Chiral Born-Infeld theory and model of baryons

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Abstract

The "bag"-like spherically-symmetrical solutions of the chiral Born-Infeld theory are studied. The properties of these solutions are obtained, and a possible physical interpretation is also discussed.

1 Introduction

From the DIS experiments we know, that baryon consists of the charged constituents (so-called constituent quarks), but the experiments with spin of proton showed that only twenty percents of the baryon spin can be associated with the charged constituents [2]! Therefore, the realistic model for low-energy baryon state should describe the quark constituents as well as chiral degrees of freedom of the meson cloud around them.

But how to combine these two opposite points of view on the baryon in the framework of one model? In order to find a model with such dualistic properties, the so-called "two-phase" models were proposed. The simplest realization of the two-phase idea is the well-known Chiral Bag Model (ChBM) [3]. In this model, constituent quarks are placed inside a spherical region (so-called "bag"), subject to the confining boundary conditions, and meson cloud lives outside. As a rule, the exterior pion fields are Skyrmionic, but the quark part is necessary in this model for the self-consistency of the model, as well as for the phenomenological point of view. Typical Lagrangian of the two-phase model consists of three parts

$$\mathcal{L}_{\mathrm{model}} = \mathcal{L}_{\mathrm{q}}\theta_V + \mathcal{L}_\pi\theta_{\bar{V}} + \mathcal{L}_{\partial V}\delta_V \tag{1}$$

where Heaviside's θ_V function cuts out the internal region for quark part of the Lagrangian L_q, inverse $\theta_{\bar{V}}$ function cuts out the external region for meson field with the Lagrangian L_π. The crucial point of this model stems from the fact that the kinematic consequence of the confinement boundary conditions for fermions in the bag is the non-zero axial current across the boundary. For self-consistency of the model, these two regions should be joined by the requirement of continuity of the axial current on the boundary. In order to guarantee such continuity, the boundary term $L_{\partial V}$ must be introduced.

Chiral Bag Model is used very widely in the phenomenology of elementary particle physics [4], but on the other hand this model has serious methodological disadvantages. First of all, as one can easily see in (1), this Lagrangian depends directly on the geometry of the confinement region V. In the case of the Chiral Bag Model, V is a sphere. But it would be very interesting to find a theory where the form of the confinement region is generated dynamically, rather than is chosen "by hand", as in the case of the ChBM. Moreover, the geometry dependence of the Lagrangian (1) directly violates the Lorenz covariance of the theory, and thus there is no a way for connection such complicated and non-invariant construction with QCD.

Another kind of methodological difficulties appears from Skyrme part of the Skyrme Model [1]. The point is that the simplest realization of the non-linear meson field (so-called Weinberg's prototype theory)

$$\mathcal{L}_{\mathrm{pr}} = \frac{f_\pi^2}{4}\mathrm{Tr}L_\mu L^\mu \tag{2}$$

has no soliton solution and any non-trivial static solutions of this theory have infinite self-energy. But on the other hand, the theory (2) correctly describes the behavior of the meson field at large distance from the source (the proton or neutron). In order to regularize the singular self-energy of the prototype theory, Skyrme proposed to modify slightly the theory [1] by means of the additional term $\mathcal{L}_{\mathrm{Sk}} = \mathrm{Tr}[L_\mu, L_\nu]^2/e$ Such term in the Lagrangian of the meson field leads to generation of the soliton solutions (skyrmions), and now is treated as the leading order of the expansion of the effective meson theory in strong limit (by analogy with the Chiral Perturbation Theory Lagrangian defined as series). But unfortunately such analogy is not so good for the soliton paradigm of baryons. Basing on the renormalization group approach, one can show that the coefficients of such expansion depend strongly on the energy scale of the process, and in the strong regime (near the nucleon) the highest terms of the expansion become more and more essential [5]. This fact is connected with the nonrenormalizable nature of chiral field theory and this situation is similar with quantum gravity.

An additional point to emphasize is that the Skyrme regularization procedure does not solve the problem of singular solutions of chiral field theory, because Skyrme model has a lot of another singular solutions along with the soliton sector.

In order to construct a suitable candidate for the role of the chiral effective theory, let us consider a very illuminative historical analogy with the problem of singular self-energy of electron. Before the Era of Quantum Electrodynamics, Born and Infeld [6] proposed the non-linear covariant action for the electro-magnetic fields with very attractive features. Firstly, in the framework of the BI theory, the problem of singular self-energy of electron can be solved. In this theory the electron is a stable finite energy solution of the BI field equation with electric charge. Second, the BI action has the scale parameter β. Using expansion in this parameter, one reduces the BI action to the usual Maxwell form in the low-energy limit. Indeed, using the analogy with the relativistic particle action, let us consider the action in the Born-Infeld form

$$\mathcal{L}_{BI} = -\beta^2\Big(1 - \sqrt{1 + \frac{1}{\beta^2}F_{\mu\nu}F^{\mu\nu}}\Big) \overset{\beta\to\infty}{\longrightarrow} \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \tag{3}$$

It can be shown that the solution for the electrostatic spherically symmetrical potential $A_0 = \phi$ reads

$$\phi(r,\beta) = C\int_0^R \frac{dr}{\sqrt{r^4 + C^2/\beta^2}} \overset{R\to\infty}{\longrightarrow} -\frac{C}{R} + \underline{O}(\frac{1}{R^5}) \tag{4}$$

And one can find that the situations with the singular self-energy in these two theories (singular self-energy of the Coulomb potential and singular self-energy of the solutions of the prototype Lagrangian (2)) are very similar and in our paper we apply the very similar procedure for the chiral prototype Lagrangian (2).

2 Chiral Bag solution of Chiral Born-Infeld Theory

Let us consider the Lagrangian

$$\mathcal{L}_{ChBI} = -f_\pi^2 \mathrm{Tr}\beta^2\Big(1-\sqrt{1-\frac{1}{2\beta^2}L_\mu L^\mu}\Big) \overset{\beta\to\infty}{\longrightarrow} -\frac{f_\pi^2}{4}\mathrm{Tr}L_\mu L^\mu, \tag{5}$$

where β is the mass dimensional scale parameter of our model. It can be easily shown that the expansion of the Lagrangian (5) gives us the prototype theory as the leading order theory in the parameter β. The theory (5) is the direct analogue of the Born-Infeld action for chiral fields.

Our theory contains only the second-order derivative terms. Thus the dynamics of this theory can be studied in detail. Now we consider the spherically symmetrical field configuration

$$U = e^{iF(r)(\vec{n}\vec{\tau})}, \qquad \vec{n} = \vec{r}/|r|. \tag{6}$$

The energy of such configuration is the functional

$$E^\beta[F] = 8\pi f_\pi^2\beta^2\int\limits_0^\infty r^2\Big(1-\sqrt{1-\frac{1}{\beta^2}\big(\frac{F'^2}{2}+\frac{\sin^2 F}{r^2}\big)}\Big)dr. \tag{7}$$

Using the variation principle, we get the equation of motion

$$(r^2-\frac{1}{\beta^2}\sin^2 F)F'' + (2rF'-\sin 2F)-$$

$$-\frac{1}{\beta^2}(rF'^3 - F'^2\sin 2F + 3\frac{1}{r}F'\sin^2 F - \frac{1}{r^2}\sin 2F\sin^2 F) = 0. \tag{8}$$

The next aim of our investigation is finding the solutions of equation (8). This equation is a very complicated nonlinear differential equation. In order to solve it, only numerical or approximation methods seem applicable. The crucial point of such analysis is that the leading derivative term in this equation contains the factor

$$\Big(r^2 - \frac{1}{\beta^2}\sin^2 F\Big). \tag{9}$$

Due to this factor, equation (8) has a singular region (singular surface) with singular behavior of solutions. One can find an equation on such surface by using the standard singular perturbation theory techniques [7]. Let r_0 belong to such singular surface. Then

$$\left\{\begin{array}{rcl} (\beta r_0)^2 - \sin^2 F_0 &=& 0 \\ F_0'\Big((F_0')^2\sin F_0 \mp F_0'\sin 2F_0 + \sin^2 F_0\Big) &=& 0 \quad \text{where } F_0 = F(r_0). \end{array}\right. \tag{10}$$

Equations (10) have only two solutions:

$$\begin{cases} r_0 & \neq & 0, \\ F_0 & = & \pm\arcsin(\beta r_0) + \pi N, \quad \text{where } N \in \mathbf{Z}, \\ F_0' & = & 0. \end{cases} \tag{11}$$

or

$$\begin{cases} r_0 & = & 0, \\ F_0 & = & \pi N, \quad \text{where } N \in \mathbf{Z}, \\ F_0' & \neq & 0. \end{cases} \tag{12}$$

Topological solitons of ChBI theory corresponding to the possibility (12) were studied in [8]. In our paper we consider more interesting possibility (11)

Using the standard singular perturbation theory procedure [7], one obtains the asymptotic behavior near the singular surface ($r \to r_0$, $F(r \to r_0) \to \arcsin(\beta r_0)$)

$$F(r \to r_0) = \arcsin(\beta r_0) + \mathbf{b}(r - r_0)^{3/2} + \underline{O}((r - r_0)^2), \tag{13}$$

where $\mathbf{b}$ is a constant. Of course, the derivative F_0' at the point $r = r_0$ is zero.

To guarantee the finiteness of the energy of our solutions, we should choose the following asymptotic at infinity ($r \to \infty$)

$$F(r) = a_\infty \, (1/r)^2 - \frac{a_\infty^3}{21} \, (1/r)^6 - \frac{a_\infty^3}{3\beta^2} \, (1/r)^8 + \underline{O}(1/r^{10}), \tag{14}$$

where a_∞ is a constant.

Notice that equation (8) has very useful symmetries. First of all, this equation is symmetric with respect to the changes $F \leftrightarrow F + N\pi$, $N \in \mathbf{Z}$ and $F \leftrightarrow -F$.

The numerical investigation of the solutions of equation (8) which have the asymptotics (11) ($a_\infty > 0$) at infinity is presented in Fig.1. Most of these solutions can be evaluated only for $r > r_0$, where r_0 is determined by $F(r_0) = \pm\arcsin(\beta r_0)$. But among this set of solutions there are solutions with $r_0 = 0$. Such solutions have the asymptotics [8]

$$F(r) = \pi N + ar - \frac{7a^2 - 4\beta^2}{30(a^2 - \beta^2)} a^3 r^3 + \underline{O}(r^5) \tag{15}$$

at the origin ($a^2 < \beta^2/3$ is a constant), and these are the topological solitons of the ChBI theory. Solitons with $B = 1, 2$ and 3 are presented in Fig.2. The scale parameter $\beta = 807\,\mathrm{MeV}$ is preliminarily defined from the hypothesis that the soliton with $B = 1$ is a nucleon.

Now we would like to draw attention to another class of solutions. These solutions are defined everywhere, except for small (~ 0.2 fm) spherical region about the origin. These solutions look like a "bubble" of vacuum in the chiral fields and are of interest for the chiral bag model of baryons [3]. In the internal region ($r < r_0$), only vacuum configuration can exist. From the mathematical point of view, such "step-like" solutions with a jump of $F(r)$ at $r = r_0$ are generalized solutions [9].

To clarify the physical nature of this "step-like" generalized solutions, let us consider the projection of the left-hand chiral current on the outward normal of the singular surface.

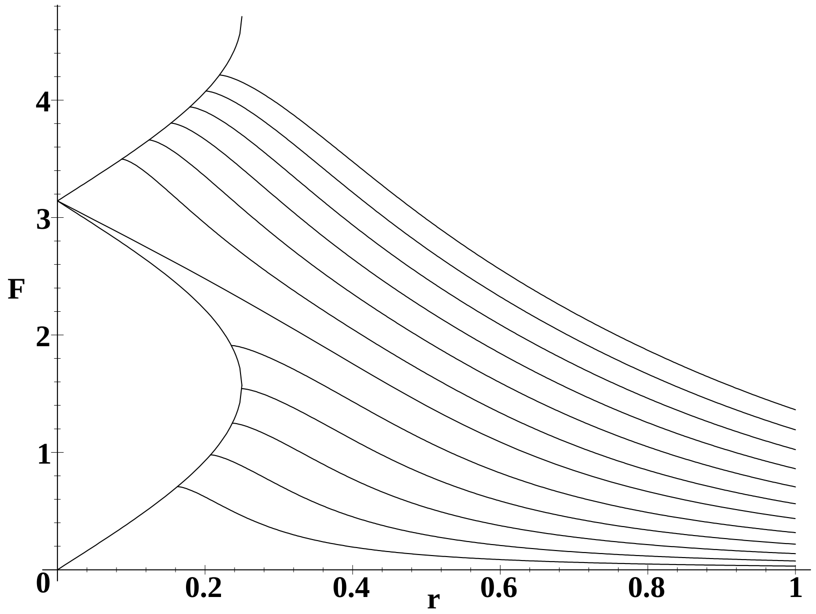

Figure 1: Solutions of equation (9) which have the asymptotics (15) ($a_\infty > 0$) at infinity. Horizontal axis: r (in fm).

As we pointed out in the Introduction, a non-zero chiral current across the boundary the kinematic is the consequence of the confinement boundary conditions for the fermions in the bag. According to our proposition, the singular surface of "step-like" solutions is a confinement boundary surface for constituent quarks. For the self-consistency of our "two-phase" picture, let us check if it is possible to compensate the non-zero quark chiral current on the confinement surface by the non-zero chiral current of ChBI "step-like" configuration that appear due to the defect on the singular surface. The projection of the chiral current on the outward normal of the singular surface for the spherically symmetrical configuration (6) reads

$$(\vec{n}\,\vec{J}_\pi)|_{\partial V} = f_\pi^2 \mathrm{Tr}\frac{\tau^a \vec{x}\vec{L}}{\sqrt{1-\frac{1}{2\beta^2}L_\mu L^\mu}} = \frac{3}{2} f_\pi^2 \frac{\mathbf{b} r_0}{\sqrt{2r_0 + 9r_0^2\mathbf{b}^2}}, \tag{16}$$

where the coefficient $\mathbf{b}$ from the asymptotic (13) is a function of $F_0 = F(r_0)$ and can be evaluated numerically.

The crucial point for such analysis stems from the fact that $\mathbf{b}(F_0)$ has the singularity at $F_0 = \pi N$ and $r_0 = 0$, or $\mathbf{b}(F(r_0 = 0)) = \infty$. This implies that the soliton solutions of the ChBI theory are solutions with point-like singular chiral source and the "bag"-like solutions are solutions with some distribution of the chiral current on the confinement surface. It is possible to show that for any internal constituent quark configuration with

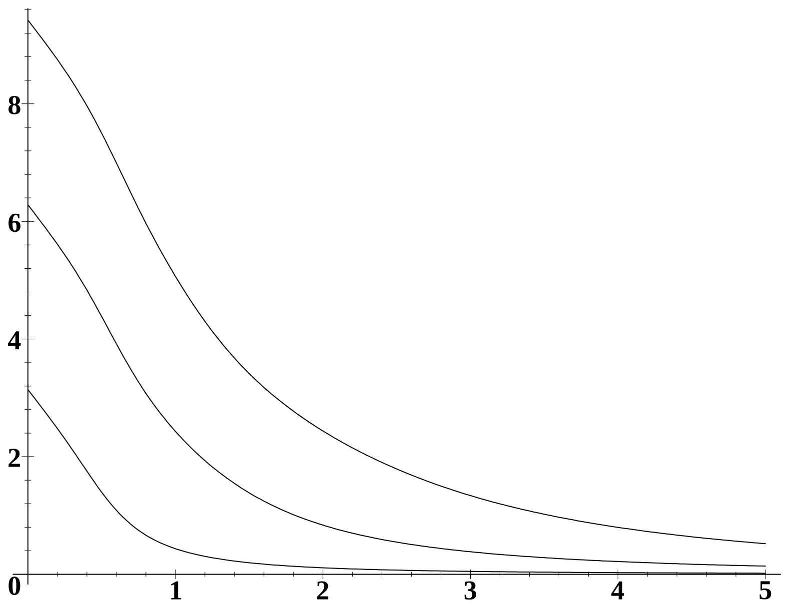

Figure 2: Solitons with $B = 1, 2$ and 3. Horizontal axis: r (in fm).

confinement inside some volume V could be defined solution of ChBI theory $U(\vec{\pi})$ which compensates the non-zero quark chiral current across the surface ∂V

$$(\vec{n}\,\vec{J}_q)|_{\partial V} = \sum_g \frac{i}{2}\bar{\Psi}_q(\vec{x}\vec{\gamma})\gamma_5\Psi_q = (\vec{n}\,\vec{J}_\pi)|_{\partial V} \tag{17}$$

Equation (17) can be considered as a condition on the coefficient $\mathbf{b}(\partial V)$, and plays the role of the self-consistency condition between quark and chiral phases.

In conclusion of this section, it would be interesting to point out that the condition (17) is not just an artifact of the spherical symmetry of our problem. This is a direct consequence of the asymptotics (13) but, as it is easy to see, such asymptotics can depend only on local characteristics of the surface ∂V. This means that the asymptotics of the field configuration along the normal in the leading order depends only on the curvature radius of the confinement surface at this point and equation (17) can be used for any geometry of ∂V.

3 Conclusions

The aim of this paper is to study the "bag"-like generalized solutions of the Born-Infeld theory for chiral fields. These solutions were firstly proposed in our work [8]. For simplicity, in this work we restricted ourselves to the spherically symmetrical configuration

(6), but, of course, the ChBI theory has solutions with another geometry of the singular surface. Physically the critical behavior in the ChBI theory appears when the chiral field strength of the prototype field theory approaches the value of the squared effective coupling constant (β^2). This is a very physical idea. Today we know that there are many theories which have the low-energy limit in the form of the Born-Infeld action [10].

The Chiral Born-Infeld theory is a good candidate to the role of the effective chiral theory and the model for the chiral cloud of the baryons. In this model one can find not only spherical "bags", it is possible also to study the "string"-like, toroidal or "**Y**-Sign"-like solutions, or some another geometry. The geometry of confinement surface depends directly on the model of color confinement, and it would be very interesting to use, for example, the lattice QCD simulations for the color degrees of freedom in combination with our model for external chiral field.

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