Charmonium production via fragmentation in k_T-factorization approach

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Abstract

The p_T-spectra for direct J/ψ in hadroproduction at Tevatron energy have been calculated within the framework of the NRQCD formalism and the fragmentation model in the collinear parton model as well as in the k_T-factorization approach. We have described the CDF data and obtained a good agreement between the predictions of the parton model and the k_T-factorization approach. We performed the calculations using the relevant leading order in α_s hard amplitudes and taking equal values of the long-distance matrix elements for the both models.

1 Introduction

During the years after the measuring of charmonium productiom cross sections and polarization effects for J/ψ mesons at the Tevatron Collider [1] the phenomenology of a quarkonium production has a phase of intensive developments. Nowadays, it is understand that the heavy quarkonium production is a very complicated physical process needs many new theoretical ideas. Starting from the Color Singlet Model [2] it was developed, so-called, the nonrelativistic QCD (NRQCD) formalism [3] to describe the nonperturbative transition of $Q\bar{Q}$-pair into a final heavy quarkonium.

The perturbative fragmentation functions for partons which split into heavy quarkonia have been obtained [4] within the framework of the NRQCD formalism. It is supposed that the fragmentation model is more adequate for the description of a quarkonium production at the large quarkonium transverse momentum $p_T \gg M_{Q\bar{Q}}$ than the fusion model.

In the charmonium production at the energy range of the Tevatron Collider we deal with the gluon distribution function from a proton which is taken at a very small x but the relevant virtuality $\mu^2 \sim M^2_{Q\bar{Q}} + p_T^2$ is large. In the region under consideration the collinear parton model can be generalized within the framework of the k_T-factorization approach [5, 6, 7, 8]. This fact leads to some interesting effects in the quarkonium production at the high energies, which were discussed ten years ago in Ref. [9] and recently in Refs. [10, 11, 12, 13].

In this paper we calculate the p_T-spectra of the unpolarized direct J/ψ mesons produced via the fragmentation mechanism. In case of direct J/ψ it is supposed that the production via the gluon fragmentation into the color-octet $Q\bar{Q}$ $[^3S_1, \underline{8}]$ state is a dominant contribution [14, 15]. We compare the predictions which are obtained as in the collinear parton model as in the k_T-factorization approach. In both cases we performed calculations with the hard amplitudes in the leading order of the QCD running constant α_s. So, in the collinear parton model we take into consideration the partonic subprocess:

$$g + g \to g + g. \tag{1}$$

In the k_T-factorization approach we take into consideration the subprocess with off-shell or reggeized initial gluons:

$$g^* + g^* \to g. \tag{2}$$

We have described the CDF data [1] and obtained a good agreement between the parton model and the k_T-factorization calculations based on the hard amlitudes for subprocesses (1) and (2) with the equal values of the long-distance matrix element $\langle \mathcal{O}^{J/\psi}[^3S_1, 8]\rangle$ in the fragmentation functions $D_{g\to J/\psi}(z,\mu^2)$ for the both models. The QCD evolution of the fragmentation function is described by the homogeneous equation with a boundary condition proportional to the delta function $\delta(1-z)$.

The results obtained in the paper within the k_T-factorization approach based on the fragmentation function model differ from results of Refs. [11, 12, 13], which were obtained using the gluon fusion model. The possible reasons of a disagreement will be discuss below.

2 NRQCD formalism

Within the framework of the NRQCD, the fragmentation function for a quarkonium H production can be expressed as a sum of terms, which are factorized into a short-distance coefficient and a long-distance matrix element [3]:

$$D(a \to H) \;=\; \sum_n D(a \to Q\bar{Q}[n])\langle \mathcal{O}^H[n]\rangle. \tag{3}$$

Here the n denotes the set of color and angular momentum numbers of the $Q\bar{Q}$ pair, which fragmentation function is $D(a \to Q\bar{Q}[n])$. Last ones can be calculated perturbatively in the strong coupling α_s. The nonperturbative transition from the $Q\bar{Q}$ state n into the final quarkonium H is described by a long-distance matrix element $\langle \mathcal{O}^H[n]\rangle$ which have to be calculated using non perturbative methods or determined from experimental data. The fit of the Tevatron data for the p_T-spectra of J/ψ, ψ' and χ_c charmonium states have been done recently by different authors (see review in Ref. [16]). As it was shown in Refs. [14, 15], at the large charmonium transverse momentum ($p_T > 7$ GeV) the gluon fragmentation into color-octet state

$$g^* \to Q\bar{Q}[^3S_1, 8] \tag{4}$$

gives dominated contribution to the direct J/ψ hadroproduction. The values of the color-octet matrix elements under consideration are following $\langle \mathcal{O}^{J/\psi}[^3S_1, 8]\rangle$=$4.4 \cdot 10^{-3}$ GeV^3 [15]. Note, that the fit of CDF data for the p_T-spectrum of a direct J/ψ production within the framework of the fusion model in the collinear parton model gives the another numerical value of the long-distance matrix element $\langle \mathcal{O}^{J/\psi}[^3S_1, 8]\rangle$=$1.2\cdot 10^{-2}$ GeV^3 [16]. It will be important to compare the results obtained in the k_T-factorization approach based on the fusion and fragmentation models.

3 Fragmentation function

The gluon fragmentation into $^3S_1$ charmonium state is determined by the color-singlet and the color-octet contributions. The previous analysis has shown that the probability of a gluon fragmentation into the color-singlet state is only a small part of the probability of a fragmentation into the color-octet state [14]. The leading order in the α_s fragmentation functions for the transition (3) are known and can be written at the scale $\mu^2 = \mu_0^2 = 4m_c^2$ as follows [4]:

$$D_{g\to J/\psi}(z,\mu^2) \simeq D^T_{g\to J/\psi}(z,\mu^2) = \frac{\pi\alpha_s(\mu_0^2)}{24m_c^3}\delta(1-z)\langle\mathcal{O}^{J/\psi}[^3S_1,8]\rangle. \tag{5}$$

The probability of the gluon fragmentation into the longitudinally polarized charmonium is negligibly small and J/ψ mesons have transverse polarization.

The fragmentation function (5) is evolved in μ^2 using the standard homogeneous DGLAP evolution equation [17]:

$$\mu^2\frac{\partial D_g}{\partial\mu^2}(z,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\int_z^1\frac{dx}{x}P_{gg}(\frac{x}{z})D_g(x,\mu^2), \tag{6}$$

where $P_{gg}(x)$ is the usual leading order gluon-gluon splitting function. To solve equation (6) we use the well known method based on Mellin transform. In a stage of convoluting of the fragmentation function $D_{g\to J/\psi}(z,\mu^2)$ with the partonic cross section for the subprocesses (1) or (2) we will use the following definition for the variable z:

$$z = (E_\psi + |\vec{p}_\psi|)/(2E_g) \tag{7}$$

We suggest also that meson has a small transverse momentum respectively initial gluon jet and we approximately can accept that in the laboratory frame

$$\eta_\psi \cong \eta_g, \tag{8}$$

where η_ψ, η_g are the meson and gluon pseudorapidities.

Within the framework of the fragmentation model, the meson production cross section and the relevant gluon production cross section are connected as follows:

$$\hat\sigma(gg\to J/\psi X) = \int dz D_{g\to J/\psi}(z,\mu^2)\hat\sigma(gg\to gg) \tag{9}$$

or

$$\hat\sigma(g^*g^*\to J/\psi X) = \int dz D_{g\to J/\psi}(z,\mu^2)\hat\sigma(g^*g^*\to g). \tag{10}$$

4 Leading order hard amplitudes

The squared amplitude for the partonic process (1) is well known and it can be presented as follows:

$$\overline{|M(gg\to gg)|^2} = 18\pi^2\alpha_s^2((\hat s^4+\hat t^4+\hat u^4)(\hat s^2+\hat t^2+\hat u^2))/(\hat s\hat t\hat u)^2, \tag{11}$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are usual Mendelstam variables.

There two approaches for calculation of a partonic amplitude for the subprocess (2) in the k_T-factorization approach [6, 8]. The effective Feynman rules for processes with off-shell gluons were suggested in Ref. [6]. The special trick is a choice of the initial gluon polarization 4-vector as follows:

$$\varepsilon^{\mu}(k_T) = k_T^{\mu}/|\vec{k}_T|. \tag{12}$$

In Ref. [8] the initial gluons are considered as reggeons (reggeized gluons) and the effective Reggeon-Reggeon-Gluon vertex function was obtained

$$C^{\lambda}(k_1,k_2) = -(k_1-k_2)^{\lambda} + P_1^{\lambda}\left(\frac{k_1^2}{(kP_1)} + 2\frac{(kP_2)}{(P_1P_2)}\right) - P_2^{\lambda}\left(\frac{k_2^2}{(kP_2)} + 2\frac{(kP_1)}{(P_1P_2)}\right), \tag{13}$$

where $P_1 = \frac{\sqrt{s}}{2}(1,0,0,1)$ and $P_1 = \frac{\sqrt{s}}{2}(1,0,0,-1)$ are the colliding protons 4-momenta, $k_1 = x_1P_1 + k_{1T}$ and $k_2 = x_2P_2 + k_{2T}$ are the initial gluons 4-momenta, $k_T = (0,\vec{k}_T,0)$, $k = k_1 + k_2$ is the final real gluon 4-momentum. It is easy to show that vertex function $C^{\lambda}(k_1,k_2)$ satisfies the gauge invariance condition $(k_1+k_2)_{\lambda}C^{\lambda}(k_1,k_2) = 0$.

Omitting the color factor f^{abc} we can write the amplitude of the subprocess (2) accordingly Ref.[6] as follows:

$$\mathcal{M} = -g\varepsilon^{\lambda}(k)\frac{k_{1T}^{\mu}k_{2T}^{\nu}}{|\vec{k}_{1T}||\vec{k}_{2T}|}\left[(k+k_1)_{\nu}g_{\lambda\mu} + (-k_1+k_2)_{\lambda}g_{\mu\nu} + (-k_1-k)_{\mu}g_{\nu\lambda}\right]. \tag{14}$$

We have obtained after simple transformations that

$$\mathcal{M} = -\frac{g\varepsilon^{\lambda}(k)}{2|\vec{k}_{1T}||\vec{k}_{2T}|}x_1x_2sC_{\lambda}(k_1,k_2). \tag{15}$$

Such a way, the approaches [6, 8] are equivalent and give the equal answer for the squared vertex function and amplitude:

$$C^{\lambda}(k_1,k_2)C_{\lambda}(k_1,k_2) = -\frac{4k_1^2k_2^2}{x_1x_2s}, \tag{16}$$

and

$$\overline{|M(g^*g^*\to g)|^2} = \frac{3}{2}\pi\alpha_s\vec{p}_T^{\,2}, \tag{17}$$

where $\vec{p}_T^{\,2} = (\vec{k}_{1T}+\vec{k}_{2T})^2 = x_1x_2s$, $\vec{p}_T$ is the transverse momentum of the final gluon.

5 Cross sections for the process $pp \to J/\psi X$

In the conventional collinear parton model it is suggested that hadronic cross section, in our case, $\sigma(pp\to J/\psi X, s)$, and the relevant partonic cross section $\hat{\sigma}(gg\to J/\psi X,\hat{s})$ are connected as follows:

$$\sigma^{PM}(pp\to J/\psi, s) = \int dx_1\int dx_2G(x_1,\mu^2)G(x_2,\mu^2)\hat{\sigma}(gg\to J/\psi,\hat{s}), \tag{18}$$

where $\hat{s} = x_1 x_2 s$, $G(x, \mu^2)$ is the collinear gluon distribution function in a proton, $x_{1,2}$ are the fraction of a proton momentum, μ^2 is the typical scale of a hard process. The μ^2 evolution of the gluon distribution $G(x, \mu^2)$ is described by DGLAP evolution equation [17].

In the k_T-factorization approach hadronic and partonic cross sections are related by the following condition [5, 6, 7]:

$$\sigma^{KT}(pp \to J/\psi X) = \int \frac{dx_1}{x_1} \int d\vec{k}_{1T}^2 \int \frac{d\phi_1}{2\pi} \Phi(x_1, \vec{k}_{1T}^2, \mu^2) \quad (19)$$

$$\int \frac{dx_2}{x_2} \int d\vec{k}_{2T}^2 \int \frac{d\phi_2}{2\pi} \Phi(x_2, \vec{k}_{2T}^2, \mu^2) \hat{\sigma}(g^* g^* \to J/\psi X, \hat{s}),$$

where $\hat{\sigma}(g^* g^* \to J/\psi X, \hat{s})$ is the J/ψ production cross section on off-shell gluons, $k_1^2 = k_{1T}^2 = -\vec{k}_{1T}^2$, $k_2^2 = k_{2T}^2 = -\vec{k}_{2T}^2$, $\hat{s} = x_1 x_2 s - (\vec{k}_{1T} + \vec{k}_{2T})^2$, $\phi_{1,2}$ are the azimuthal angles in the transverse XOY plane between vectors $\vec{k}_{1T}(\vec{k}_{2T})$ and the fixed OX axis ($\vec{p}_\psi \in XOZ$). The unintegrated gluon distribution function $\Phi(x, \vec{k}_T^2, \mu^2)$ satisfies the BFKL evolution equation [19].

Our calculation in the parton model is done using the GRV LO [20] and CTEQ5L [21] parameterizations for a collinear gluon distribution function $G(x, \mu^2)$. In case of the k_T-factorization approach we use the following parameterizations for an unintegrated gluon distribution function $\Phi(x, \vec{k}_T^2, \mu^2)$: JB by Bluemlein [22], JS by Jung and Salam [23], KMR by Kimber, Martin and Ryskin [24]. The direct comparison between different parameterizations as functions of x, $\vec{k}_T^2$ and μ^2 was presented in paper [10].

6 The results

We compare our predictions with the CDF data [1] for unpolarized direct J/ψ production. The direct J/ψ cross section does not include contributions from B meson decays into J/ψ as well as from χ_c radiative decays. Our results obtained in the collinear parton model do not depend on a choice of the parameterization for the gluon distribution function and coincide approximately (±10-20%) to the results obtained in Ref. [15] using a similar approch. We can see in Fig. 1 that the curve denoted as "GRV", which were obtained in the collinear parton model, are below the data especially at the small p_T, were the gluon fusion into ${}^1S_0$ and ${}^3P_J$ color octet states is dominant [15, 16].

The curves which were obtained in the k_T-factorization approach strongly depend on a choice of the unintegrated gluon distribution function. In the region of a large p_T, where the fragmentation approach is more adequate, the results obtained with JB [22] and KMR [24] parameterizations coincide well. However, JS parameterization [23] predicts the values smaller by factor 2, which are near the values obtained in the collinear parton model with the GRV gluon distribution function. Such a way, the p_T spectra of direct J/ψ mesons obtained in the collinear parton model and in the k_T-factorization approach are approximately coincide. Note, that our conclusions disagree with the previous results obtained in Refs. [11, 12, 13] using the fusion model. Opposite our result, the fit of the CDF data accordingly [11, 12, 13] needs strong suppression (in 10-30 times) for the long-distance matrix elements $\langle \mathcal{O}^{J/\psi}[^3S_1, \underline{8}] \rangle$ to compare the values obtained in the collinear parton model.

There are several reasons of a such disagreement. At first, we used the fragmentation functions, which take into account effectively high order corrections via the DGLAP evolution equation. At second, in Refs. [11, 12, 13] the argument μ^2 of the strong coupling constant $\alpha_s(\mu^2)$ is equal to $\vec{k}_{1T}^2$ or $\vec{k}_{2T}^2$, our choice is $\mu^2 = \vec{p}_T^2 + 4m_c^2$. This fact gives the additional factor 3 in a cross section [13]. At third, in Refs.[11, 12] the KMS[25] parameterization for an unintegrated gluon distribution function was used. We have shown (Fig. 1) that the difference between predictions based on the different parameterizations may be about factor 2-3.

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References

[1] CDF Coll., F.Abe et al., Phys.Rev.Lett. **79** (1997) 572; ibib. **79** (1997) 578

[2] E.L. Berger and D. Jones, Phys.Rev. **D23** (1981) 1521;
R. Baier and R. Ruckl, Phys.Lett. **B102** (1981) 364;
S.S. Gershtein, A.K. Likhoded, S.R. Slabospiskii, Sov.J.Nucl.Phys. **34** (1981) 128

[3] G.T. Bodwin, E. Braaten, G.P. Lepage, Phys.Rev. **D51** (1995) 1125

[4] E. Braaten, T.C. Yuan, Phys.Rev. **D50** (1994) 3176;
E. Braaten, T.C. Yuan,Phys.Rev. **D52** (1995) 6627;
E.Braaten, K.Cheung, T.C.Yuan, Phys.Rev. **D48** (1993) 4230;
E.Braaten, J.Lee, Nucl.Phys. **B586** (2000) 427;
J.P.Ma, Nucl.Phys. **B447** (1995) 405

[5] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rep. **100** (1983) 1

[6] J.C. Collins and R.K. Ellis, Nucl.Phys. **B360** (1991) 3

[7] S. Catani, M. Ciafoloni, F. Hautmann, Nucl.Phys. **B366** (1991) 135

[8] V.S. Fadin, L.N. Lipatov, Nucl.Phys. **B477** (1996) 767

[9] V.A. Saleev and N.P. Zotov, Mod.Phys.Lett. **A9** (1994) 151

[10] V.A. Saleev, Phys.Rev. **D65** (2002) 054041;
V.A. Saleev, D.V. Vasin, Phys.Lett. **B548** (2002) 161

[11] Ph. Hagler et al., Phys.Rev. **D62** (2000) 071502;
Ph. Hagler et al., Phys.Rev.Lett. **86** (2001) 1446

[12] F. Yuan, K.-T. Chao, Phys.Rev. **D63** (2001) 034006;
F. Yuan et al., Phys.Rev.Lett. **87** (2001) 022001

[13] S.P. Baranov, Phys.Rev. **D66** (2002) 114003

[14] E. Braaten, S. Fleming, Phys.Rev.Lett. **74** (1995) 3327

[15] E. Braaten, B.A. Kniehl, J. Lee, Phys.Rev. **D62** (2000) 094005

[16] M. Kramer, Prog.Part.Nucl.Phys. **47** (2001) 141

[17] V.N. Gribov and L.N. Lipatov, Sov.J.Nucl.Phys. **15** (1972) 438;
Yu.A. Dokshitser, Sov.Phys.JETP **46** (1977) 641;
G. Altarelli and G. Parisi, Nucl.Phys. **B126** (1977) 298

[18] B.A. Kniehl, L. Zwirner, DESY preprint **99-124**, (1999); hep-ph/9909517.

[19] E. Kuraev, L. Lipatov, V. Fadin, Sov.Phys.JETP **44** (1976) 443;
Y. Balitskii and L. Lipatov, Sov.J.Nucl.Phys. **28** (1978) 822

[20] M. Gluck, E. Reya, A. Vogt, Z.Phys. **C67** (1995) 433

[21] CTEQ Coll., H.L. Lai et al. Eur.Phys.J. **C12** (2000) 375

[22] J. Blumlein, DESY preprint **95-121**, 1995

[23] H. Jung, G. Salam, Eur.Phys.J. **C19** (2001) 351

[24] M.A. Kimber, A.D. Martin and M.G. Ryskin, Phys.Rev. **D63** (2001) 114027

[25] J. Kwiecinski, A. Martin, A. Stasto, Phys.Rev. **D56** (1997) 3991

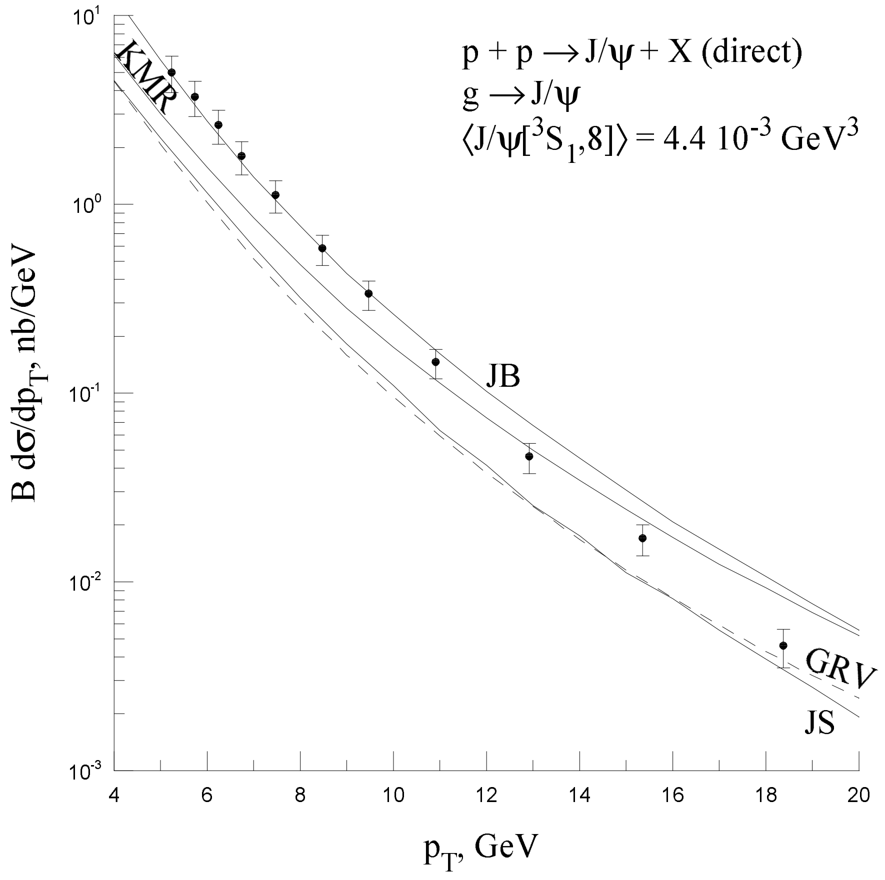

Figure 1: The spectrum of direct J/ψ on p_T at the $\sqrt{S}$=1800 GeV and $|\eta| < 0.6$. The data points are from [1]. The B is the J/ψ lepton branching.