Squeezed and entangled collinear gluon states at the non-perturbative stage of QCD jet evolution

V.I. Kuvshinov, V.A. Shaparau

Institute of Physics, National Academy of Sciences of Belarus
F. Skaryna av. 68, 220072 Minsk, Belarus
E-mail: kuvshino@dragon.bas-net.by; shaporov@dragon.bas-net.by

Abstract

At the non-perturbative stage of jet evolution, fluctuations of collinear gluons are smaller than those for coherent states providing indication of gluon squeezed states. We show that gluon entangled states which are closely related to the two-mode squeezed states of gluon fields can appear in analogy with the corresponding photon states in quantum optics.

1 Introduction

Analogies between multiple hadron in HEP and photon production in quantum optics (QO) have been discussed long ago [1]-[4]. It was shown, in particular, that the general distribution that characterizes $e^+e^-, p\bar{p}$, neutrino-induced collisions is a k-mode squeezed state distribution [5, 6], the multiplicity distribution of the pions has been explained by the formalism of the squeezed isospin states [7].

Squeezed states (SS), introduced by Stoler [8] and named by Hollenhorst [9], provoke great interest in connection with their uncommon properties: can have both sub-Poissonian (for coincide phases) and super-Poissonian (for antiphases) statistics corresponding to antibunching and bunching of photons [10]-[13] and characteristic behavior of the factorial and cumulant moments [14]. Moreover oscillatory behavior of the multiplicity distribution of the photon SS [11] is different from Poissonian and Negative binomial distributions (NBD). The squeezed light is generated from the coherent one by nonlinear devices and is pure quantum non-perturbative phenomenon [11]-[13].

Studying non-perturbative evolution of gluon states prepared by perturbative cascade stage in jets [15] we have proved within quantum chromodynamics (QCD) that this stage of jet evolution can be a source of gluon SS in analogy with nonlinear devices in QO for photon [16]-[18]. In these papers we investigated single-mode squeezing effect for virtual gluons with definite color and vector components during small temporal evolution. Using the local parton-hadron duality it is easy to show that in this case behavior of hadron multiplicity distribution in jet events is different from the negative binomial one, that is confirmed by experiments for $pp, p\overline{p}$-collisions [19]-[21].

At the same time two-mode photon SS [12] in the limit of infinite squeezing are isomorphic to the Bell states [22] which have been introduced in relation to the Einstein-Podolsky-Rosen (EPR) paradox [23], and they are one of the examples of the entangled states for two polarizations [24]. The states

$$\begin{aligned} &|\Phi^+\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 + |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2}, \qquad &|\Phi^-\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 - |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2}, \\ &|\Psi^+\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2}, \qquad &|\Psi^-\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2} \end{aligned} \tag{1}$$

are the basis of the Bell states. Each of these entangled states, for example, $|\Phi^{\pm}\rangle$, has a uncommon property: if one photon is registered with defined polarization (for example, with polarization $\updownarrow$), the other photon immediately becomes opposite polarized (longitudinal polarization). Thus a measurement over one particle has an instantaneous effect on the other, possibly located at a large distance. In particular, recently it was shown that it is possible for two photons to travel a total of 600 metros through free space and still remain "entangled" [25].

At finite squeeze factor r (r is positive) a continuous variables entangled state is known from quantum optics as a two-mode squeezed state [12, 24]

$$|\mathrm{f}\rangle = S(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle_1|n\rangle_2, \tag{2}$$

where $S(r) = \exp\{r(a_1^+a_2^+ - a_1a_2)\}$ is the operator of two-mode squeezing. Rewriting the expression (2) at small squeezing parameter r

$$|\mathrm{f}\rangle = |0\rangle_1|0\rangle_2 + \,r\,|1\rangle_1|1\rangle_2, \tag{3}$$

it is easy to show that the state vector $|\mathrm{f}\rangle$ describes the entangled state. Indeed, the entanglement condition of considering photon state can be verified by investigation of the conditional probability $P\,(Y_j/X_i)$ $(i,j=\overline{1,2})$

$$P\,(Y_j/X_i) = \frac{\|\langle Y_j|\,\langle X_i\,|\mathrm{f}\rangle\|^2}{\langle \mathrm{f}|\,X_i\rangle\,\langle X_i\,|\mathrm{f}\rangle} = \begin{cases} \delta_{ij} \\ \text{or} \\ 1-\delta_{ij}, \end{cases} \tag{4}$$

where $|X_1\rangle = |0\rangle_1, |X_2\rangle = |1\rangle_1$, $|Y_1\rangle = |0\rangle_2, |Y_2\rangle = |1\rangle_2$.

The dimensionless coefficient

$$y = \left[\frac{|\overline{a_1a_2^+}|^2 + |\overline{a_1a_2}|^2}{2(\overline{a_1^+a_1}+1/2)(\overline{a_2^+a_2}+1/2)}\right]^{1/2} \tag{5}$$

is the measure of entanglement for two-mode states [26], $0 \leq y < 1$ (entanglement is not observe when $y=0$). Here $\overline{a_ia_j^+} = \langle a_ia_j^+\rangle - \langle a_i\rangle\langle a_j^+\rangle$, a_i, a_j^+ are the annihilation and creation operators correspondingly. Averaging the annihilation and creation operators in the expression (5) over the vector $|\mathrm{f}\rangle$ (3) at small squeeze factor we have

$$y = \sqrt{2}r. \tag{6}$$

It is obvious that maximal coefficient of the entanglement is defined by maximal possible value of the squeeze factor r: $y_{max} \approx 0.462$ at $r = 0.327$.

Two-mode gluon states with two different colors can lead to $q\bar{q}$- entangled states. Interaction of the quark entangled states with stochastic vacuum (quantum measurement) has a remarkable property, namely, as soon as some measurement projects one quark onto a state with definite color, another quark also immediately obtains opposite color that leads to coupling of quark-antiquark pair, string tension inside $q\bar{q}$-pare and free propagation of colorless hadrons. Therefore, investigation of the gluon entangled states is an issue of the day.

2 Two-mode squeezed gluon states

In order to verify whether the gluon state vector describes the two-mode squeezed state on colors h and g, it is necessary to introduce the phase-sensitive Hermitian operators $(X_\lambda^{h,g})_1 = [b_\lambda^h + b_\lambda^g + b_\lambda^{h+} + b_\lambda^{g+}]/(2\sqrt{2})$ and $(X_\lambda^{h,g})_2 = [b_\lambda^h + b_\lambda^g - b_\lambda^{h+} - b_\lambda^{g+}]/(2i\sqrt{2})$ in analogy with quantum optics [12] and to establish conditions under which the variance of one of them can be less than the variance of a coherent state. Here $b_{(\lambda)}^h, b_\lambda^g (b_\lambda^{h+}, b_\lambda^{g+})$ are operators annihilating (creating) gluons of colors $h, g = \overline{1,8}$ and polarization index $\lambda = \overline{1,3}$.

Mathematically, the condition of two-mode squeezing of colors h, g is expressed in the form of the inequality

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle < 0. \tag{7}$$

Here N is the normal-ordering operator such as

$$\begin{aligned}
\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{1}{8}\Big\{&\left\langle\left(b_\lambda^h\right)^2\right\rangle - \left\langle b_\lambda^h\right\rangle^2 + \left\langle\left(b_\lambda^g\right)^2\right\rangle - \left\langle b_\lambda^g\right\rangle^2 + \left\langle\left(b_\lambda^{h+}\right)^2\right\rangle - \left\langle b_\lambda^{h+}\right\rangle^2 \\
+&\left\langle\left(b_\lambda^{g+}\right)^2\right\rangle - \left\langle b_\lambda^{g+}\right\rangle^2 + 2\Big[\left\langle b_\lambda^h b_\lambda^g\right\rangle - \left\langle b_\lambda^h\right\rangle\left\langle b_\lambda^g\right\rangle + \left\langle b_\lambda^{h+} b_\lambda^{g+}\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^{g+}\right\rangle\Big] \\
\pm 2&\Big[\left\langle b_\lambda^{h+} b_\lambda^h\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^h\right\rangle + \left\langle b_\lambda^{g+} b_\lambda^g\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^g\right\rangle + \left\langle b_\lambda^{h+} b_\lambda^g\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^g\right\rangle \\
&+ \left\langle b_\lambda^{g+} b_\lambda^h\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^h\right\rangle\Big]\Big\}.
\end{aligned} \tag{8}$$

The expectation values of the creation and annihilation operators for gluons with specified color and polarization indices are taken with the vector $|\mathrm{f}\rangle$ which describes the evolution of the virtual gluon field during a small time interval Δt within interaction representation

$$|\mathrm{f}\rangle \simeq |\mathrm{in}\rangle - i\,\Delta t\, H_{\mathrm{I}}(t_0)\,|\mathrm{in}\rangle, \tag{9}$$

where $H_{\mathrm{I}}(t_0) = H_{\mathrm{I}}^{(3)}(t_0) + H_{\mathrm{I}}^{(4)}(t_0)$ are the Hamiltonians of three-gluon ($H_{\mathrm{I}}^{(3)}$) and four-gluon ($H_{\mathrm{I}}^{(4)}$) self-interactions the explicit form of which are given in Appendix in momentum representation, $|\mathrm{in}\rangle$ is an initial state vector of the virtual gluon field, $\Delta t = t - t_0$ (below we will assume $t_0 = 0$ and consequently $\Delta t = t$).

We choose a product of the coherent states of the gluons with different colors and polarization indices as the initial state vector, that is $|\mathrm{in}\rangle \equiv |\,\alpha\rangle = \prod_{\lambda=1}^{3}\prod_{b=1}^{8}|\alpha_\lambda^b\rangle$, because any vector may be decomposed on these basis vectors and coherent states are widely used in QO [12, 13]. In analogy with QO gluon coherent state vector $|\alpha_\lambda^b\rangle$ is an eigenvector of the corresponding annihilation operator b_λ^b with the eigenvalue $\alpha_{\lambda_1}^b$ which can be written in terms of the gluon coherent field amplitude $|\alpha_{\lambda_1}^b|$ and phase γ_λ^b of a given gluon field $\alpha_{\lambda_1}^b = |\alpha_{\lambda_1}^b| e^{i\,\gamma_{\lambda_1}^b}$. In each gluon coherent state $|\alpha_\lambda^b\rangle$ the gluon number with fixed color b and polarization λ is arbitrary (the average multiplicity of a given gluon is equal to the square of the gluon coherent field amplitude $\langle n_\lambda^b\rangle = |\alpha_\lambda^b|^2$), and the phase of the state γ_λ^b is fixed.

Averaging the annihilation and creation operators $b^h_\lambda, b^g_\lambda, b^{h+}_\lambda, b^{g+}_\lambda$ in (8) with the evolved vector $|\mathrm{f}\rangle$ which is defined according to (9) and taking into account of chosen initial state vector we write the two-mode squeezing condition in the form

$$\begin{aligned}\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle &= \pm\frac{i\,t}{8}\Big\{\langle\alpha|[[H_{\mathrm{I}}(0), b^{h+}_\lambda], b^{h+}_\lambda]|\alpha\rangle - \langle\alpha|[b^h_\lambda,[b^h_\lambda, H_{\mathrm{I}}(0)]]|\alpha\rangle \\ &+ \langle\alpha|[[H_{\mathrm{I}}(0), b^{g+}_\lambda], b^{g+}_\lambda]|\alpha\rangle - \langle\alpha|[b^g_\lambda,[b^g_\lambda, H_{\mathrm{I}}(0)]]|\alpha\rangle \\ &+ 2\langle\alpha|[[H_{\mathrm{I}}(0), b^{h+}_\lambda], b^{g+}_\lambda]|\alpha\rangle - 2\langle\alpha|[b^g_\lambda,[b^h_\lambda, H_{\mathrm{I}}(0)]]|\alpha\rangle\Big\} < 0. \quad (10)\end{aligned}$$

It is easy to show that the three-gluon self-interaction (as for single-mode squeezing of gluons [18]) does not lead to squeezing effect since

$$\left.\begin{array}{lll}[[H^{(3)}_{\mathrm{I}}(0), b^{h+}_\lambda], b^{h+}_\lambda] = 0, & [[H^{(3)}_{\mathrm{I}}(0), b^{g+}_\lambda], b^{g+}_\lambda] = 0, & [[H^{(3)}_{\mathrm{I}}(0), b^{h+}_\lambda], b^{g+}_\lambda] = 0, \\ [b^g_\lambda,[b^h_\lambda, H^{(3)}_{\mathrm{I}}(0)]] = 0, & [b^h_\lambda,[b^h_\lambda, H^{(3)}_{\mathrm{I}}(0)]] = 0, & [b^g_\lambda,[b^g_\lambda, H^{(3)}_{\mathrm{I}}(0)]] = 0.\end{array}\right\} \quad (11)$$

Thus only the four-gluon self-interaction can yield a two-mode squeezing effect. Indeed, the two-mode squeezing condition can be written in the explicit form as

$$\begin{aligned}&\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}g^2(2\pi)^3\int d\tilde{k}_1 d\tilde{k}_2\sum_{\lambda_1,\lambda_2}\times\Big\{\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2) - b^b_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)|\alpha\rangle \\ &\times\Big[\delta(2\vec{k}-\vec{k_1}-\vec{k_2}) - \delta(2\vec{k}+\vec{k_1}+\vec{k_2})\Big]\Big[(f_{ahb}f_{ahc}+f_{agb}f_{agc}+2f_{ahb}f_{agc})\Big(\varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_{\lambda_2}(k_2) \\ &\times\varepsilon^\lambda_\nu(k)\,\varepsilon^\nu_\lambda(k) - \varepsilon^{\lambda_1}_\mu(k_1)\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\,\varepsilon^\nu_\lambda(k)\Big) - 2f_{ahg}f_{abc}\varepsilon^{\lambda_1}_\mu(k_1)\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\varepsilon^\nu_\lambda(k)\Big] \\ &+2\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)|\alpha\rangle\times\Big[\delta(2\vec{k}-\vec{k_1}+\vec{k_2}) - \delta(2\vec{k}+\vec{k_1}-\vec{k_2})\Big](f_{ahb}f_{ahc}+f_{agb}f_{agc} \\ &+f_{ahc}f_{agb}+f_{ahb}f_{agc})\Big(\varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_{\lambda_2}(k_2)\varepsilon^\lambda_\nu(k)\,\varepsilon^\nu_\lambda(k) - \varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\,\varepsilon^\nu_\lambda(k)\Big)\Big\} < 0. \\ &\hspace{30em}(12)\end{aligned}$$

Here g is a self-interaction constant, $d\tilde{k} = \dfrac{d^3k}{(2\pi)^3 2k_0}$, k_0 is gluon energy, ε^μ_λ is polarization vector, f_{ahb} are the structure constants of $\mathrm{SU_c}(3)$ group.

For visualization, let us investigate the obtained two-mode squeezing condition (12) for collinear gluon. In this case the corresponding squeezing condition is

$$\begin{aligned}\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle &= \pm t\,\frac{\alpha_s\pi}{4k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc}) \\ &\times\sum_{\lambda_1\neq\lambda}|\alpha^b_{\lambda_1}||\alpha^c_{\lambda_1}|\sin(\gamma^b_{\lambda_1}+\gamma^c_{\lambda_1}) < 0. \quad (13)\end{aligned}$$

Here we have taken into account that $\alpha^b_{\lambda_1} = |\alpha^b_{\lambda_1}|e^{i\,\gamma^b_{\lambda_1}}$ and $\alpha^c_{\lambda_1} = |\alpha^c_{\lambda_1}|e^{i\,\gamma^c_{\lambda_1}}$, $\alpha_s = g^2/(4\pi)$ is a coupling constant[1]. The two-mode squeezing condition is fulfilled for all cases except for $\gamma^b_{\lambda_1}+\gamma^c_{\lambda_1} = 0, \pi$. In particular, if all initial gluon coherent fields are real or imaginary, then the two-mode squeezing condition is not fulfilled as in the single-mode case. Obviously, the larger are both amplitudes of the initial gluon coherent fields with different color and polarization indices and coupling constants, the larger is two-mode squeezing effect.

Thus non-perturbative gluon evolution (at large coupling constant) is very important for investigation of the squeezing effect.

[1] Here we also take into consideration gluons with momenta directed toward the jet development.

3 Entangled collinear gluon states

In analogy with QO we assume that two-mode gluon SS with fixed colors h, g is closely connected to corresponding entangled states of gluons.

At small value of the squeeze factor we have

$$r = 2 \left| \left\langle N \left(\Delta (X_\lambda^{h,g})_{\frac{1}{2}} \right)^2 \right\rangle \right|. \tag{14}$$

As initial states we can take a vector including $|0_\lambda^h\rangle\, |0_\lambda^g\rangle$ evolution of which at small time can be written in terms of the squeeze factor as

$$|\mathrm{f}\rangle = |0_\lambda^h\rangle |0_\lambda^g\rangle + r\, |1_\lambda^h\rangle |1_\lambda^g\rangle, \tag{15}$$

where the squeeze factor for the collinear gluons is defined as

$$\begin{aligned} r &= t\, \frac{\alpha_s \pi}{2k_0} (f_{ahb} f_{ahc} + f_{agb} f_{agc} + f_{ahb} f_{agc} + f_{agb} f_{ahc}) \\ &\times \sum_{\lambda_1 \neq \lambda} |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c). \end{aligned} \tag{16}$$

From the expression (16) it follows that the squeeze factor is not equal to zero for all cases except for $\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c = 0, \pi$.

The entanglement condition of gluon states with colors h, g and polarization λ can be verified by investigation of the conditional probability $P\left(Y_j / X_i\right) (i, j = \overline{1,2})$ in analogy with the corresponding condition for photons (4) assuming that $|X_1\rangle = |0_\lambda^h\rangle$, $|X_2\rangle = |1_\lambda^h\rangle$, $|Y_1\rangle = |0_\lambda^g\rangle$, $|Y_2\rangle = |1_\lambda^g\rangle$. It is not complicated to see that the condition (4) is fulfilled for the case of the two-mode squeezing for the state vector $|\mathrm{f}\rangle$ (15) which describes the non-perturbative evolution of the gluon fields during small time.

In case two-mode gluon states, rewriting the expression (5) for the entanglement coefficient as

$$y = \left[\frac{|\overline{b_\lambda^h b_\lambda^{g+}}|^2 + |\overline{b_\lambda^h b_\lambda^g}|^2}{2(\overline{b_\lambda^{h+} b_\lambda^h} + 1/2)(\overline{b_\lambda^{g+} b_\lambda^g} + 1/2)} \right]^{1/2} \tag{17}$$

we can write the condition of the entanglement with account of the evident form (16) for r

$$0 < t\, \frac{\alpha_s \pi}{\sqrt{2} k_0} (f_{ahb} f_{ahc} + f_{agb} f_{agc} + f_{ahb} f_{agc} + f_{agb} f_{ahc}) \sum_{\lambda_1 \neq \lambda} |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) < 1. \tag{18}$$

Thus in analogy with quantum optics, as a result of four-gluon self-interaction we obtain two-mode squeezed gluon states which are also entangled.

4 Conclusion

Investigating the gluon fluctuations we have proved theoretically the possibility of existence of the gluon two-mode squeezed states. The emergence of such remarkable states becomes possible owing to the four-gluon self-interaction. The three-gluon self-interaction does not lead to squeezing effect.

We have shown that non-perturbative QCD jet evolution leads both to squeezing and entanglement of gluons. It should be noted that the larger are both amplitudes of the initial gluon coherent fields with different color and polarization indices and coupling constants, the larger is squeezing effect of the color gluons. We have demonstrated that entanglement condition of the gluon states with fixed two colors and polarizations is defined by the corresponding squeeze factor which is dependent on the amplitudes and phases of the initial coherent gluon fields.

Two-mode gluon states with two different colors can lead to $q\bar{q}$-entangled states which role could be very significant for explanation of the confinement phenomenon.

Appendix

$$
\begin{aligned}
H_{\mathrm{I}}^{(3)}(t) = {} & ig(2\pi)^3 f_{abc} \sum_{\lambda_1,\lambda_2,\lambda_3} \int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3 \times \Big\{ \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\
& \times [b^a_{\lambda_1}(k_1) b^b_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\, e^{-2i(k_{01}+k_{02}+k_{03})t} - b^{a+}_{\lambda_1}(k_1) b^{b+}_{\lambda_2}(k_2) b^{c+}_{\lambda_3}(k_3)\, e^{2i(k_{01}+k_{02}+k_{03})t}] \\
& + b^{a+}_{\lambda_1}(k_1) b^b_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\, e^{2i(k_{01}-k_{02}-k_{03})t}\, \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3) \\
& \times \Big[\vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_3}_{\nu}(k_3) + \vec{k}_2 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\nu}(k_2) - \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \Big] \\
& + b^{a+}_{\lambda_1}(k_1) b^{b+}_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\, e^{2i(k_{01}+k_{02}-k_{03})t}\, \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3) \\
& \times \Big[\vec{k}_1 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\nu}(k_2) + \vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_3}_{\nu}(k_3) - \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \Big] \Big\}, \quad (A.1)
\end{aligned}
$$

$$
\begin{aligned}
H_{\mathrm{I}}^{(4)}(t) = {} & \frac{g^2}{4}(2\pi)^3 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3 d\tilde{k}_4 \Big\{ \varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\mu}(k_3) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_4}_{\nu}(k_4) f_{abc} f_{ade} \\
& \times \Big[\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \Big(b^b_{\lambda_1}(k_1) b^c_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\, e^{-2i(k_{01}+k_{02}+k_{03}+k_{04})t} \\
& + b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^{d+}_{\lambda_3}(k_3) b^{e+}_{\lambda_4}(k_4)\, e^{2i(k_{01}+k_{02}+k_{03}+k_{04})t} \Big) \\
& + 4 b^{b+}_{\lambda_1}(k_1) b^c_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\, e^{2i(k_{01}-k_{02}-k_{03}-k_{04})t}\, \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \\
& + 4 b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^{d+}_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\, e^{2i(k_{01}+k_{02}+k_{03}-k_{04})t}\, \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}_4) \Big] \\
& + 2 b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\, e^{2i(k_{01}+k_{02}-k_{03}-k_{04})t}\, \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \\
& \times \Big[\varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\mu}(k_3) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_4}_{\nu}(k_4) (f_{abc} f_{ade} + f_{abe} f_{adc}) \\
& \quad + \varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\mu}(k_2) \varepsilon^{\nu}_{\lambda_3}(k_3) \varepsilon^{\lambda_4}_{\nu}(k_4) f_{abd} f_{ace} \Big] \Big\}. \qquad (A.2)
\end{aligned}
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