Semiphenomenological model of unstable particles

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Abstract

A model of unstable particles based on uncertainty principle is discussed in quantum field approach. We show that the simplest quantum field description of mass uncertainty makes it possible to account for instability effects for particles with large decay widths.

1 Introduction

Quantum field description of unstable particles (UP) has some peculiarities connected with finite lifetime due to decay channels. The history of quantum field approach starts from 50-s (20 century) [1] - [8].

At that time two directions of investigations were founded - "propagator" and "mass smearing" approaches. The first approach was developed better, and now is a standard way of investigations [9] - [11]. The UP in intermediate states is described by renormalized propagator with self-energy insertion [5] - [8]. The position of pole $s_p = M_p^2 - iM_p\Gamma_p$ determines the mass M_p and width Γ_p of UP.

The second approach was formulated by Matthews and Salam [3, 4]. It is based on the uncertainty relation for unstable quantum systems, which leads to uncertainty of the mass of unstable particle or, so called, "mass smearing" effect:

$$\delta m\,\delta\tau \sim 1 \quad \longrightarrow \quad \delta m \sim \Gamma_{tot} \;\; (c = \hbar = 1) \tag{1}$$

In this work we discuss a simple model based on the uncertainty principle.

2 A model of unstable particles

Proposed model of UP is the simplest description of "mass smearing" effect in the frame of quantum field approach. Field operator function is superposition of ordinary ones weighted by $\omega(\mu)$:

$$\varphi(x) = \int \omega(\mu)\varphi(x)\,d\mu\,, \tag{2}$$

where:

$$\varphi_\mu(x) = \frac{1}{(2\pi)^{3/2}}\int \varphi_\mu(k)\delta(k^2-\mu)e^{ikx}\,d^4k\,. \tag{3}$$

Parameter μ has the status of m^2: $k^2 = \mu$.

Commutation relations have an additional δ - function connected with parameter μ:

$$[\varphi^-_\mu(\vec{k}), \dot{\varphi}^+_{\mu'}(\vec{k}')]_- = \delta(\vec{k}-\vec{k}')\delta(\mu-\mu')\,, \tag{4}$$

where $\varphi^{+,-}_{\mu}(\vec{k}) = \varphi^{+,-}_{\mu}(k)/\sqrt{2k^0}$ - creation and annihilation operators.

Two-point function is determined in the standard way and weighted by a model probability density $\rho(\mu)$:

$$i\langle T[\varphi(x)\varphi(y)]\rangle_0 = D^C(x-y) = \int \rho(\mu) D^C_{\mu}(x-y)\, d\mu\,, \tag{5}$$

where D^C_{μ} - causal function, $\rho(\mu) = |\omega(\mu)|^2$ and

$$D^C_{\mu}(x) = \frac{1}{(2\pi)^4}\int \frac{e^{-ikx}}{\mu - k^2 - i\epsilon}\, dk. \tag{6}$$

In momentum representation we have also a standard casual Green function weighted by probability density $\rho(\mu)$:

$$D^C(k) = \int \frac{\rho(\mu)\, d\mu}{\mu - k^2 - i\epsilon} = \int \rho(\mu) D^C_{\mu}(k)\, d\mu\,, \tag{7}$$

where D^C_{μ} - standard casual function when $\mu = m^2$, and $\rho(\mu)$ plays a part of spectral function in Lehmann representation.

There is one undetermined yet element in the model - the probability density $\rho(\mu)$. One of the ways to find $\rho(\mu)$ is identification of model $D^C(k)$ with standard renormalized propagator by means of analytical continuation to complex plane:

$$\int_{\mu_0}^{\infty} \frac{\rho(\mu)\, d\mu}{\mu - k^2 - i\epsilon} \quad \longrightarrow \quad \frac{1}{k^2 - M_0^2 - \Sigma(k^2)}\,, \tag{8}$$

where μ_0 - threshold, $\Sigma(k^2)$ - self-energy, $\Sigma(k^2 \pm i\epsilon) = Re\Sigma(k^2) \mp iIm\Sigma(k^2)$. From this identification it follows the expression for $\rho(\mu)$ in a general case:

$$\rho(\mu) = \frac{1}{\pi}\frac{Im\Sigma(\mu)}{[\mu - M_0^2 - Re\Sigma(\mu)]^2 + [Im\Sigma(\mu)]^2}. \tag{9}$$

Near the peak $\mu \approx M^2$ we have the usual Breight-Wigner type (Lorentzian) distribution:

$$\rho(\mu) = \frac{1}{\pi}\frac{M\Gamma}{(\mu - M^2)^2 + M^2\Gamma^2}\,, \tag{10}$$

where $M^2 = M_0^2 + Re\Sigma(M^2)$ and $M\Gamma = -Im\Sigma(M^2)$.

Now we shall discuss a transition amplitude $A(i \to f)$ when UP is in initial $|i\rangle$ or in final $\langle f|$ states. The state vector is determined in the standard way:

$$|\vec{k}, \mu\rangle = \dot{\varphi}^{+}_{\mu}(\vec{k})|0\rangle\,,\ \varphi^{-}_{\mu}(\vec{k})|0\rangle. \tag{11}$$

Then the transition amplitude contains $\omega(\mu)$ as a result of the commutator

$$[\varphi^-(x), \dot{\varphi}^{+}_{\mu}(\vec{k})]_- = \frac{1}{(2\pi)^{3/2}}\frac{\omega(\mu)}{\sqrt{2k^0_{\mu}}}\exp{(-ikx)}\,, \tag{12}$$

where $k^0_\mu = \sqrt{(\vec{k})^2 + \mu}$ and $\omega(\mu)$ - a weight function. From this result it follows that the transition amplitude is determined as:

$$A(k,\mu) = \omega(\mu) A^{st}(k, \mu = m^2). \tag{13}$$

The expression for differential probability of transition has the same structure:

$$dP(\mu) = \rho(\mu) |A^{st}(\mu)|^2 \, d\mu \,, \tag{14}$$

where $\rho(\mu) = |\omega(\mu)|^2$ is the model probability density.

3 Model description of decays

From the expression for probability of transition we can get the formula for the decay width in the form of standard one weighted by $\rho(\mu)$:

$$\Gamma(\bar{\mu}, \sigma) = \int_{\mu_0} \rho(\mu; \bar{\mu}, \sigma) \Gamma^{st}(\mu) \, d\mu \,, \tag{15}$$

where $\mu_0 = (\Sigma m_f)^2$ is threshold, $\bar{\mu} \approx M^2$ and $\sigma \approx \Gamma_{tot}/2$.

The expression for Γ leads to some important phenomenological consequences: "smearing" of threshold, modification of phase space and of branchings, s - dependence of widths. Finite width effects in decays of fundamental UP with large Γ_{tot}, such as Z, W - bosons and t - quark, should be taken into account in precise measurements. To illustrate the deviation of model predictions from conventional ones we choose Gaussian approximation for $\rho(\mu)$ due to suitable asymptotic behavior in infinity (large μ_{max}). In a case of heavy boson decay to two fermions, when $M_f \ll M_V$ and $\bar{\mu} = M_V^2$, $\sigma = \Gamma_V^{tot}/2$ we get

$$\Gamma^M(\bar{\mu}, \sigma)/\Gamma^{st}(\bar{\mu}) \approx 1 + 3\Gamma^2_{tot}/4M^2 \,, \tag{16}$$

where Γ^M and Γ^{st} are the model and standard predictions for discussed partial widths. The deviation of $Br^M(Z \to f\bar{f})/Br^{st}(Z \to f\bar{f})$ from unity according to (16) is equal to $3\Gamma^2_{tot}/4M_Z^2 \approx 6 * 10^{-4}$. So, the effect of instability in such channels gives corrections of an order of 0.1% which is an order of least experimental errors in Z - physics and much less than errors in W - physics. We need more precise both experimental data and theoretical calculations in Standard Model. However large "smearing" effect can be registered in the processes near threshold when $(M_i - M_f) \sim \Gamma_{tot}$. Finite width contribution to $Br(t \to ZWb)$ increases the value of this decay rate by orders of magnitude near threshold [12]. The rough model evaluation of correction caused by finite width effect in rare decay channel $Z \to Wbc$ gives us the value $\approx 10\%$. This correction should be taken into account when precision measurements are compared with theoretical prediction.

The effects of "mass smearing" are important in the processes of generations and decays of hadrons with large total widths. Hadrons are not fundamental particles and quantum field approach cannot be applied in general case. But effects of "mass smearing" follow from fundamental uncertainty principle and take place at various hierarchy levels. Proposed model does not describe the process of hadron decay, but gives a simple way to take into account "smearing" effects in traditional approach.

The effects of "mass smearing" in hadron decays were first accurately evaluated in the [13] - [16]. In these works the phenomenological Breight-Wigner weighting of widths was applied to B^0 and Λ_b^0 decays :

$$B^0 \to D^- \rho^+, D^- a_1^+; \quad \Lambda_b^0 \to \Lambda_c^+ a_1^- .$$

Here UP in final states $\rho(770)$ and $a_1(1260)$ have large total widths. Summary of results is presented in the Table 1.

Decay	Br^{st}	Br^M	Br^{exp}
$B^0 \to D^- \rho^+$	$1,1 \cdot 10^{-2}$	$5,8 \cdot 10^{-3}$	$(7,9 \pm 1,4)10^{-3}$
$B^0 \to D^- a_1^+$	$9,34 \cdot 10^{-3}$	$6,6 \cdot 10^{-3}$	$(6,00 \pm 3,3)10^{-3}$
$B_s^0 \to D^- a_1^+$	$8,12 \cdot 10^{-3}$	$5,8 \cdot 10^{-3}$	$6 \cdot 10^{-3}$
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	$3,34 \cdot 10^{-2}$	$2,8 \cdot 10^{-2}$	(seen)

Table 1. Theoretical predictions (Br^{st} - standard, Br^M -model) and experimental (Br^{exp}) data on branchings.

From this table we can see that the effects of "smearing" are large (20-40%) and their account improves considerably the conformity of experimental data and theoretical predictions.

With the help of proposed model we can consider widely discussed decay channels:

$$\phi(1020) \quad \to \quad K^+K^-, K_L K_S. \tag{17}$$

The ratio of branchings does not depend on hadron factors in good approximation and is mainly determined by the ratio of phase space and kinematic factors:

$$k = \frac{Br(\phi \to K^+K^-)}{Br(\phi \to K_L K_S)} = \frac{g_+^2}{g_0^2}\left(\frac{1 - 4m_+^2/m_\phi^2}{1 - 4m_0^2/m_\phi^2}\right)^{3/2} \tag{18}$$

There is a discrepancy between the experimental and standard theoretical values of the k [17]:

$$k^{exp} = 1.456 \pm 0.033, \quad k^{th} = 1.528.$$

Various corrections to k^{th} have been evaluated in [18], but discrepancy has increased (Fermy "gold rule" puzzle). The result of model evaluation depends on μ_{max} and in Breight-Wigner approximation:

$$k^M = 1.42 - 1.49 \text{ when } \mu_{max} = (1-3)\,Gev, \;\; k^M = k^{exp} \text{ when } \mu_{max} = 2\,Gev.$$

So the model can resolve the problem with the help of a reasonable assumption concerning μ_{max}.

Proposed model can be applied to verification of universality of coupling hypothesis in vector meson decays. We shall discuss the decays $V \to \varphi_1\varphi_2$ where $V = (\rho, \phi, \omega, K^*)$ and $\varphi = (\pi, \eta, \eta', K)$ - vector and pseudoscalar nonets consist of $1 \oplus 8$ representation of

$SU(3)$ group. In quantum field approach decays $V \to \varphi_1\varphi_2$ are described by interaction Lagrangian:

$$L_{int}(V_i\varphi_\alpha\varphi_\beta) = g_{i\alpha\beta}V_i^\mu(\varphi_\alpha\partial_\mu\varphi_\beta - \varphi_\beta\partial_\mu\varphi_\alpha) \tag{19}$$

In this approach we shall describe decays $\rho \to 2\pi, 2K;\ K^* \to \pi K, \eta K, \eta' K;\quad \phi \to 2K$. Here $\rho \to 2K$ and $K^* \to \eta K, \eta' K$ are "mass forbidden" decay channels.

Decay constants as functions of a single parameter g we have from $U(1)SU(3)$ - gauge model of low energy meson interactions [19]:

$$g_{\rho\pi\pi} = g_{\rho KK} = g,\quad g_{\phi KK} = \sqrt{3}g\sin\theta/2,\quad g_{K^*\pi K} = g/2,$$

$$g_{K^*\eta K} = \sqrt{3}g\cos\varphi/2,\quad g_{K^*\eta' K} = \sqrt{3}g\sin\phi/2,$$

where θ and φ are the $(\omega - \rho)$- and $(\eta - \eta')$ - mixing angles which have fixed values.

We have fulfilled fitting of experimental data on ρ, K^* and ϕ decays by minimization:

$$\frac{\partial\epsilon(g^2)}{\partial g^2} = 0,\quad \epsilon = [\Sigma_i\frac{(\Gamma_i^{exp} - \Gamma_i^M(g^2))^2}{(\Gamma_i^{exp})^2}]^{1/2}, \tag{20}$$

where $\Gamma_i^M(g^2)$ is determined by

$$\Gamma_i^M(g^2) = \int_{\mu_0}\rho(\mu)\Gamma_i^{st}(\mu, g^2)\,d\mu. \tag{21}$$

and $\rho(\mu)$ has Breight-Wigner approximation. The summary of experimental data and theoretical predictions (standard and model 1 and 2 - without and with account of "mass forbidden" channels) is presented in the Table 2.

Data (Mev)	Γ_ρ	Γ_{K^*}	Γ_ϕ	$\bar{\epsilon}(\%)$
Exp.	150.2 ± 0.8	50.8 ± 0.9	3.70 ± 0.03	1.66
Standard	175.6	50.4	3.32	6.64
Model 1	159.2	48.9	3.55	2.59
Model 2	154.5	49.3	3.65	1.25

Table 2. Experimental and theoretical values of total $(\Gamma_\rho, \Gamma_{K^*})$ and partial (Γ_ϕ) widths.

From the table we can see that the account of "smearing" effect considerably improves conformity of experimental data and theoretical results. From this result we should make conclusions that hadron factors, which describe the decays of vector mesons, contain some universal parameter.

4 Conclusions

Instability effects can be considered in the frameworks of two quantum field approaches - "propagator" and spectral function ("mass smearing") approaches. Proposed model of UP is connected with spectral function approach and describes "mass smearing" effects in a simple way. "Smearing" of mass naturally leads to modification of branchings, total widths and s - dependence of widths. The model does not contradict the data on Z -, W - bosons and t - quark decays and is in quantitative agreement with data on some hadron decays. The effects of "mass smearing" can be observed in the follow processes:

1. Hadron decays (large total widths);
2. Rare meson decays (semileptonic and nonleptonic);
3. Near-threshold decays of t-quark and Z-boson;
4. Mixing in neutral systems of mesons $M^0 - \bar{M}^0$ (when $\Delta m \sim \Gamma_S$);
5. Z, W-bosons and t-quark physics (precision test of SM).

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