On the SU(2) Yang-Mills fields turbulence at the thermal and quantum fluctuations

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Abstract

The heavy ions experiments at low and high energies stimulate the statistical physics approach to the various hadron and quark matter processes - vortexes, solitons etc. [1-3]. In particular, there are developed models of the nuclear matter fragmentation and multifragmentation to the product-particles in the heavy ions reactions. Here we are mostly interested in the models employing analogies of the hadron production with the phase transitions [4] with vortex production in the turbulent liquids [5,6].

The bifurcate model (BM) [6] satisfactory explains the intermittency, observed in the multiple hadron production reactions, the power increase of the mean charged hadron multiplicity $< n_{ch} >\sim (\sqrt{s})^{\Delta}$ and step-like character of the $< P_{tr}(n_{ch}) >$ dependence. The BM calculated exponent $\Delta = 0.449 \pm 0.018$ is in good correspondence with the exponent $\Delta = ln2/ln\delta = 0.4499806...$, where δ is the Feigenbaum constant [7]. The BM and other similar models are phenomenological ones, but in view of the well-developed nonlinear dynamics approach to the turbulence [7,8] - models are rather perspective. Therefore, it is of interest to continue the Yang-Mills (YM) turbulent dynamics investigations [9-11] and to consider conformity of the QCD turbulence evolution scenario to succession of doubling period bifurcations (DPB) of the quark-gluon system phase trajectory and to find foundation of the BM in the QCD frame. The DPB scenario of liquid turbulence evolution picture [12] constitutes the ground to BM .

To proceed, we consider in steps complications of the model approach the QCD: first, we consider the turbulence evolution of purely gluon fields, then take into consideration the Higgs mechanism, quark freedom degrees of hadronic matter and thermal/quantum thermostats fluctuations. Finally, it is interesting to consider the formation, evolution and emission possibility of the space structures - vortexes (hadrons?) from the "wilding" quark-gluon system. The latter, possibly, will make the notions of quark-gluon matter hadronization mechanism more precise.

Some of these problems were considered in the works [13,14] for the Higgs particles-gluons and the quark-gluon systems. It was shown that the Higgs particles and quarks, do not visibly disturb the YM- fields phase transition scenario to the well-developed turbulence which is the infinite succession of doubling periods of the corresponding phase trajectory bifurcations.

In the present work, following this program, by the methods of Poincare maps and maximum Lyapunov exponents the turbulence evolution scenario of the homogeneous SU(2) YM-fields at non-zero temperature is investigated. It is necessary to note that the determined chaos of the YM-fields was studied in some aspects in the works [9-11], and here we use these results.

The gluon system lagrangian has the following form:

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \tag{1}$$

here $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g/2\varepsilon_{ank}(A^k_\mu A^k_\nu - A^n_\nu A^k_\mu)$ is the tensor of the YM-fields with potential A^a_μ.

From Eq.1 we have equations of motion of the gluon system:

$$\partial_\nu F^a_{\nu\mu} + g\epsilon_{abc} A^b_\nu F^c_{\nu\mu} = 0. \tag{2}$$

We limit investigations of dynamics of the system Eqs.2 by the long-wave approach, essentially reducing the mathematical difficulties and entirely successful from the physical view point. In this case information about QCD IR-regime is kept and also we can estimate from above [15] the energy threshold of appearance of the chaos in dynamics of the essentially non-linear field system Eqs.2.

In the space-homogeneous case Eqs.2 are (in the gauge $A^a_0 = 0$):

$$\frac{d^2 A^a_j}{dt^2} - g^2(A^k_i A^k_i A^a_j - A^a_i A^k_i A^k_j) = 0. \tag{3}$$

Further some simplification is reached with the help of the reduction $A^a_j = O^a_j R^a$, $R^{1,2} = X, Y/g$, $R^3 = 0$ ($O^a_j O^b_j = \delta_{ab}$, without summation over a) [8]. We study the SU(2) YM-field dynamics at non-zero temperature T in the framework of the Lan'jeven normal sources formalism $< R_{x,y} >= 0, < R^2 >\sim T$ [10] and Eq.3 takes form:

$$\frac{d^2 X}{dt^2} + Y^2 X = R_x - \beta\dot{X}, \frac{d^2 Y}{dt^2} + X^2 Y = R_y - \beta\dot{Y}. \tag{4}$$

Here the friction force $\beta\dot{X}, \beta\dot{Y}$ ensure no YM system overheating by the thermostat fluctuations.

With the help of these equations we considered [10] the stabilizing thermostat fluctuations influence on the YM fields turbulent dynamics under the condition that the correlation time $t_R \sim \beta T < R^2 >^{-1}$ is much less than the correlation time $t_{\dot{X},\dot{Y}} \sim \beta^{-1}$. These conditions are satisfied at the small energy of YM fields E and $\beta \ll 1$.

In this work we use Eqs.4 for numerical ($g = 1$) investigation of the gluon turbulence evolution scenario, the small YM-fields system energy $E < 1$ being the bifurcation parameter, $\beta = 0.0001$. The maximum Lyapunov's exponents $\lambda_{max}(t \to \infty)$ were calculated on the base of time dependence of the non-stability increment $\lambda_{max}(t)$ of nonlinear dynamical system Eqs.4. The question about conformity of chaos evolution scenario in quark-gluon fields dynamics to the succession of doubling periods bifurcations of the field system phase trajectory was solved on the basis of calculation of the crossing number N of Poincare surface with the plane $(Y, \dot{Y})$ at $X \oplus \dot{X} = 0$, see [7], by the YM fields trajectory for fixed enough large time interval $\Delta t \sim 70$. Here the starting point of Δt counting was selected using the time dependence of the non-stability increment $\lambda_{max}(t)$ of non-linear quark-gluon system and was large enough $t_0 \geq 40$ for considerable relaxation of the influence of initial conditions on the motion frequencies of this system.

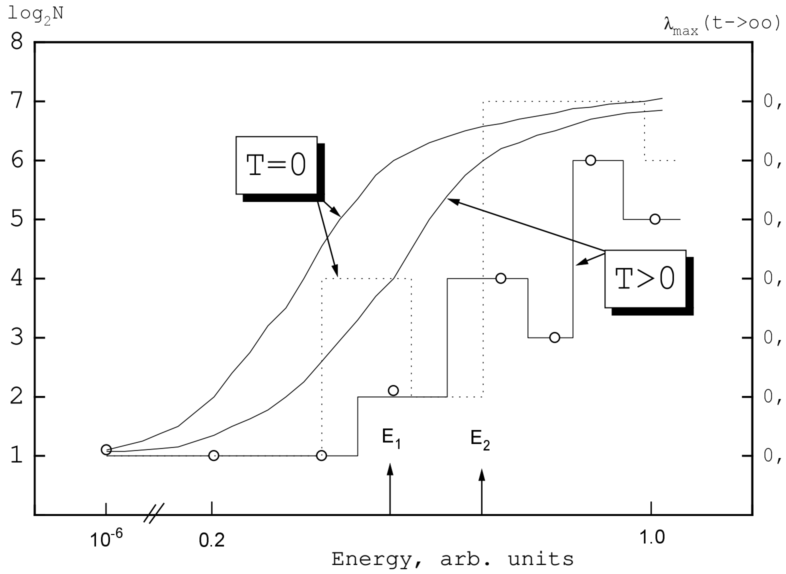

Figure 1: *The crossing number N(E) of Poincare surface by the phase trajectory of the systems: YM-fields (1, $T = 0$), YM-fields (2, $\beta T = 5$). Circles and triangles - calculation for these systems, the break lines - approximation 2^n, the fluent lines - the maximum Lyapunov exponents versus E. $E_{1,2}$ is the energy of these systems phase transition to well developed turbulence.*

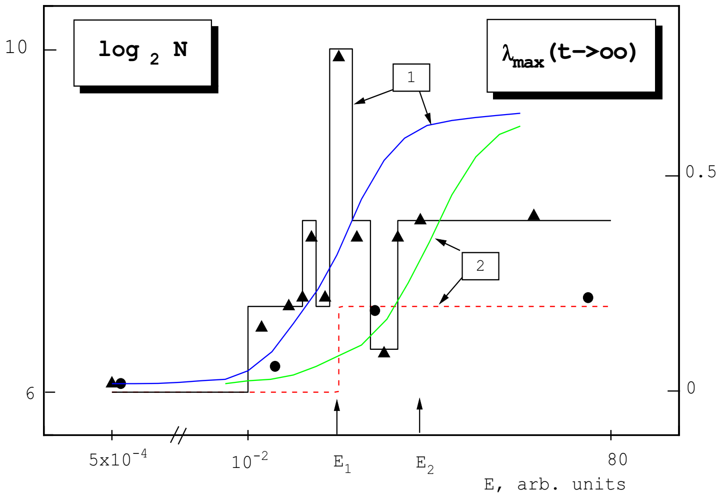

Figure 2: *The 2D gluon field turbulent dynamics: the upper figure corresponds to the case $T = 0$; the low figure corresponds to the case $T > 0$ as on the figure 1.*

Results of calculation performed following the described methods are presented in Fig.1.

One can see, that for the free YM-fields and YM-fields at the $0 < \beta T < 5$ temperature region transition scenario (at $E \sim E_{1,2}$) to well-developed chaos which corresponds the maximum Lyapunov exponents value $\lambda_{max}(t \to \infty) \sim 0.5$ and destruction of the invariant torus described by phase trajectories of these dynamical systems is satisfactory good approximated by the succession of direct and reverse doubling period bifurcations. In particular, the direct bifurcations are prevailed (in average) over the reverse bifurcations. Also we can see from the Fig.1 that the above scenario is not‘t visibly more simple at the non-zero temperature weakly stabilized YM-fields system [13].

We also investigated equation (2) in 2D - case with the covariant Lan‘jeven normal sources. Results are represented on figures 2. It can be seen that the thermostat fluctuations stabilize also the gluon field space chaos.

Finally, it is important to say what we can expect, if we take into consideration the quantum "thermostat" fluctuations. The YM-fields "quantum chaos" problem now isn‘t satisfactory investigated and there is no physical necessity to quantize the gluon (massless particles) wave dynamics ("classical" dynamics of the massless particles is unknown !). Nevertheless, the estimations of the stabilizing influence of the quantum vacuum fluctuations on the YM fields dynamics were made in the work [11]. It was shown, that the thermal fluctuations strongly stabilize the YM-fields dynamics at the temperatures compared with the YM-fields energy, and the stabilizing influence of the quantum vacuum fluctuations is very weak. We can expect, therefore, the corresponding weak quantum vacuum fluctuations influence on the YM-fields turbulence evolution scenario. However, the more definite conclusions about this scenario may be made only after the corresponding more detailed calculations. These calculations and investigations of the vortex dynamics in quark-gluon system are planned to realize in the next works.

Thus, in this work by the methods of the Poincare maps and the maximum Lyapunov exponents it is established, that in the Lan‘jeven sources formalism the thermal fluctuations do not visibly disturb the YM-fields phase transition scenario to the well-developed turbulence. This scenario is the infinite succession of doubling periods of the corresponding phase trajectory bifurcations. Also is discussed the quantum thermostat fluctuations influence on the YM-fields turbulence evolution scenario.

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