the fine structure constant at $z \sim 10^{10}$. The standard model of BBN is very sensitive to the abundances of light elements like ^{2}H, ^{4}He, ^{7}Li. The ^{4}He mass fraction is given by:

$$Y_\mathrm{p} = 2\frac{1}{1 + \left(\frac{n_\mathrm{p}}{n_\mathrm{n}}\right)_\mathrm{f}} = \frac{2}{1 + e^{\Delta m/T_\mathrm{f}}},$$

where $\Delta m \equiv m_\mathrm{n} - m_\mathrm{p} \approx 1.29$ MeV is the neutron-proton mass difference anf T_f is the freeze-out temperature. Using the Cottingham formula $\Delta m \approx (1.29 - 0.76(\Delta\alpha/\alpha))$MeV and the experimental value $Y_\mathrm{p} = 0.244(2)$ the bound

$$\frac{\Delta\alpha}{\alpha} = (-7 \pm 9) \times 10^{-3}$$

was derived [16]. The combined CMB and BBN likelihood analysis is presented in Fig. 2. These results do not support evidence for variation of α at more than 1σ level at either epoch, they also show no direct contradiction with the quasar data.

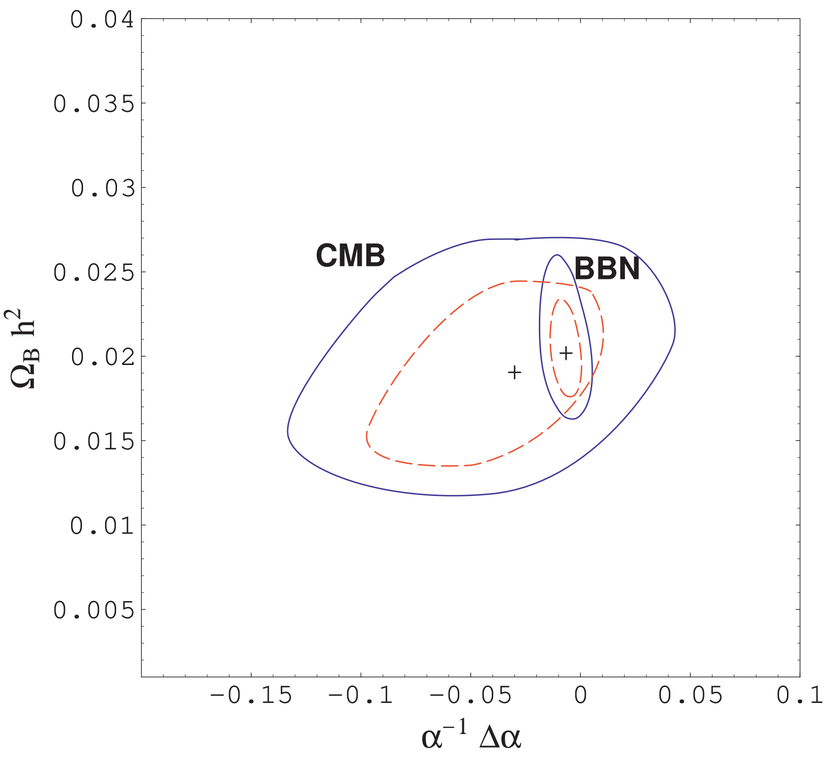

Figure 2: CMB and BBN likelihood analysis. The curves represent 95% C.L. (solid line) and 68% C.L. Crosses correspond to the maxima of the likelihood functions. Adapted from [15].

2.2 Variation of G

There is a number of observational constraints on the time variation of the gravitational Newton constant G. Below we list some of them.

1. Paleonthological and geophysical arguments [17] show that the temperature on the Earth is in the acceptable range only if $|\Delta G/G| < 0.1$.

2. Lunar Laser ranging gives $|\dot{G}/G| \lesssim 3 \times 10^{-11}$ yr^{-1}.

3. Observations of the period of pulsars yield even stronger bounds of the rate of variation of G. For instance, examining the data of the pulsar PSR 1913+16 [18, 19, 20] it was concluded that $\dot{G}/G = (4 \pm 5) \times 10^{-12}$ yr^{-1}.

4. Any possible variation of G leads to a modification of the main sequence evolutionary time scale of stars in globular clusters. Using this constraints the rate of variation of G have been obtained by several authors. For example, in Ref. [21] it was obtained that $\dot{G}/G = (-1.4 \pm 2.1) \times 10^{-11}$ yr^{-1}, whereas in Ref. [22] a negative rate of change was obtained: $0 > \dot{G}/G > -4 \times 10^{-11}$ yr^{-1}.

5. Another stellar constraint comes from observations of the cooling of white dwarfs. Using the fact that the variation of G causes an additional release of gravitational

energy which modifies the energy balance in the star in [23] a tight constraint, $-(3^{+1}_{-3}) \times 10^{-11}$ yr$^{-1} < \dot{G}/G < 0$, was obtained.

6. Helioseismological data also provide a constraint on $\dot{G}/G$. In particular, the sensitivity of the periods of solar oscillations to the He/H ratio in the interior of the Sun, which depends on the precise value of G provides the current best upper bound. In [24] an upper limit $\dot{G}/G \leq -1.6 \times 10^{-12}$ yr^{-1} was obtained.

In summary, several constraints to the *local* rate of change of the gravitational constant have been obtained up to now using very different methods. However, we emphasize that all these upper bounds are local. Let us discuss now the constraints obtained from the analysis of the Hubble diagram of type Ia Supernovae (SNIa). Observations carried out by the High-z Supernova Search Team [25] and the Supernova Cosmology Project [26] indicate that distant SNIa appear to be *dimmer* than local ones. The explosion of a thermonuclear supernova is described by the Chandrasekhar mass model which, in particular, predicts that peak luminosity is proportional to the Chandrasekhar mass $M_{\rm Ch} = (\hbar c)^{3/2} m_{\rm p}^{-1} G^{-3/2}$. The apparent magnitude is then given by

$$m(z) = M_0 + \frac{15}{4} \log \frac{G(z)}{G_0} + 5 \ln d_L(z; \Omega_{\rm M}, \Omega_\Lambda; G) + 25,$$

where M_0 is the (intrinsic) absolute magnitude of SNIa, d_L is the luminosity distance and G_0 is the present day value of the gravitational constant. A variation of G affects both the luminosity distance and the Chandrasekhar mass. In Ref. [27] it was argued that the former dependence is small and can be ommitted, whereas the latter one was taken into account within a generalized Brans-Dicke model. Moreover, using the 2σ confidence contours for $z = 0.5$ obtained from the fit to the Hubble diagram of SNIa obtained in [27] bounds on $\dot{G}/G$ have been computed. The bounds depend on the cosmological scenario. For example, for the currently favoured cosmological scenario $(\Omega_{\rm M}, \Omega_\Lambda) = (0.3, 0.7)$ and for the flat matter-dominated Universe with $(\Omega_{\rm M}, \Omega_\Lambda) = (1.0, 0.0)$ the following estimates [28]:

$$-1.4 \times 10^{-11}\ \mathrm{yr}^{-1} < \frac{\dot{G}}{G} < +2.6 \times 10^{-11}\ \mathrm{yr}^{-1}$$

and

$$-2.9 \times 10^{-11}\ \mathrm{yr}^{-1} < \frac{\dot{G}}{G} < -0.3 \times 10^{-11}\ \mathrm{yr}^{-1}$$

were respectively obtained.

2.3 Variation of μ.

For electron-vibro-rotational lines the wavelength λ depends on the reduced mass of the molecule and therefore its variation is related to the variation of μ:

$$\Delta\lambda = \lambda K \frac{\Delta\mu}{\mu}.$$

Here K is a sensitivity coefficient which was calculated theoretically for some molecules, for example for H_2. Thus

$$\lambda^{\mathrm{obs}}(z) = \lambda^{\mathrm{em}}(1+z)\left(1 + K\frac{\Delta\mu}{\mu}\right).$$

Observing absorption lines from quasars one can determine z and $\Delta\mu/\mu$. In Ref. [29] it was obtained that

$$\frac{\Delta\mu}{\mu} = (5.02 \pm 1.82) \times 10^{-5} \quad \text{at} \quad z \simeq 3 \tag{4}$$

3 Theoretical models

3.1 General features

As we have already discussed there are nowadays a few puzzling astrophysical observations which challenge theorists. Clearly, one of them is the variation of the fine structure constant at $z \gtrsim 1$ discussed in the previous section. The question now is whether or not this experimental result can really be interpreted as a sign for the need of new physics beyond the Standard Model. Within an efective four-dimensional field theory the only consistent way to make Lagrangian parameters time dependent is through promoting them into functions of a dynamical scalar field $\phi(x)$. Then α, G, and other effective couplings should be understood as functions $\alpha(\phi)$, $G(\phi)$, respectively. In the string theory ϕ is a dilaton field, or a moduli field. Then, the constant value of, say, the fine structure constant, can be considered as $\alpha_0 \equiv \alpha(\phi)|_{\phi=\langle\phi\rangle}$, where $\langle\phi\rangle$ is the vacuum expectation value of the scalar field. It turns out that string models and theories with extra dimensions contain built-in mechanisms for the variation of (effective) constants in four dimensions. In the vicinity of $\phi = \langle\phi\rangle$ the function $\alpha(\phi)$ can be written as $\alpha(\phi) = \alpha_0 + \lambda\varphi/M_{\mathrm{Pl}}$, where $\varphi = \phi - \langle\phi\rangle$ and λ is some constant. Then, the variation $\Delta\alpha$ is related to the corresponding variation of the scalar field as

$$\frac{\Delta\alpha}{\alpha} = \frac{\lambda}{\alpha_0}\frac{\Delta\phi}{M_{\mathrm{Pl}}} \tag{5}$$

Such models have three general features [30]. Firstly, the mass of the scalar field driving the change of α is tiny: $m_\varphi \sim H_0 \sim 10^{-33}$ eV. Secondly, since from the quasar observations $\Delta\alpha/\alpha \sim -10^{-5}$ from Eq. (5) one gets that

$$\left|\lambda\frac{\Delta\varphi}{M_{\mathrm{Pl}}}\right| \sim 10^{-7} \quad \text{within} \quad \Delta t \sim H_0^{-1}. \tag{6}$$

Finally, electromagnetic corrections to the nucleon masses:

$$\delta m_{\mathrm{p}}(\varphi) = B_{\mathrm{p}}\alpha(\varphi) \approx 0.63 \text{ MeV}, \quad \delta m_{\mathrm{n}}(\varphi) = B_{\mathrm{n}}\alpha(\varphi) \approx -0.13 \text{ MeV}$$

induce a new force of long range $r \sim m_\varphi^{-1} \sim H_0^{-1}$ which is isotope-dependent. The corresponding term in the Lagrangian in the leading order is

$$\delta L = \sum_{\mathrm{N=n,p}} \delta m_{\mathrm{N}}(0)\bar{\psi}_{\mathrm{N}}\psi_{\mathrm{N}} + \sum_{\mathrm{N=n,p}} \frac{B_{\mathrm{N}}\lambda}{M_{\mathrm{Pl}}}\varphi\bar{\psi}_{\mathrm{N}}\psi_{\mathrm{N}}\ldots$$

This leads to a violation of the equivalence principle and may be detected in tests of the universality of free fall. Indeed, the acceleration of a test body in this case is given by the sum of the standard gravity acceleration and that caused by the additional force, $a = a + a_\phi$. Therefore, the Eötvös ratio for two test bodies with accelerations a_1 and a_2 is

$$\eta \equiv 2\left|\frac{a_1 - a_2}{a_1 + a_2}\right| \approx \frac{a_{\phi 1} - a_{\phi 2}}{a} \approx 10^{-3}\lambda^2.$$

The recent experimental bound $\eta < 10^{-13}$ implies that the constant λ must be smaller then 10^{-5}. Combining this bound with Eq. (6) and assuming that the variation of the scalar field $\Delta\phi \lesssim M_{\mathrm{Pl}}$ one gets $10^{-7} \lesssim \lambda < 10^{-5}$. Therefore, the new interaction is likely to be detected either through astronomical observations or from planned Eötvös type experiments which will improve the bound on η till $\eta \sim 10^{-15}$ (MICROSCOPE experiment) or even $\eta \sim 10^{-18}$ (STEP).

3.2 The Bekenstein-Sandvik-Barrow-Magueijo model

The Bekenstein-Sandvik-Barrow-Magueijo model [31, 32, 33] is a generalization of the Brans-Dicke model. It includes two scalar fields, the Brans-Dicke field ϕ and the "dielectric" field ψ, and is described by the action

$$\begin{aligned} S &= \int d^4x\sqrt{-\gamma}\left[\phi\mathcal{R} - w_{\mathrm{BD}}\frac{\phi_{;\mu}\phi^{;\mu}}{\phi}\right.\\ &+ \left. 16\pi\left(L_{\mathrm{m}} + e^{-4\psi}L_{\mathrm{em}} - \frac{w}{2}\psi_{;\mu}\psi^{;\mu}\right)\right], \end{aligned}$$

where $\mathcal{R}$ is the scalar curvature and L_{m} and L_{em} are the matter and electromagnetic Lagrangians respectively. Then the gravitational and fine structure coulpings are given by $G \propto 1/\phi, \quad \alpha = \alpha_0 e^{2\psi}$. The scale of new physics is set by w satisfying the bounds $10\ \mathrm{MeV} \lesssim w \lesssim M_{\mathrm{Pl}}$. In Ref. [31] cosmological solutions for different epochs were found. In particular, it was shown that for the dust era the product $G\alpha$ remains constant. Also, the no-oscillatory theorem stating that in this model $\alpha(\phi(t))$ cannot display oscillatory behaviour in time in a Friedmann universe of any curvature was proven. The scenario complies with the Oklo constraint and is consistent with the $\Delta\alpha/\alpha \neq 0$ result inferred from observations of quasars.

3.3 String inspired models and GUTs

Within the theoretical framework of string models it is shown that the low-energy action which includes string-loop effects is of the form

$$S = \int d^4x\sqrt{-\gamma}\left[B_g(\phi)\mathcal{R} - B_\phi(\phi)(\nabla\phi)^2 - \frac{1}{4}B_F(\phi)F^2 + \ldots\right],$$

where ϕ is the dilaton field and the coefficient functions behave as

$$B_i(\phi) = e^{-\phi} + a_0^{(i)} + a_1^{(i)} e^{\phi} + a_2^{(i)} e^{2\phi} + \ldots$$

for $g_s^2 = e^{\phi} \to 0$. Here $i = g,\ \phi,\ F$. It has been shown [34] that at certain values $\phi = \phi_{\rm m}$, which are fixed points of the theory, the dilaton decouples from the matter. One of this fixed points [35] can be $\phi_{\rm m} = +\infty$ (the so-called runaway dilaton). In this case $B_i(\phi)|_{\phi\to+\infty} = C_i + \mathcal{O}\left(e^{-\phi}\right) \to C_i$. In this approach all couplings (including, for instance, α and G) are related to *the same* scalar field ϕ. Thus, in the Einstein frame, where we denote the scalar field as φ, the fine structure coupling and the hadronic coupling are given by

$$\begin{aligned} \alpha(\varphi) &= \alpha(\infty)\left[1 - b_F e^{-\kappa\varphi}\right], \\ \alpha_{\rm h}(\varphi) &\approx 40 b_F e^{-\kappa\varphi}, \end{aligned}$$

where b_F is some constant. Then, the Eötvös ratio for two test bodies is equal to $\eta \approx 5.2 \times 10^{-5}\alpha_{\rm h}^2 \lesssim 10^{-12}$, and it is related to the variation of the fine structure constant by the expression

$$\left|\frac{\Delta\alpha}{\alpha}\right| \approx 3.5 \times 10^{-6}\sqrt{10^{12}\eta}\langle\varphi'\rangle_z \ln(1+z), \tag{7}$$

where $\langle\varphi'\rangle_z$ is the averaged change of the field φ during the period of evolution. After imposing the Oklo and Rhenium constraints Eq. (7) results in

$$\left|\frac{\Delta\alpha}{\alpha}\right| \approx 0.7 \times 10^{-6}\ln(1+z).$$

If no special fine tuning is made the above mentioned constraint is valid for all z, but it is 10 times smaller than the result obtained from quasar observations. Hence, there is no *natural* explanation of the this result within the class of string inspired models under consideration. Conversely, if one imposes the observed value of the fine structure variation in Eq. (7), then the Eötvös ratio η violates the existing experimental bound $\eta \approx 10^{-13}$.

We see that in string inspired models the variations of different couplings are given by the change of the same dilaton field ϕ. Thus, they predict correlated variations of various couplings and, consequently, this correlation can potentially give insights into the underlying theory. It turns out that the correlation of the variations of constants is a rather generic feature which takes place, for example, in theories of Grand Unification and models with extra dimensions. Thus, in the $SU(5)$ GUT the three couplings of the Standard Model $\alpha_1, \alpha_2, \alpha_3$ at the scale $M_Z \approx 100$ GeV are related to the unique fundamental constant $\alpha_{\rm GUT}$ by the following 1-loop renormalization group relations:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{\rm GUT}} + \frac{b_i}{2\pi}\ln\frac{M_Z}{M_{\rm GUT}}, \quad i = 1, 2, 3, \tag{8}$$

where $M_{\rm GUT} \approx 3 \times 10^{16}$ GeV is the scale of Grand Unification, and b_i are some constants calculated in this theory. In particular, the fine structure constant at the electroweak scale is given by

$$\alpha^{-1}(M_Z) = \frac{5}{3}\alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z) \approx 127.9$$

If the GUT constant $\alpha_{\rm GUT}$ varies with time then the couplings of the Standard Model experience corresponding correlated variations. Such scenario with the scales M_Z and $M_{\rm GUT}$ kept constant was studied in Ref. [36], where it was shown that

$$\frac{\Delta\alpha}{\alpha} \approx 0.49\frac{\Delta\alpha_{\rm GUT}}{\alpha_{\rm GUT}}, \quad \frac{\Delta\alpha_{\rm strong}}{\alpha_{\rm strong}} \equiv \frac{\Delta\alpha_3}{\alpha_3} \approx 5.8\frac{\Delta\alpha}{\alpha}, \quad \frac{\Delta\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} \approx 34\frac{\Delta\alpha}{\alpha}.$$

3.4 Theories with extra dimensions

There is a large amount of work on the variation of fundamental constants within the multidimensional approach, starting with [37, 38, 39, 40]. Within this approach the interactions are described by a fundamental theory formulated in the $(4+d)$-dimensional space-time with d compact extra dimensions and the metric of the form

$$ds^2 = -dt^2 + a^2(t)\sum_{i,j=1}^{3}\hat{\gamma}_{ij}dx^i dx^j + R^2(t)\sum_{m,n=4}^{d+3}\hat{\gamma}_{mn}dy^m dy^n.$$

The four-dimensional theory appears as a result of the dimensional reduction. Parameters in four dimensions are determined by a set of a few new constants of the multidimensional theory and the size $R(t)$ of the space of extra dimensions (see Refs. [41, 42] for reviews). The multidimensional constants are assumed to be genuinely fundamental and, consequently, do not vary with time. On the other hand, in the cosmological or astrophysical context it is natural to assume that $R(t)$ varies with time, very much in the same way that the scale factor $a(t)$ of the three-dimensional space does. Dynamical solutions for $a(t)$ and $R(t)$ are determined by the parameters of a cosmological scenario of the evolution of the Universe. The change of the scale factors with t gives rise to the time variation of parameters of the effective four-dimensional theory, like the gravitational constant G and of the fine structure constant α which are correlated between themselves. We will discuss the correlated variations of G and α in three types of models with extra dimensions following Ref. [28].

3.4.1 Kaluza-Klein theories

Kaluza and Klein [43, 44] formulated the essential elements of the multidimensional approach to the description of fundamental interactions which was later called the Kaluza-Klein approach. They considered the pure Einstein gravity in five-dimensional space time $M^4 \times S^1$ and showed that the sector of zero modes of the dimensionally reduced theory includes the four-dimensional gravity and Maxwell theory with the electromagnetic potential given by the (4μ)-components of the multidimensional metric tensor $\hat{\gamma}_{MN}$. Later this construction was generalized [45, 46, 47] to more complicated compact spaces of extra dimensions K_d. In this case the reduced theory contains the Einstein gravity and Yang-Mills fields with the gauge group determined by the isometry group of K_d. The action of the multidimensional theory is given by

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{\gamma}}\frac{1}{16\pi G_{(4+d)}}\mathcal{R}^{(4+d)}, \tag{9}$$

where $\mathcal{R}^{(4+d)}$ is the scalar curvature in the $(4+d)$-dimensional space-time and $G_{(4+d)}$ is the multidimensional gravitational parameter which is assumed to be constant.

To obtain the four-dimensional effective theory, the $\mu\nu$-components of the metric tensor $g_{\mu\nu}$ are first identified as the four-dimensional metric tensor. In a second step, certain combinations of the rest of the components are identified as the gauge and scalar fields. Later on, the mode expansion of all these fields is performed. The coefficients of the expansion only depend on x^μ and are interpreted as four-dimensional fields. In general there is an infinite number of them. Here we are interested in the sector corresponding to the leading terms of the mode expansion. Its action is given by

$$S_{\rm LO} = \int d^4x\sqrt{-\gamma}\left[\frac{1}{16\pi G}\mathcal{R}^{(4)} + \sum_i \frac{1}{4g_i^2}TrF^{(i)}_{\mu\nu}F^{(i)\mu\nu}\right], \tag{10}$$

where $G \equiv G_{(4)}$ is the four-dimensional gravitational constant. The $g_i \equiv g_{(4)i}$ are the gauge couplings, and the index i labels the simple subgroups of the gauge group. The scalar fields usually give highly non-linear interaction terms and are coupled non-minimally to the gravitational and gauge fields. They are supposed to be frozen out and their contribution is neglected. A simple calculation gives the following expressions for the couplings of the four-dimensional theory in terms of $G_{(4+d)}$:

$$G = \frac{G_{(4+d)}}{V_d}, \tag{11}$$

$$g^2 = \kappa_i\frac{G_{(4+d)}}{R^2V_d}, \tag{12}$$

where $V_d(t) \propto R^d(t)$ is the volume of the space of extra dimensions and κ_i are coefficients which depend on the isometry group of K_d. We assume that the dimensionally reduced theory includes the electrodynamics. The fine structure constant α is given by a linear combination of g_i^2, the specific relation depending on the model, namely on the gauge group and the scheme of the spontaneous symmetry breaking. It is easy to show using Eq. (12) that the time variation of the fine structure and of the gravitational constants are related by

$$\frac{\dot{\alpha}}{\alpha} = \frac{d+2}{d}\frac{\dot{G}}{G}. \tag{13}$$

3.4.2 Einstein-Yang-Mills theories

These are theories on the $(4+d)$-dimensional space-time $M_{(4)} \times K_d$ that include gravity and the Yang-Mills field with the action

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{\gamma}}\left[\frac{1}{16\pi G_{(4+d)}}\mathcal{R}^{(4+d)} + \frac{1}{4g^2_{(4+d)}}Tr\hat{F}_{MN}\hat{F}^{MN}\right], \tag{14}$$

where, as above, $G_{(4+d)}$ is the multidimensional gravitational constant, and $g_{(4+d)}$ is the multidimensional gauge coupling. Both are supposed to be constants in time. The dimensionally reduced theory includes the Einstein gravity, the four-dimensional gauge fields and scalar fields with the action of type (10). The explicit form of the dimensionally reduced theory depends on the topology and geometry of the space of extra dimensions and the multidimensional gauge group. The case of K_d being a homogeneous space was studied in detail in the literature [48, 49]. The four dimensional couplings are given by

$$\begin{aligned} G &= \frac{G_{(4+d)}}{V_d}, & (15) \\ \alpha &= \frac{\alpha_{(4+d)}}{V_d}. & (16) \end{aligned}$$

From these expressions we obtain the following relation between the time variations of G and α:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{G}}{G}. \tag{17}$$

3.4.3 Randall-Sundrum-type models with gauge fields in the bulk.

Another multidimensional setting motivated by string and M-theories was proposed and studied in [50, 51]. The model is formulated in the five-dimensional space $M^4 \times S^1/Z_2$ with the space of extra dimensions compactified to the orbifold S^1/Z_2. There are two 3-branes located at the fixed points of the orbifold, which play the role of the macroscopic three-dimensional spaces. In the initial version of the model only gravity propagates in the five-dimensional bulk, whereas the fields of the Standard Model are localized on the branes [51]. If the tensions of the branes are fine tuned to be equal in absolute value, but of the opposite sign, then the model possesses a background solution for the metric with the exponential warp factor. Due to this feature the model provides an elegant geometric solution to the hierarchy problem and predicts new interesting physical effects which may be observed at the current and future collider experiments. This assumes that the visible brane, i.e. the brane we live on, is the negative tension brane. The form of the relation between G and $G_{(4+d)}$ depends on the coordinates on this brane. We argue that the relationship which gives the correct formula for the time variation of G is obtained in the Galilean coordinates on the visible brane [52] and is given by

$$G = \frac{k}{16\pi M^3} \frac{1}{e^{2k\pi R(t)} - 1} \approx \frac{k}{16\pi M^3} e^{-2k\pi R(t)}, \tag{18}$$

where M is the fundamental mass scale and k is a parameter of the dimension of mass. It is related to the tension of the branes and the five-dimensional cosmological constant. In Eq. (18) we have taken into account that to generate the correct hierarchy between the Planck scale and the TeV-scale the product kR must be $kR_0 \approx 11 \div 12$. The time variation of the Newton constant is given by

$$\frac{\dot{G}}{G} = -2\pi kR \frac{\dot{R}}{R} \frac{1}{1 - e^{-2k\pi R}} \approx -2\pi kR \frac{\dot{R}}{R} \tag{19}$$

However, since the fields of the Standard Model are localized on the brane and do not depend on R this scenario does not explain the variation of the fine structure constant. To describe this effect one has to consider bulk gauge and fermionic fields. Such a model has been studied in a number of papers, see for example [53]. Simple calculations show that

$$\alpha \propto \frac{g^2_{(5)}}{R}, \tag{20}$$

From Eqs. (19) and (20) we get the following relation:

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{2\pi kR}\frac{\dot{G}}{G}\frac{1}{1-e^{-2k\pi R}} \approx \frac{1}{2\pi kR}\frac{\dot{G}}{G}. \tag{21}$$

3.4.4 Comparison with observations

We have seen that within the three classes of models with extra dimensions the variability of $R(t)$ implies the *correlated variations* of α and G. Let us use the detected variation of the fine structure constant discussed above to set some constraint on the variation of G. The multidimensional models analyzed above predict that the gravitational constant G was *smaller* in the past and, therefore, $\dot{G}/G$ should be *positive*. Assuming for the sake of simplicity a constant rate of variation and using a typical age of the Universe of $\sim$ 14 Gyr, we get the estimate $\dot{G}/G \sim +10^{-15}$ yr^{-1} for the Kaluza-Klein and the Einstein-Yang-Mills theories, whereas it is a factor of 10^2 larger for the Randall-Sundrum scenario. Let us note now that the existing bounds on the variation of G (see Sect. 2) — regardless if they are local or they are obtained at moderately high redshifts — are negative and, consequently, positive values of $\dot{G}/G$ seem to be not allowed by the present astrophysical data. To be more precise, for models within the preferred scenario, namely $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$, a *positive* value of $\dot{G}/G$ makes distant supernovae to appear *brighter*. This effect is just the opposite found observationally. However, the effect of a varying G at this rate is indistinguishable from $\dot{G}/G = 0$ within the experimental errors.

3.5 Final remarks

We have seen that there are a number of generalizations of the Standard Model which contain mechanisms of variation of fundamental constants. This can be viewed as another outcome of the interface between cosmology and particle physics. The variation of the fine structure constant induces the violation of the Universality of Free Fall which can be detected by precision tests. Either a positive or a negative result of such detection will give additional information about types of mechanisms and models beyond the Standard Model.

We also would like to mention the arguments presented in Ref. [54]. There it is assumed that the variation of α is modelled by coupling a dynamical scalar field ϕ with a potential $V(\phi)$ to $F^2_{\mu\nu}$. A variation $\delta\phi$ around a ground state value ϕ_0 generates not only the variation of the fine structure constant, but also a corresponding change of the potential $\delta V(\phi) = V_0'\delta\phi + \frac{1}{2}V_0''(\delta\phi)^2 + \ldots$, where it is supposed that the cosmological constant $\Lambda_{\rm C} = V(\phi_0) = 0$. The variation of the potential is interpreted as the variation of the cosmological constant. By simple and rather general arguments it can be shown that this variation is related to the variation of α as $\delta\Lambda_{\rm C} = \delta V = C\delta\alpha\Lambda^4$, where C is a constant and Λ is a typical physical scale, for example $\Lambda = \Lambda_{\rm QCD} \sim 100$ MeV. The variation of the potential cannot exceed the energy density ρ_{m*} in the Universe at the time of galaxy formation, when the quasar light was emitted. This gives the condition

$$\delta V = \left| C\frac{\delta\alpha}{\alpha} \right| \times 10^{29}\ {\rm eV}^4 < \rho_{m*} \sim 10^{-8}\ {\rm eV}^4$$

or

$$\left| C\frac{\delta\alpha}{\alpha} \right| \lesssim 10^{-37}.$$

To describe the variation $|\delta\alpha/\alpha| \sim 10^{-5}$ an extreme fine tuning is needed. The problem is essentially the huge back-reaction produced by varying couplings on the vacuum energy. This difficulty could be intimately related to the long-standing cosmological constant problem. Hence, its satisfactory solution could also provide a mechanism to suppress the enourmous variation of the vacuum energy due to the time variation of ϕ. However, consistent and "natural" quantative descriptions of this sort of new physics are still missing and represent a big challenge for theorists.

4 Conclusions

We have reviewed the case of a hypothetical variation of the fundamental constants. Along the first section of this review we have assembled some of the relevant observational work done in the field so far. In particular, we have paid special attention to the most recent astrophysical methods used to determine — or, at least, to set upper bounds to — the rate of variation of effective couplings. For this purpose we have focused on the three best studied cases: the case of the fine structure constant, that of the gravitational constant and, finally, that of the proton-to-electron mass ratio. For all three there are several independent constraints to their rate of change either coming from local environments or at cosmological scales. We have argued that although the local constraints turn out to be very tight, the bounds at cosmological distances are rather loose. Moreover, some recent observational works suggest that indeed the fine structure constant was smaller in the past. Although this is still the subject of a strong debate it is important to realize that if the reported variations of α and μ are finally confirmed, then they would be a clear indication of the need of new physics. This is precisely the reason why the second section of this review was devoted to summarize the current theoretical approaches beyond the Standard Model which provide a natural way to recover a variation of the effective couplings. In particular we have elaborated on several models, like the Bekenstein-Sandvik-Barrow-Magueijo model, some string inspired models and some models with extra dimensions. All of them predict correlations in the variations of the couplings. Hence, correlations in the variations of constants, if detected, would be a valuable piece of information for building theories beyond the Standard Model, if needed. Moreover, one should bear in mind that the interpretation of the observational data on the variation of the fundamental constants depends on the underlying theoretical layout and is based on a number of assumptions that should be eventually confronted with observations. Clearly, the last word on this subject has not been told yet and, therefore, more observational and theoretical efforts are needed.

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