The Randall-Sundrum model as it is

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Abstract

We discuss the origin and the structure of the Randall-Sundrum model with two branes. The second variation Lagrangian of this model is derived, the corresponding equations of motion for the metric fluctuations are studied and decoupled, and the physical degrees of freedom of the model are found. We argue that in order to have a physically meaningful model on the negative tension brane, the inverse size of the extra dimension must be of the order of $1TeV$. We also present exact solutions to the equations of motion for linearized gravity in the model with matter on the branes and calculate the Newtonian limit in it. The effects of "shadow" matter, which lives on the other brane, are considered and compared with those of ordinary matter for both positive and negative tension branes. In particular, we calculate light deflection and Newton's law in the zero mode approximation and explicitly distinguish the contribution of the radion field. We also consider an example of stabilized Randall-Sundrum model and discuss the problem of the radion mass.

1 Introduction

The present-day approach to Kaluza-Klein theories is based on the idea due to Rubakov and Shaposhnikov of localization of fields on a domain wall [1, 2]. In a first step it is natural to drop the mechanism of localization of fields and to treat the domain wall as an infinitely thin object, i.e. as a membrane, and to consider the effects due to the gravitational interaction of such 3-branes.

The main advantage of such scenario is a solution to the hierarchy problem. It turns out that within the framework of this scenario extra dimensions may be much larger, than the Planck length [3] (or even infinite). In this case the Planck mass in multidimensional space (in the bulk) may be much less, than the Planck mass in four-dimensional space-time. In other words, gravity in multidimensional space-time becomes "strong" not at the energies of the order of $10^{19}\,GeV$, but at much lower energies, maybe of the order of $1 \div 10\,TeV$. The effects due to interaction of this "strong" quantum gravity with the fields of the standard model can be observable already at the energies, which are available at the existing colliders.

There were numerous attempts to calculate such effects by considering quantum linearized gravity, interacting with the fields of the standard model located on a brane, in the flat background in space-time $E = M \times T^d$, T^d being a d-dimensional torus. The assumption that the multidimensional background metric can be taken to be flat means that the proper gravitational field of the brane can be neglected. The validity of the results obtained within this approach depends upon whether this approximation is good or not so good. In any case, studying the equations of motion for multidimensional gravity interacting with a brane of non-zero tension, it is not difficult to understand that if extra dimensions are compact, there should exist at least two branes, and the background metric must be essentially non-flat.

A particular realization of this scenario was proposed in paper [4]. In this paper an exact solution for a system of two branes interacting with gravity in a five-dimensional space-time was found. It turned out that this model also allowed a solution to the hierarchy problem due to an exponential warp factor appearing in the metric [4, 5], and there are many papers on the effects of linearized gravity in the background of the Randall-Sundrum solution. However, almost all the papers on the phenomenology of the model assume the size of extra dimension to be of the order of the Planck mass, which is absolutely erroneous. Another problem is the explicit identification of the physical degrees of freedom of the model. Again most of the papers on the phenomenology of the model either completely drop the radion field, or use the Gaussian normal gauge, where this degree of freedom is absorbed by the four-dimensional gravitational field. The Gaussian normal gauge being local, i.e. defined not in the whole multidimensional space-time, this approach cannot provide for a consistent quantization of all the physical degrees of freedom of the model.

In the present contribution we are going to discuss these issues in detail. To this end, we consider the Lagrangian of quadratic fluctuations around the Randall-Sundrum solution and discuss its properties. In particular, we put a special emphasis on the choice of a convenient gauge. We also carry out a diagonalization of this Lagrangian and of the corresponding equations of motion, so that the physical degrees of freedom are distinguished explicitly and can be consistently quantized by the standard methods. This allows us to study the interaction of the linearized gravity with matter on the branes in more detail than it was done in other papers. In particular, we show that the model on the negative tension brane is physically acceptable only if the radion field acquires a mass, or, equivalently, the separation distance is stabilized. We discuss the Goldberger-Wise stabilization mechanism, consider an example of the stabilized RS model and give an estimate of the radion mass.

The paper is organized as follows. In Sect. 2 we derive an expression for the Lagrangian of quadratic fluctuations in the Randall-Sundrum model. Here we also discuss the gauge transformations in the model and the identification of the physical degrees of freedom. In Sect. 3 we calculate the couplings of the fields, arising from the five-dimensional metric, to matter. We emphasize that, for the proper interpretation of the interactions, Galilean coordinates on the branes should be used. Sect. 4 is devoted to the discussion of gravity theories on the branes. The Goldberger-Wise mechamism and an example of the stabilized RS model are discussed in Sect. 5.

2 Second variation Lagrangian

The Randall-Sundrum model [4] describes gravity in a five-dimensional space-time E with two branes embedded into it. We denote the coordinates by $\{x^M\} \equiv \{x^\mu, y\}$, $M =$

$0,1,2,3,4$, $\mu = 0,1,2,3$, the coordinate $x^4 \equiv y$ parameterizing the fifth dimension. It forms the orbifold S^1/Z_2, which is realised as the circle of the circumference $2R$ with points y and $-y$ identified. Correspondingly, we have the usual periodicity condition in space-time E, which identifies points (x,y) and $(x, y+2nR)$, and the metric g_{MN} satisfies the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x,-y) &= g_{\mu\nu}(x,y), \\ g_{\mu 4}(x,-y) &= -g_{\mu 4}(x,y), \\ g_{44}(x,-y) &= g_{44}(x,y). \end{aligned} \tag{1}$$

The branes are located at the fixed points of the orbifold, $y = 0$ and $y = R$.

The action of the model is

$$S = S_g + S_1 + S_2, \tag{2}$$

where S_g, S_1 and S_2 are given by

$$\begin{aligned} S_g &= \frac{1}{16\pi \hat{G}} \int_E (R - \Lambda)\sqrt{-g}\, d^4x dy, \\ S_1 &= V_1 \int_E \sqrt{-\tilde{g}} \delta(y) d^4x dy, \\ S_2 &= V_2 \int_E \sqrt{-\tilde{g}} \delta(y-R) d^4x dy. \end{aligned} \tag{3}$$

Here $\tilde{g}_{\mu\nu}$ is the induced metric on the branes and the subscripts 1 and 2 label the branes. We also note that the signature of the metric g_{MN} is chosen to be $(-1,1,1,1,1)$.

The Randall-Sundrum solution for the metric is given by

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{4}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y) = -k|y|$ in the interval $-R \leq y \leq R$. The parameter k is positive and has the dimension of mass, the parameters Λ and $V_{1,2}$ are related to it as follows:

$$\Lambda = -12k^2, \quad V_1 = -V_2 = -\frac{3k}{4\pi\hat{G}}.$$

We see that brane 1 (also called the Planck brane) has a positive energy density, whereas brane 2 (the TeV brane) has a negative one. The function σ has the properties

$$\partial_4 \sigma = -k\, sign(y), \quad \frac{\partial^2 \sigma}{\partial y^2} = -2k(\delta(y) - \delta(y-R)) \equiv -2k\tilde{\delta}. \tag{5}$$

Here and in the sequel $\partial_4 \equiv \frac{\partial}{\partial y}$.

We denote $\hat{\kappa} = \sqrt{16\pi\hat{G}}$, where $\hat{G}$ is the five-dimensional gravitational constant, and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat{\kappa} h_{MN}, \tag{6}$$

h_{MN} being the metric fluctuations. Substituting this parameterization into (2) and retaining the terms of the zeroth order in $\hat{\kappa}$, we get the second variation Lagrangian of this model [6]:

$$\begin{aligned} L/\sqrt{-\gamma} &= -\frac{1}{4}\left(\nabla_R h_{MN}\nabla^R h^{MN} - \nabla_R h \nabla_R h + 2\nabla_M h^{MN}\nabla_N h - \right. \\ &- \left. 2\nabla_M h^{MN}\nabla^R h_{RN}\right) + \frac{k^2}{2}(h_{MN}h^{MN} + hh) + \\ &+ \left[-2k h_{MN}h^{MN} + kh\tilde{h} - kh_{M\nu}h^{M\nu} + 3k(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}\tilde{h}\tilde{h})\right]\tilde{\delta}. \end{aligned} \tag{7}$$

The action built with this Lagrangian is invariant under the gauge transformations

$$h'_{MN}(x,y) = h_{MN}(x,y) - (\nabla_M \xi_N(x,y) + \nabla_N \xi_M(x,y)), \tag{8}$$

where ∇_M is the covariant derivative with respect to the background metric γ_{MN}, and the functions $\xi_N(x,y)$ satisfy the orbifold symmetry conditions

$$\begin{aligned} \xi^\mu(x,-y) &= \xi^\mu(x,y) \\ \xi^4(x,-y) &= -\xi^4(x,y). \end{aligned} \tag{9}$$

With the help of these gauge transformations we can impose the gauge

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x), \tag{10}$$

which will be called the *unitary gauge* (see [6]). The field ϕ is called the radion field. If we calculate in the linear approximation the separation distance between the branes along the geodesic $x^\mu = const$ of the metric γ, we get

$$\int_0^R ds = \int_0^R \sqrt{1+\hat{\kappa}h_{44}}dy \sim \int_0^R 1 + \frac{\hat{\kappa}h_{44}}{2}dy = R(1+\frac{\hat{\kappa}\phi}{2}).$$

Thus, the radion describes the fluctuations of the branes with respect to each other.

We would like to emphasize once again that the branes remain straight in this gauge, i.e. we *do not* use the bent-brane formulation, which allegedly destroys the structure of the model (this problem was discussed in [7]).

The general form of interaction with matter is standard,

$$\frac{\hat{\kappa}}{2}\int_{B_1} h^{\mu\nu}(x,0)T^1_{\mu\nu}dx + \frac{\hat{\kappa}}{2}\int_{B_2} h^{\mu\nu}(x,R)T^2_{\mu\nu}\sqrt{-\gamma_2}dx, \tag{11}$$

where $T^1_{\mu\nu}$ and $T^2_{\mu\nu}$ are energy-momentum tensors of the matter on brane 1 and brane 2 respectively:

$$T^{1,2}_{\mu\nu} = 2\frac{\delta L^{1,2}}{\delta\gamma^{\mu\nu}} - \gamma^{1,2}_{\mu\nu}L^{1,2}.$$

As follows from formula (11), $h_{\mu\nu}$ is the only physical field of the model, since only this field interacts with matter on the branes. Obviously, the unitary gauge conditions

(10) do not fix the gauge of this field. In fact, after imposing these gauge conditions there remain gauge transformations of the form

$$\xi_\mu = e^{2\sigma}\epsilon_\mu(x), \tag{12}$$

which change the longitudinal components of the field $h_{\mu\nu}$. Nevertheless, it turns out that it is convenient to solve the equations of motion for linearized gravity in the unitary gauge and then to choose an appropriate gauge in our four-dimensional world on the brane. We will use the de Donder gauge for the field $h_{\mu\nu}$ on the branes, which corresponds to the choice of *harmonic coordinates*.

The equations of motion for different components of the metric fluctuations in the unitary gauge take the form (see [6]):

1) $\mu\nu$-component

$$\begin{aligned} & \frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial {x^4}^2}\right) - \\ - & \ 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \frac{1}{2}\partial_\mu\partial_\nu\phi + \\ + & \ \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \frac{\partial^2\tilde{h}}{\partial {x^4}^2} - 4\partial_4\sigma\partial_4\tilde{h} - \partial_\rho\partial^\rho\phi + 12k^2\phi\right) + \\ + & \ [2kh_{\mu\nu} - 3k\gamma_{\mu\nu}\phi]\,\tilde{\delta} = -\frac{\hat{\kappa}}{2}T_{\mu\nu} \end{aligned} \tag{13}$$

2) $\mu 4$-component,

$$\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = 0, \tag{14}$$

which plays the role of a constraint,

3) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = 0, \tag{15}$$

with $T_{\mu\nu}$ being the energy-momentum tensor of the matter and $\tilde{h} = \gamma^{\mu\nu}h_{\mu\nu}$. In what follows, we will also use an auxiliary equation, which is obtained by multiplying the equation for 44-component by 2 and subtracting it from the contracted equation for $\mu\nu$-component. This equation contains $\tilde{h}$ and ϕ only and has the form:

$$\frac{\partial^2\tilde{h}}{\partial {x^4}^2} + 2\partial_4\sigma\partial_4\tilde{h} - 8k^2\phi + 8k\phi\tilde{\delta} + \partial_\mu\partial^\mu\phi = \frac{\hat{\kappa}}{3}T^\mu_\mu. \tag{16}$$

If $T_{\mu\nu} = 0$, the physical degrees of freedom of the model can be extracted by the substitution [6]

$$h_{\mu\nu} = b_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial_\nu\phi. \tag{17}$$

with $c = \frac{kR}{e^{2kR}-1}$. It turns out that the field $b_{\mu\nu}(x^\mu, y)$ describes the massless graviton [4, 5] and massive Kaluza-Klein spin-2 fields, whereas $\phi(x)$ describes the scalar radion field.

Apparentely, the radion field was first identified in Ref. [8] (see also [9]) and discussed in [10, 11, 12, 13].

The remaining gauge transformations are sufficient to impose the conditions

$$\partial^\mu b_{\mu\nu} = 0, \quad \tilde{b} = b = 0. \tag{18}$$

These conditions define the gauge often called the transverse-traceless (TT) gauge. Having imposed this gauge, we are still left with residual gauge transformations

$$\xi_\mu = e^{2\sigma}\epsilon_\mu(x), \quad \Box\epsilon_\mu = 0, \quad \partial^\mu\epsilon_\mu = 0, \tag{19}$$

which are important for determining the number of degrees of freedom of the massless mode of $b_{\mu\nu}$.

In this gauge the constraint equation and the 44-equation are satisfied trivilly. The equation for $\mu\nu$-components takes the form:

$$\frac{e^{-2\sigma}}{2}\Box b_{\mu\nu} + \frac{1}{2}\frac{\partial^2 b_{\mu\nu}}{\partial y^2} - 2k^2 b_{\mu\nu} + 2k b_{\mu\nu}\tilde{\delta} = 0. \tag{20}$$

To find an effective Lagrangian for the system it remains to decompose the field $b_{\mu\nu}$ into the modes with definite masses and to integrate the Lagrangian over y.

To do this, we expand the field $b_{\mu\nu}$ as follows:

$$b_{\mu\nu}(x,y) = \sqrt{2}\sum_n b^n_{\mu\nu}(x)\psi_n(y), \tag{21}$$

where $b^n_{\mu\nu}(x)$ are ordinary spin-2 transverse traceless 4-dimensional fields with masses m_n, and $\psi_n(y)$ and m_n are eigenfunctions and eigenvalues of the problem

$$\left[\frac{1}{2}\frac{d^2}{dy^2} + 2k(\delta(y) - \delta(y-R)) - 2k^2\right]\psi_n(y) = -\frac{m_n^2}{2}e^{2k|y|}\psi_n(y). \tag{22}$$

This equation can be solved exactly in terms of the Bessel functions, and there exists a complete orthonormal system $\{\psi_n(y), n = 0, 1, \ldots\}$ of solutions of the eigenvalue problem, satisfying

$$\int_{-R}^{R} e^{2k|y|}\psi_m(y)\psi_n(y)dy = \delta_{mn}.$$

The mass eigenvalues are given by $m_n = \beta_n k \exp(-kR)$, with β_n for small n being approximately the roots of the equation $J_1(\beta_n) = 0$.

Substituting decomposition (21) into Lagrangian (7) in the TT gauge, passing to the corresponding action, integrating explicitly over the orbifold and rescaling the radion field, we get an effective action for the physical degrees of freedom of the 5-dimensional gravitational field

$$S_{eff} = \int\left(-\frac{1}{2}\sum_n(\partial_\mu b^n_{\rho\sigma}\partial^\mu b^{n,\rho\sigma} + m_n^2 b^n_{\rho\sigma}b^{n,\rho\sigma}) - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)dx. \tag{23}$$

Thus, we have a massless spin-2 tensor field, an infinite tower of massive spin-2 tensor fields and just one massless scalar radion. A very important point is that, due to the form of the solution with zero mass, the remaining gauge transformations (19) result in the following gauge transformations of the field $b^0_{\mu\nu}(x)$:

$$b'^0_{\mu\nu}(x) = b^0_{\mu\nu}(x) - (\partial_\mu \zeta_\nu(x) + \partial_\nu \zeta_\mu(x)), \quad \partial^\mu \zeta_\mu(x) = 0, \quad \Box \zeta_\nu(x) = 0. \tag{24}$$

This guarantees that the field $b^0_{\mu\nu}(x)$ has only two degrees of freedom, and, therefore, can be identified with the field of the massless graviton.

The Lagrangian in (23) is a sum of the free Lagrangians for the physical degrees of freedom and therefore it can be consistently quantized by the standard methods. To study the interaction of these quantized fields with the fields of the standard model, one has to find the corresponding interaction Lagrangians. This problem is addressed in the next section.

3 Physical interpretation of the RS model

Using the interaction Lagrangian (11), we find that the interaction of the graviton fields $b^n_{\mu\nu}(x)$ and the massless canonically normalized radion field φ with matter on brane 1 is given by

$$\frac{1}{2}\int_{B_1} \left(\kappa_1 \, b^0_{\mu\nu}(x) T^{\mu\nu} + \kappa_1 \sum_{n=1}^{\infty} \omega_n^{(1)} b^n_{\mu\nu}(x) T^{\mu\nu} - \frac{\kappa_2}{\sqrt{3}} \varphi \, T^\mu_\mu \right) dx. \tag{25}$$

Here

$$\kappa_1 = \left(\frac{k}{(1-e^{-2kR})M^3} \right)^{1/2} \approx \frac{\sqrt{k}}{M^{3/2}} \quad \kappa_2 = \kappa_1 e^{-kR}, \quad \omega_n^{(1)} \sim 1, \tag{26}$$

and $M = (16\pi \hat{G})^{-1/3}$ is the fundamental mass scale of the theory. Since κ_1 is the coupling constant of the massless graviton, it can be expressed in terms of the 4-dimensional gravitational constant G_1 on brane 1 as $\kappa_1 = \sqrt{16\pi G_1}$, which gives the relation

$$G_1 = \frac{\hat{G} k}{1 - e^{-2kR}}$$

between the constants on brane 1 [4, 14].

To find the interaction of the 5-dimensional gravity with matter on brane 2 turns out to be a more complicated problem, because the coordinates $\{x^\mu\}$ in (4), in which we work, are not Galilean on this brane (coordinates are called Galilean, if $g_{\mu\nu} = diag(-1,1,1,1)$, see [15]). This fact was also noted in papers [14, 16].

To solve this problem, we follow a simple logic and introduce the Galilean coordinates $\{z^\mu\}$ on brane 2 related to $\{x^\mu\}$ by $x^\mu = e^{kR} z^\mu$. In order to find the interaction of the 5-dimensional gravity with matter on brane 2 we have to pass to the Galilean coordinates $\{z^\mu\}$ in the effective action (23) as well. This results in the following expression:

$$S_{eff} = \int \left(-\frac{1}{2} \sum_n (\partial_\mu u^n_{\rho\sigma} \partial^\mu u^{n,\rho\sigma} + (m_n e^{kR})^2 u^n_{\rho\sigma} u^{n,\rho\sigma}) - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) dz. \tag{27}$$

Now we are able to write down the interaction of these fields with matter on brane 2. The interaction Lagrangian on this brane in (11) can be rewritten so that the coupling strengths are evident:

$$\frac{1}{2}\int_{B_2}\left(\frac{1}{M_{Pl}}u^0_{\mu\nu}(z)T^{\mu\nu}-\sum_{n=1}^{\infty}\frac{\omega_n}{\Lambda_\pi}u^n_{\mu\nu}(z)T^{\mu\nu}-\frac{1}{\Lambda_\pi\sqrt{3}}\varphi T^\mu_\mu\right)dz. \tag{28}$$

Here $1/(16\pi M_{Pl}^2)=G_2=\hat{G}k/(e^{2kR}-1)$, $\Lambda_\pi=M_{Pl}e^{-kR}\approx\sqrt{M^3/k}$, and the parameter ω_n is of the order of unity for small n. The masses of KK-excitations, which can be found from (27), turn out to be $m_n=k\beta_n$, so that $m_n\sim k\sim 1$ TeV for small n.

Formulas (23), (25) and (27), (28) enable one to calculate the processes due to the interaction of the linearized quantum gravity in RS1 model with the fields of the Standard Model both on brane 1 and brane 2. The energy-momentum tensor of the Standard Model can be found, for example, in [17].

4 Gravity on the branes

Up to now we have considered the RS model without mater on the branes. However, the situation is rather different, when there is matter on the branes. Let us first consider the case, where matter is located on the brane at 0, i.e. the energy-momentum tensor is of the form $T_{\mu\nu}=t_{\mu\nu}(x)\delta(y)$. The substitution, which allows one to decouple the equations, looks like

$$h_{\mu\nu}=u_{\mu\nu}+\gamma_{\mu\nu}(\sigma-c)\phi+\frac{1}{2k^2}\left(\sigma-c+\frac{1}{2}+ce^{2kR}\right)\partial_\mu\partial_\nu\phi. \tag{29}$$

The auxiliary equation results in

$$\Box\phi=\frac{\hat{\kappa}}{6R}t,\quad t\equiv t^\mu_\mu \tag{30}$$

and a rather complicated equation for $u_{\mu\nu}$. An imporatant point is that we can impose the de Donder gauge condition on the field $u_{\mu\nu}$

$$\partial^\nu\left(u_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}u\right)=0 \tag{31}$$

and solve the equation for it exactly:

$$\begin{aligned} u_{\mu\nu} &= -\hat{\kappa}\sum_{n=0}^{\infty}\psi_n(0)\psi_n(y)(\Box-m_n^2)^{-1}[t_{\mu\nu}-\frac{1}{3}(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box})t]+ \\ &+ \frac{\hat{\kappa}k}{6(1-e^{-2kR})}e^{2\sigma}\Box^{-1}(\eta_{\mu\nu}+2\frac{\partial_\mu\partial_\nu}{\Box})t \end{aligned} \tag{32}$$

Let us consider a static point mass with energy-momentum tensor $t_{00}=M\delta(\vec{x}), t_{0k}=t_{ij}=0$ (and $\tilde{t}_{00}=2\pi M\delta(p_0)$), M denoting the inertial mass. Then for an observer on brane 1 the gravitational potential is

$$V\approx-\frac{M}{r}G_1\left(1+\frac{4\cos(kr)}{3\pi kr}-\frac{4}{3\pi k^2r^2}\left[sin(kr)-si(kr)\right]\right), \tag{33}$$

where $G_1 = \frac{\hat{G}k}{1-e^{-2kR}}$ and

$$si(b) = \int_0^b \frac{\sin t}{t}\, dt.$$

If the observer is on brane 2, the potential is

$$\begin{aligned} V &\approx -G_1 e^{-kR} Mk \frac{8}{3\pi}\left(1 + \frac{3\pi}{4\sqrt{2}} - 0.2\sqrt{\frac{\pi}{2}} + 0.099\sqrt{\frac{\pi}{2}}\frac{1}{6}(kr)^2\right) \approx \qquad (34) \\ &\approx -G_1 e^{-kR} Mk \left(2.05 + 0.02(kr)^2\right)\ kr << 1; \\ V &\approx -\frac{4}{3} G_1 e^{-kR} \frac{M}{r}\left(1 + \sqrt{\frac{2}{\pi}} e^{-1} \frac{\cos(kr)}{kr}\right),\ 1 << kr, \qquad (35) \end{aligned}$$

r being the distance from the mirror point.

When matter is on brane 2 (at $y = R$), all the reasonings are the same, as presented above. The coresponding substitution looks like

$$h_{\mu\nu} = u_{\mu\nu} + e^{2\sigma}\eta_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma + \frac{1}{2}\right)\partial_\mu\partial_\nu\phi. \qquad (36)$$

The solution for $u_{\mu\nu}$ is also given by (32), provided $\psi_n(0)$ is replaced by $\psi_n(R)$.

If a static point source is on brane 2, the results for the gravitational field are:

I) The observer is on brane 2:

$$V \approx -G_2 \frac{M}{r}\left(1 - e^{2kR} + \frac{8e^{2kR}}{3\pi} si(kr) + \frac{2e^{2kR}}{3\pi}\left[\frac{\cos(kr)}{kr} + \frac{\sin(kr)}{(kr)^2}\right]\right), \qquad (37)$$

where $G_2 = \frac{\hat{G}k}{e^{2kR}-1}$ is the effective gravitational constant on brane 2.

2) The observer is on brane 1 ("shadow" matter also):

$$\begin{aligned} V &\approx -\frac{4}{3} e^{kR} G_2 \frac{M}{r}\left(1 + \sqrt{\frac{2}{\pi}} e^{kR} e^{-1} \frac{\cos(kre^{-kR})}{kr}\right),\ e^{kR} << kr; \qquad (38) \\ V &\approx -G_2 Mk \frac{8}{3\pi}\left(1 + \frac{3\pi}{4\sqrt{2}} - 0.2\sqrt{\frac{\pi}{2}} + 0.099\, e^{-2kR}\sqrt{\frac{\pi}{2}}\frac{1}{6}(kr)^2\right) \approx \qquad (39) \\ &\approx -G_2 Mk \left(2.05 + 0.02\, e^{-2kR}(kr)^2\right)\ kr << e^{kR}. \end{aligned}$$

We would like to note that in all formulas for the Newtonian limit, presented above, parameter M denotes the physical mass in Galilean coordinates on the brane the mass is located on.

In paper [6] possible values of parameter k were discussed. It was shown that the value $k \sim 1Tev$ is the only one admissible for a correct physical interpretation of the theory on brane 2. This means, that for $r \sim 1cm$ the value $kr \sim 10^{17}$ and the terms proportional to $\sin(kr)$ and $\cos(kr)$ can be dropped as negligible. For example, since $si(t \to \infty) \to \frac{\pi}{2}$, equation (37) takes the form

$$V \approx -G_2 \frac{M}{r}\left(1 + \frac{1}{3} e^{2kR}\right). \qquad (40)$$

The same arguments can be applied to other cases of matter and observer disposition. Since the contribution of massive modes are negligible for $r >> 10^{-17}cm$, only the zero modes constitute the Newtonian limit. Unfortunately, in formulas (33), (35), (37) and (38) the contributions of the massless graviton and of the radion are "mixed". This problem can be solved with the help of the zero mode approximation.

5 Zero mode approximation

Suppose we have a matter source on brane 1. Substitution (29) represents the physical gravitational field $h_{\mu\nu}$ as a combination of the field $u_{\mu\nu}$ and of the radion field ϕ. It is well known that the field $u_{\mu\nu}$ in the presence of matter is a combination of zero and massive modes [19], whose eigenfunctions are orthogonal. In particular, the zero mode can be represented as $u^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu}$, where $\alpha_{\mu\nu}$ depends on x only. It also means that with the help of the residual gauge transformations $\xi_\mu = e^{2\sigma}\epsilon_\mu(x)$ it is possible to impose the gauge condition

$$\begin{aligned} \partial^\nu u^{mass}_{\mu\nu} &= 0, \\ u^{mass\,\mu}_{\mu} &= 0, \end{aligned} \tag{41}$$

on the massive modes $u^{mass}_{\mu\nu}$ and the de Donder gauge condition on the zero mode

$$\partial^\nu \left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) = 0. \tag{42}$$

After imposing this gauge, we are still left with residual gauge transformation (12), which, in addition, satisfy the four-dimensional D'Alambert equation. Zero mode truncation of equation (32) gives

$$\alpha = \frac{\hat{\kappa} c e^{2kR}}{R} t. \tag{43}$$

Now let us consider equation (13). Making substitution (29) with condition (30) and passing to the gauge (41), (42) we get

$$\begin{aligned} \frac{1}{2}\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) + \frac{1}{2}e^{-2\sigma}\Box u^{mass}_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^{mass}_{\mu\nu} - 2k^2 u^{mass}_{\mu\nu} - \partial_4\partial_4\sigma u^{mass}_{\mu\nu} = \\ = -\frac{\hat{\kappa}}{2} t_{\mu\nu}\delta(y) + \frac{\hat{\kappa} k e^{2kR}}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right) t - \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right) t\,\delta(y). \end{aligned} \tag{44}$$

We note that equation (44) has been solved exactly in Section 2. But it turned out to be impossible to evaluate analytically all the arising integrals even for a simple form of $t_{\mu\nu}$ (for example, for a static point mass).

To find an equation for the field $\alpha_{\mu\nu}$, we multiply equation (44) by $e^{2\sigma}$, integrate it over y and take into account the orthonormality condition for the wave functions of the modes, which gives

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat{\kappa} k}{(1 - e^{-2kR})} t_{\mu\nu}. \tag{45}$$

This equation, together with equation (30) for the radion field, provides a complete descriotion of the gravitational field in the zero mode approximation.

When matter is on brane 2 (at $y = R$), all the reasonings are the same, as presented above. The substitution has the form (36). The gauge conditions are the same as in case of matter on brane 1 too. The equations of motion for the fields $\alpha_{\mu\nu}$, ϕ in the presence of matter on brane 2 can be derived analogously and read

$$\begin{aligned} \Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) &= -\frac{\hat{\kappa}k}{(e^{2kR}-1)}t_{\mu\nu}, \\ \Box\phi = \frac{\hat{\kappa}}{6R}t^{\mu}_{\mu}. \end{aligned} \tag{46}$$

We would like to note once again that these equations are written in the coordinates $\{x^\mu\}$, which are *Galilean on brane 1* and are inappropriate for studying physical effects on brane 2.

Now we are ready to find the Newtonian limit and light deflection by a point-like static mass. Let us first make it for brane 1. Substitution (29) in the zero mode approximation is

$$h^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu} - \frac{kR}{e^{2kR}-1}\eta_{\mu\nu}\phi + \frac{R}{2k}\partial_\mu\partial_\nu\phi + \frac{1}{4k^2}\partial_\mu\partial_\nu\phi. \tag{47}$$

The equation for $h_{\mu\nu}$ on brane 1 in the zero mode approximation looks like

$$\begin{aligned} \Box h^0_{\mu\nu} &= -\frac{\hat{\kappa}ke^{2kR}}{e^{2kR}-1}\left(t_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}t^\rho_\rho\right) - \\ &- \frac{\hat{\kappa}k}{6(e^{2kR}-1)}\eta_{\mu\nu}t + \frac{\hat{\kappa}}{12k^2R}\left(kR + \frac{1}{2}\right)\partial_\mu\partial_\nu t. \end{aligned} \tag{48}$$

Let us consider a static point mass with energy-momentum tensor $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$, M denoting the inertial mass again. Then we can find a solution of equation (48) for h_{00}-component

$$h_{00} = \frac{\hat{\kappa}k(e^{2kR} + \frac{1}{3})}{8\pi(e^{2kR}-1)}\frac{M}{r}, \tag{49}$$

where $r = \sqrt{\vec{x}^2}$. Thus, we get

$$V = -\hat{G}k\frac{(e^{2kR} + \frac{1}{3})}{(e^{2kR}-1)}\frac{M}{r} = -G_1\left(1 + \frac{1}{3}e^{-2kR}\right)\frac{M}{r}. \tag{50}$$

Now let us show that the Randall-Sundrum model in the zero mode approximation is equivalent to the linearized Brans-Dicke theory (apparently, it was first noted in [19]). Appropriate formulas for the light deflection in the Brans-Dicke theory are well known and can be found, for example, in [20].

The equation for the fluctuations of metric in the linearized Brans-Dicke theory looks like

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G\left(t_{\mu\nu} - \frac{1}{2\omega+3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right), \tag{51}$$

where ω is the BD-parameter. It is easy to see that $h_{\mu\nu}$ satisfies the de Donder gauge condition $\partial^\nu\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=0$. The light deflection angle is given by [20]

$$\Delta\varphi \approx \frac{4MG}{r_0}\left(\frac{2\omega+3}{2\omega+4}\right), \tag{52}$$

with r_0 being the impact parameter and M being the mass of the static point-like source.

Now let us examine the four-dimensional effective theory on brane 1. It is easy to see that $h^0_{\mu\nu}$ (47) does not satisfy the de Donder gauge condition. But we can impose it with the help of appropiate gauge functions.

In this gauge equation (48) takes the form

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G_1\left(t_{\mu\nu}-\frac{e^{-2kR}}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{53}$$

Here and in what follows we drop the superscript "0" of the zero mode approximation. By comparing (53) with (51), we get $\omega=\frac{3}{2}\left(e^{2kR}-1\right)$. Substituting this into (52), we find the angle of light deflection by a static point-like source on brane 1

$$\Delta\varphi \approx \frac{4MG_1}{r_0}\left(\frac{1}{1+\frac{1}{3}e^{-2kR}}\right) \tag{54}$$

The second term in the denominator of formula (54) and the second term in formula (50) correspond to the contribution of the radion. One can see that the contribution of the radion is e^{2kR} times smaller than the contribution of the massless graviton.

Direct calculations lead us to the following results for the remaining cases. All the reasonings concerning Galilean coordinates are the same as in the previous sections.

1) We live on brane 1, the mass is located on brane 2 ("shadow" matter)

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G_1 e^{-kR}\left(t_{\mu\nu}-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right) \tag{55}$$

The light deflection angle is

$$\Delta\varphi \approx \frac{3MG_1e^{-kR}}{r_0} = 4MG_1e^{-kR}\left(\frac{1}{1+\frac{1}{3}}\right)\frac{1}{r_0} \tag{56}$$

Newton's Law looks like

$$V = -\frac{4}{3}G_1e^{-kR}\frac{M}{r} = -\left(1+\frac{1}{3}\right)G_1e^{-kR}\frac{M}{r} \tag{57}$$

The second term in the denominator of formula (56) and the second term in the brackets in formula (57) correspond to the contribution of the radion. One can see that the contribution of the radion is of the same order, as the contribution of the massless graviton, but the effect of the "shadow" matter is e^{kR} times smaller, than the effect of the ordinary matter (compare with (54) and (50)). It means that the effective theory on brane 1 is phenomenologically acceptable, but not interesting (see [6]).

2) We live on brane 2, and the mass is located on the same brane

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2\left(t_{\mu\nu}-\frac{e^{2kR}}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right) \tag{58}$$

with $G_2=\hat{G}k\frac{1}{e^{2kR}-1}$.

The light deflection angle is

$$\Delta\varphi\approx\frac{4MG_2}{r_0}\left(\frac{1}{1+\frac{e^{2kR}}{3}}\right) \tag{59}$$

Newton's Law looks like

$$V=-G_2\left(1+\frac{e^{2kR}}{3}\right)\frac{M}{r} \tag{60}$$

The second term in the denominator of formula (59) and the second term in the brackets in formula (60) correspond to the contribution of the radion. Now the contribution of the radion is e^{2kR} times stronger than the contribution of the massless graviton. It means that, in the case of the massless radion, scalar gravity is realized on brane 2. However, if one assumes some mechanism for generating the radion mass, for example, the Goldberger-Wise mechanism [18], gravity in the zero mode approximation becomes tensor and the hierarchy problem solves.

3) We live on brane 2, the mass is located on brane 1 ("shadow" matter also)

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2e^{kR}\left(t_{\mu\nu}-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right) \tag{61}$$

The light deflection angle is

$$\Delta\varphi\approx\frac{4MG_2e^{kR}}{r_0}\left(\frac{1}{1+\frac{1}{3}}\right)=\frac{3MG_1e^{-kR}}{r_0} \tag{62}$$

Newton's Law looks like

$$V=-\left(1+\frac{1}{3}\right)G_2e^{kR}\frac{M}{r}=-\frac{4}{3}G_1e^{-kR}\frac{M}{r} \tag{63}$$

As before, the second term in the denominator of formula (62) and the second term in the brackets in formula (63) correspond to the contribution of the radion. We can see that the contribution of the radion is of the same order, as contribution of the massless graviton, but the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times smaller, than the contribution of the ordinary matter, because of the interaction with the massless radion (compare with (60)). But if the radion is massive (otherwise we have scalar gravity in "our" world on brane 2), the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times stronger, than the contribution of the ordinary matter. It could lead to some observable effects, resulting from the distribution of matter in the "mirror" world (brane 1).

One can show that equations (50), (57), (60) and (63) coincide with the zero-mode parts of equations (33), (38), (37) and (35) respectively.

In paper [19] the effects of the "shadow" matter were also considered. In this paper the deflection angles with the same impact parameters and by the same Newtonian masses were compared (Newtonian mass is the coefficient in front of the $\frac{1}{r}$ term in Newton's Law). We have compared the effects of the "shadow" and the ordinary matter, as it was made in [19], and we have got the following results: 44% instead of 25% in [19] for brane 1 and the difference of the order of e^{4kR} instead of 25% in [19] for brane 2. This discrepancy appears because we use the Galilean coordinates on brane 2. In the case of the massive radion there is no difference in the zero mode approximation at all for both branes.

In particular, we see that the model with the massless radion is phenomenolagically inacceptable. We have to endow th radion with a mass, or, as it is formulated equivaliently, to stabilize the value of R. This problem is discussed in the next section.

6 Goldberger-Wise mechanism and the radion mass

To stabilize the value of R one can add to the model a scalar field with the action [18]

$$\begin{aligned} S_f &= -\frac{1}{2}\int_E d^5x\sqrt{-g}(\partial_\mu\Phi\partial^\mu\Phi + m^2\Phi^2) - \\ &- \lambda_1\int_E d^5x\sqrt{-\tilde{g}}(\Phi^2 - v_1^2)^2\delta(y) - \lambda_2\int_E d^5x\sqrt{-\tilde{g}}(\Phi^2 - v_2^2)^2\delta(y-R), \end{aligned} \tag{64}$$

$g_{\mu\nu}$ being the induced metric on the branes. The vacuum solution for this field is given by

$$\Phi(y) = e^{2\sigma}(Ae^{\nu\sigma} + Be^{-\nu\sigma}), \quad \nu = \sqrt{4 + \frac{m^2}{k^2}}, \tag{65}$$

the values of A and B being defined by the boundary conditions on the branes. Thus, vacuum configuration of the scalar field has a vacuum energy density, which, when averaged over y, depends on R. If λ_1 and λ_2 are large, this energy density has the minimum at

$$R = \frac{4}{\pi}\frac{k}{m^2}\ln(\frac{v_1}{v_2}). \tag{66}$$

As long as $\hat{G}v_1^2$ and $\hat{G}v_2^2$ are small, the energy-momentum tensor of the field Φ can be neglected in comparison to the 5-dimensional cosmological constant Λ, thus the RS solution for the metric remains almost inaffected. But it is exactly the backreaction of the scalar field on the metric, which generates the radion mass. Therefore, to find the radion mass we have to find, or at least to estimate, this backreation of the scalar field on the background geometry. This can be done in a generalized RS model, which was proposed in [21]. The action of the generalized model including the Goldberger-Wise scalar field is

$$S = S_g + S_f \tag{67}$$

$$S_g = \frac{1}{16\pi\hat{G}}\int_E R\sqrt{-g}d^5x \tag{68}$$

$$S_f = -\frac{1}{2}\int_E d^5x\sqrt{-g}(\partial_\mu\Phi\partial^\mu\Phi + V(\Phi)) - \tag{69}$$
$$- \int_E d^5x\sqrt{-\tilde{g}}\lambda_1(\Phi)\delta(y) - \int_E d^5x\sqrt{-\tilde{g}}\lambda_2(\Phi)\delta(y-R).$$

One looks for a vacuum solution to this system of the form

$$\Phi(x,y) = \Phi_0(y) \tag{70}$$
$$ds^2 = e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu + (dy)^2. \tag{71}$$

If the potential $V(\Phi)$ can be represented as

$$V(\Phi) = \frac{1}{8}\left(\frac{\partial W(\Phi)}{\partial\Phi}\right)^2 - \frac{\hat{\kappa}^2}{12}W(\Phi)^2, \tag{72}$$

then the solution of the Einstein and the scalar field equations satisfy the following first order equations [21]:

$$\Phi_0'(y) = \frac{1}{2}\frac{\partial W}{\partial\Phi}, \quad A' = \frac{\hat{\kappa}^2}{12}W(\Phi_0). \tag{73}$$

We choose the superpotential $W(\Phi)$ to be

$$W(\Phi) = \frac{12k}{\hat{\kappa}^2} - u\Phi^2, \tag{74}$$

the brane potentials being

$$\lambda_1(\Phi) = W(v_1) + W'(v_1)(\Phi - v_1) + \gamma_1^2(\Phi - v_1)^2, \tag{75}$$
$$\lambda_2(\Phi) = -W(v_2) + W'(v_2)(\Phi - v_2) + \gamma_2^2(\Phi - v_2)^2. \tag{76}$$

We note that though potential (72) is not bounded from below, the brane potentials (75), (76) prevent the scalar field from lapsing to infinity.

The exact solution of equations (73) is

$$\Phi_0 = v_1 e^{-u|y|}, \tag{77}$$
$$A(y) = k|y| + \frac{\hat{\kappa}^2 v_1^2}{24}e^{-2u|y|}. \tag{78}$$

The separation distance is fixed by the relation

$$e^{-uR} = \frac{v_2}{v_1}. \tag{79}$$

The authors of [22] suggest that the spin-0 fluctuations of the system can be parameterized in the following way

$$\Phi(x,y) = \Phi_0(y) + \phi(x,y), \tag{80}$$
$$ds^2 = e^{-2A-2F(x,y)}\eta_{\mu\nu}dx^\mu dx^\nu + (1 + 2F(x,y))^2(dy)^2. \tag{81}$$

It turns out that all the spin-0 fluctuations can be described by the field $F(x, y)$ only, the equation for this field being

$$F'' - 2A'F' - 4A''F + 2uF' - 4uA'F + e^{2A}\Box F = 0. \tag{82}$$

This equation is similar to equation (20), which defines the mass spectrum of the vector particles in the RS model. In analogy to decomposition (21), the general solution to this equation has the form

$$F(x, y) = \sum_{n=0}^{\infty} \alpha_n F_n(x, y), \quad \Box F_n(x, y) = m_n^2 F_n(x, y), \tag{83}$$

$F_n(x, y)$ being the mass eigenmodes of the scalar particles. The lowest eigenmode $F_0(x, y)$ is the radion. An important poit is that $F_n(x, y)$ for $n > 0$ exist only if the backreaction of the Goldberger-Wise scalar field on the metric is taken into account.

Assuming $l = \frac{\hat{\kappa} v_1}{2}$ to be a small parameter, one can solve the equation for $F_0(x, y)$ perturbatively. Then the radion mass is found to be

$$m_{rad} = \frac{\hat{\kappa} v_2}{2} u. \tag{84}$$

The coupling of the radion to the SM fields is governed by the normalization of the kinetic term and is estimated to be of the order

$$-\frac{1}{\Lambda_\pi \sqrt{3}} F_0 T^\mu_\mu, \tag{85}$$

i.e. exactly as in the case of the unstabilized model (cf. (27)). The couplings of the heavier scalar modes are suppressed by a factor u/k.

7 Conclusion

In this contribution we have presented a Lagrangian description of linearized gravity in the Randall-Sundrum model with two branes, which enabled us to find easily the physical degrees of freedom of this model and to construct an effective Lagrangian for them. A very important point of our considerations is the observation that the 5-dimensional gravity looks different, when viewed from different branes: the masses of the gravitational KK modes and the coupling constants of the massless fields differ by the exponential factor.

If one assumes that our brane is brane 2 (the TeV brane) with the negative energy density, then, as follows from (28), the interaction of massless radion field is not suppressed. In this case the constant G_2 can be identified with Newton's constant, $G_2 = 1/16\pi M_{Pl}^2$, provided the radion field acquires a mass by some mechanism. Then we get the following relation between the Planck mass and the fundamental mass scale M in the theory defined as $M^3 = 1/16\pi\hat{G}$:

$$M_{Pl}^2 = \frac{M^3}{k}\left(e^{2kR} - 1\right) \approx \frac{M^3}{k} e^{2kR},$$

which can be satisfied by choosing $M \sim k \sim 1$TeV and the argument in the exponential $kR \approx 30 \div 35$. We would like to note that the value of the parameter $k \sim 1TeV$ is the only one admissible for a correct physical interpretation of the theory on brane 2.

The stabilized RS model predicts, besides a tower of vector particles, also a tower of scalar particles with masses in the range of several hundred GeV to several TeV, which can be, in principle, observed on the future colliders. The present day experimental data do not exclude the possibility that the lightest scalar particle of the model, the radion, can have the mass of the order of 100 GeV.

Thus, nowadays the RS model is a most realistic model with extra dimensions, and its predictions are of interest for the future collider experiments.

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