Structures of double-Λ hypernuclei and $\Lambda\Lambda$ interaction

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Abstract

Energy levels of double-Λ hypernuclei ${}^{7}_{\Lambda\Lambda}$He, ${}^{7}_{\Lambda\Lambda}$Li, ${}^{8}_{\Lambda\Lambda}$Li, ${}^{9}_{\Lambda\Lambda}$Li, ${}^{9}_{\Lambda\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be are predicted on the basis of an α+x+Λ+Λ four-body model, where $x = n, p, d, t, {}^3$He and α, respectively. Interactions between constituent particles are determined so as to reproduce reasonably the observed low-energy properties of $\alpha + x$ nuclei and the existing data for Λ-binding energies of $x + \Lambda$ and $\alpha + x + \Lambda$ hypernuclei. An effective $\Lambda\Lambda$ interaction is constructed so as to to reproduce the $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He extracted from the NAGARA event observed in emulsion. With no adjustable parameters for $\alpha + x + \Lambda + \Lambda$ systems, $B_{\Lambda\Lambda}$ of ground and bound excited states of double-Λ hypernuclei with $A = 7 - 10$ are calculated within the Gaussian-basis coupled-rearrangement-channel method.

1 Introduction

In strangeness nuclear physics, the most fundamental problem is to understand the different facets of interactions among the octet of baryons (N, Λ, Σ, Ξ) in a unified way. Our detailed knowledge of the $S = 0$ NN sector is based on the rich data base of NN scattering. Recent studies for $S = -1$ many-body systems such as Λ hypernuclei have clarified interesting features of the ΛN and ΣN interactions in spite of scarce data for the free-space scattering. On the other hand, for the baryon-baryon interactions with $S = -2$ sector, there is presently no experimental information from two-body scattering experiments. Therefore, the observed $\Lambda\Lambda$ bond energies of double-Λ hypernuclei should be the provide the only reliable source for information on the $S = -2$ interaction, and such data play a pivotal role in determining the strength of the underlying $\Lambda\Lambda$ interaction.

Historically, in the 1960's, there appeared two reports on the observation of double-Λ hypernuclei, ${}^{10}_{\Lambda\Lambda}$Be [1] and ${}^{6}_{\Lambda\Lambda}$He [2], but the validity of the latter case was considered doubtful [3]. Two decades later the modern emulsion-counter hybrid technique has been applied in the KEK-E176 experiment [4], in which a new double-Λ hypernucleus event was found but no unique identification has been so far: One explanation as ${}^{10}_{\Lambda\Lambda}$Be leads to a repulsive $\Lambda\Lambda$ interaction ($\Delta B_{\Lambda\Lambda} < 0$), while the other possibility involving ${}^{13}_{\Lambda\Lambda}$B leads to an attractive $\Lambda\Lambda$ interaction [5]. In the latter case, the extracted strength of the $\Lambda\Lambda$ interaction is attractive with $\Delta B_{\Lambda\Lambda} \simeq 4$ MeV, which seems consistent with the old data of ${}^{10}_{\Lambda\Lambda}$Be [1]. Under this situation, it became the usual understanding that the $\Lambda\Lambda$ interaction is substantially attractive.

A recent observation of the double-Λ hypernucleus ${}^{6}_{\Lambda\Lambda}$He, which is called the NAGARA event in the KEK-E373 experiment, should have a great impact on the study of baryon-baryon interactions in the strangeness $S = -2$ sector and on the study of dynamics of many-body systems with multi-strangeness. The importance of this event derives from the precise experimental value of the $\Lambda\Lambda$ binding energy $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})= 1.01 \pm 0.20\pm^{0.18}_{0.11}$ MeV [6]. This value suggests that $\Lambda\Lambda$ interactions are substantially weaker than those derived from the old data. Because the value of $\Delta B_{\Lambda\Lambda}$ is related to an averaged strength of the underlying $\Lambda\Lambda$ interaction, the above reliable data gives a crucial input for parametrizations of various interaction models. In view of this, the NAGARA event is an epoch-making one which provides us with a new basis for modeling the $\Lambda\Lambda$ interaction and understanding all other double-Λ hypernuclei.

Stimulated by this exciting experimental situation, let us perform careful theoretical calculations of double-Λ hypernuclei from a new viewpoint. We extract the $\Lambda\Lambda$ interaction from the above NAGARA datum within the framework of the $\alpha+\Lambda+\Lambda$ three-body model. In order to utilize the extracted $\Lambda\Lambda$ interaction, hypernuclear calculations should be complete and sufficiently realistic to make structural ambiguity as negligible as possible. All the dynamic changes due to ΛN interaction should be fully taken into account. To satisfy these requirements we explore light p-shell double-Λ hypernuclei ($A = 6-10$) comprehensively using microscopic four-body models, consisting of $\alpha+x+\Lambda+\Lambda$ systems with $x = n, p, d, t, {}^{3}\mathrm{He}$ and α (${}^{7}_{\Lambda\Lambda}\mathrm{He}$, ${}^{7}_{\Lambda\Lambda}\mathrm{Li}$, ${}^{8}_{\Lambda\Lambda}\mathrm{Li}$, ${}^{9}_{\Lambda\Lambda}\mathrm{Li}$, ${}^{9}_{\Lambda\Lambda}\mathrm{Be}$ and ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$), where nuclear core parts are quite well represented by $\alpha+x$ cluster models. As a result of these systematic and realistic calculations, we will give reliable predictions of both the ground-state binding energies and possible excited-state energies, which should encourage double-Λ hypernuclear spectroscopic studies in the near future.

2 Model and interactions

Our four-body calculations are accurately performed by using the Jacobian-coordinate Gaussian-basis method [7]. Then, our four body wave function is described as a sum of amplitudes of the rearrangement channels ($c = 1-9$) in the LS coupling scheme:

$$\begin{aligned}\Psi_{JM} & ({}^{A}_{\Lambda\Lambda}\mathrm{Z}) = \sum_{c=1}^{9} \sum_{n,N,\nu} \sum_{l,L,\lambda} \sum_{S,\Sigma,I,K} C^{(c)}_{nlNL\nu\lambda S\Sigma IK} \\ \times & \; \mathcal{A}_{\Lambda}\mathcal{S}_{\alpha}\Big[\Phi(\alpha)\,\Big[\Phi_s(x)\,[\chi_{\frac{1}{2}}(\Lambda_1)\chi_{\frac{1}{2}}(\Lambda_2)]_S\Big]_{\Sigma} \\ \times & \; \Big[[\,\phi^{(c)}_{nl}(\mathbf{r}_c)\psi^{(c)}_{NL}(\mathbf{R}_c)]_I\;\xi^{(c)}_{\nu\lambda}(\boldsymbol{\rho}_c)\Big]_K\Big]_{JM} . \end{aligned} \tag{1}$$

Here the operator $\mathcal{A}_{\Lambda}$ stands for antisymmetrization between the two Λ particles, and $\mathcal{S}_{\alpha}$ is the symmetrization operator for exchange of α clusters when $x=\alpha$. $\chi_{\frac{1}{2}}(\Lambda_i)$ is the spin function of the i-th Λ particle. Then, the four-body Schrödinger equation represented in the Gaussian base is solved by the Rayleigh-Ritz variational method. For the angular-momentum component of the wave function, the approximation with $l, L, \lambda \leq 2$ was found to be sufficient to obtain in getting satisfactory convergence of the binding energies for the states concerned.

In our $\alpha+x+\Lambda+\Lambda$ four-body model, the αx, Λx and $\Lambda\Lambda$ interactions should be chosen so as to describe well the αx, Λx and $\Lambda\Lambda\alpha$ subsystems. In reliable cluster-model calculations, it is a necessary condition that threshold energies for every partition into sub-cluster systems are reproduced.

(i) αx interactions

They should reproduce well energies of the low-lying states and scattering phase shifts of the $\alpha + x$ nuclear systems. The potentials are described in the parity-dependent form with the central and spin-orbit terms. The Pauli principle between nucleons belonging to α and x clusters is taken into account by the orthogonality condition model.

(ii) Λx interactions

They should be taken so as to reproduce B_Λ of hypernuclei composed of $x + \Lambda$ and $\alpha + x + \Lambda$, x being $d, t, {}^3$He, α. We derive the interaction between the Λ particle and the x cluster by folding the G-matrix type YN interaction into the density of the x cluster [8], which are adjusted finely in the respective cases of Λ hypernuclei.

(iii) $\Lambda\Lambda$ interaction

We take it so as to reproduce $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He $= \alpha + \Lambda + \Lambda$, the experimental value of which is given by the NAGARA datum. We employ here the $\Lambda\Lambda$ interaction represented in the three-range Gaussian form, keeping some linkage to the Nijmegen OBE models [9] in determining the interaction parameters: The outer two components of the Gaussian potential are determined so as to simulate the $\Lambda\Lambda$ sector of the Nijmegen model-F (NF), and then the strength of the core part is adjusted so as to reproduce the experimental value of $B_{\Lambda\Lambda}$. It is worthwhile to say that the strength of the attractive OBE part is about a half of that of the Nijmegen model D (ND), which is attractive appropriately for the old understanding of $\Delta B_{\Lambda\Lambda} \simeq 4$ MeV before the finding of the NAGARA event.

Thus, our four-body calculations can be performed with no adjustable parameters at this stage, expecting high reliability of the results.

3 Results and discussion

Table I summarizes the calculated ground-state energies for the double-Λ hypernuclei including the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be. The results are expressed in terms of two quantities: One is the total energy measured from the breakup threshold of $\alpha + x + \Lambda + \Lambda$ which is denoted as $E_{\Lambda\Lambda}$. The other is $B_{\Lambda\Lambda}$ which is the binding energy of two Λ particles with respect to the ground-state nuclear core $\alpha + x$. Then, the $\Lambda\Lambda$ bond energy determined essentially by the strength of the $\Lambda\Lambda$ interaction is given by the difference $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}{\rm Z}) \equiv B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}{\rm Z}) - B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}{\rm Z}; V_{\Lambda\Lambda} = 0)$.

The calculated values of $B_{\Lambda\Lambda}$ can be compared with some experimental data, though the data are quite limited at present. The most recent and precise data of the NAGARA event is used as a basic input of our model, so that our $\Lambda\Lambda$ interaction is adjusted to reproduce the experimental value $B^{\rm exp}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}{\rm He}) = 7.25 \pm 0.19 \pm^{0.18}_{0.11}$ MeV [6].

It is of particular interest to compare the present result with another datum which is not used in the fitting procedure. There is an event found in the E373 experiment, named the *Demachi-Yanagi* event [10]; the most probable interpretation of this event is a bound state of ${}^{10}_{\Lambda\Lambda}$Be having $B^{\rm exp}_{\Lambda\Lambda} = 12.33 \pm^{0.35}_{0.21}$ MeV, which is obtained by assuming $B^{\rm exp}_{\Xi} = 0.15 \pm^{0.3}_{0.1}$ MeV. In the emulsion analysis there is no direct evidence for the production of ${}^{10}_{\Lambda\Lambda}$Be in an excited state. However, if the produced ${}^{10}_{\Lambda\Lambda}$Be is interpreted to be in the ground state, the resultant $\Lambda\Lambda$ bond energy becomes repulsive, contradicting the NAGARA event. From the viewpoint of the present study, the *Demachi-Yanagi* event can be interpreted most probably as the observation of the 2^+ excited state in ${}^{10}_{\Lambda\Lambda}$Be; our calculated value of $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+))$ is 12.28 MeV, which agrees with the above experimental value. This good agreement suggests that our systematically calculated level structures are predictive and useful for interpreting coming events expected to be found in the further analysis of the E373 data. Now it should be stressed that the above experimental datum on ${}^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+)$ leads to no information about the ground-state value of $B_{\Lambda\Lambda}$ unless the theoretical value (2.86 MeV in our case) of the excitation energy of ${}^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+)$ is utilized.

Table I. Calculated energies of the ground states of $A = 6-10$ double-Λ hypernuclei based on the $\alpha + x + \Lambda + \Lambda$ four-body model ($x = 0, n, p, d, t, {}^3$He, and α). $E_{\Lambda\Lambda}$ are measured from the $\alpha + x + \Lambda + \Lambda$ threshold. The $\Lambda\Lambda$ bond energy $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ is defined in the text. Information on the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be is specially added so as to demonstrate the agreement with the experimental result.

	J^π	$E_{\Lambda\Lambda}$ (MeV)	$B_{\Lambda\Lambda}$ (MeV)	$B^{\rm exp}_{\Lambda\Lambda}$ (MeV)	$\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ (MeV)
${}^{6}_{\Lambda\Lambda}$He	0^+	-7.25	7.25	7.25 ± 0.19 [a]	0.88
${}^{7}_{\Lambda\Lambda}$He	$\frac{3}{2}^-$	-8.47	9.36	-	0.96
${}^{7}_{\Lambda\Lambda}$Li	$\frac{3}{2}^-$	-7.48	9.45	-	0.95
${}^{8}_{\Lambda\Lambda}$Li	1^+	-12.10	11.44	-	0.98
${}^{9}_{\Lambda\Lambda}$Li	$\frac{3}{2}^-$	-17.05	14.55	-	0.98
${}^{9}_{\Lambda\Lambda}$Be	$\frac{3}{2}^-$	-16.00	14.40	-	0.97
${}^{10}_{\Lambda\Lambda}$Be	0^+	-15.05	15.14	17.7 ± 0.4 [b] 14.6 ± 0.4 [b]	0.93
${}^{10}_{\Lambda\Lambda}$Be	2^+	-12.19	12.28	$12.33{\pm}^{0.35}_{0.21}$ [c]	0.93

[a] Ref.[6].
[b] Ref.[1]. Also see text for the second value.
[c] Ref.[10].

On the other hand, the earlier experiment by Danysz *et al.* [1] on the pionic decay of ${}^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+) \to {}^{9}_{\Lambda}\mathrm{Be}(1/2^+) + p + \pi^-$ gave $B^{\rm exp}_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+)) = 17.7 \pm 0.4$ MeV. This value has been used for a long time, which implies a strongly attractive $\Lambda\Lambda$ interaction. However, it should be noted that the authors also suggested the possibility of another decay ${}^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+) \to {}^{9}_{\Lambda}\mathrm{Be}(3/2^+, 5/2^+) + p + \pi^-$ (Table 5 of Ref.[1]). In this case, the value of $B^{\rm exp}_{\Lambda\Lambda}$ (${}^{10}_{\Lambda\Lambda}$Be (0^+)) is modified to 14.6 ± 0.4 MeV, which is obtained by using 3.05 MeV

[11] as the excitation energy of ${}^{9}_{\Lambda}$Be$(3/2^+, 5/2^+)$. This modified value turns out to not contradict our calculated value, 15.14 MeV. A similar re-interpretation, with the hypernuclear excited states taken into account, may be needed also for the E176 event which was identified as ${}^{13}_{\Lambda\Lambda}$B (${}^{10}_{\Lambda\Lambda}$Be) with a strongly attractive (repulsive) $\Lambda\Lambda$ interaction.

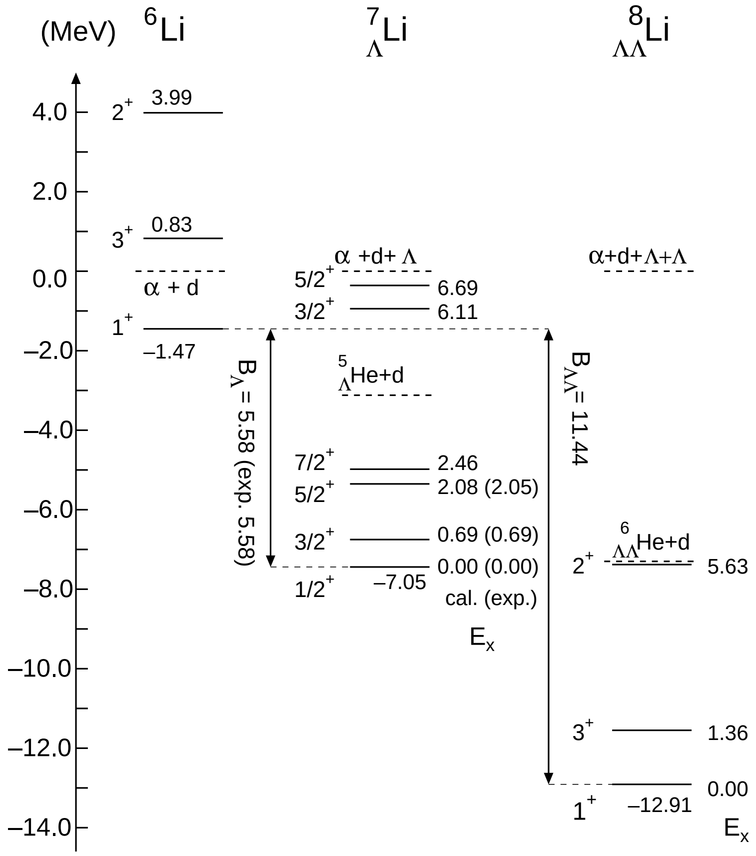

Figure 1: Calculated energy levels of ^{6}Li, ${}^{7}_{\Lambda}$Li and ${}^{8}_{\Lambda\Lambda}$Li on the basis of the $\alpha+d$, $\alpha+d+\Lambda$ and $\alpha+d+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle break-up thresholds or are given by excitation energies $E_{\rm x}$.

In order to understand the role of two Λ particles attached to the core nuclei, it is useful to compare the level structures of the $\alpha+x+\Lambda+\Lambda$ double-Λ hypernuclei with those of the corresponding $\alpha+x$ nuclei and the $\alpha+x+\Lambda$ single-Λ hypernuclei. Then, we can see clearly how the ground and excited states of $\alpha+x$ nuclei are changed due to the addition of the Λ particles. In Fig. 1 we demonstrate the obtained result in the case of $x=d$. The calculated level structure of $\alpha+d$ core nuclei, $\alpha+d+\Lambda$ hypernuclei and $\alpha+d+\Lambda+\Lambda$ hypernuclei are illustrated side by side. All the ground and bound excited states of ${}^{8}_{\Lambda\Lambda}$Li predicted in the present model are exhibited. It should be noted here that the energy levels of ${}^{7}_{\Lambda}$Li observed in the γ-ray experiments are nicely fitted in our model, which assures the reliability of the predicted levels of ${}^{8}_{\Lambda\Lambda}$Li. In this figure, one sees clearly that injection into an $\alpha+d$ core nucleus of one and two Λ particles leads to stronger binding of the whole system and a prediction of more bound states in most systems. Thus, the Λ particle turns out to play a glue-like role, which makes a whole system more strongly bound. This interesting features can be seen also in the other cases of $x = n, p, t, {}^3$He, α.

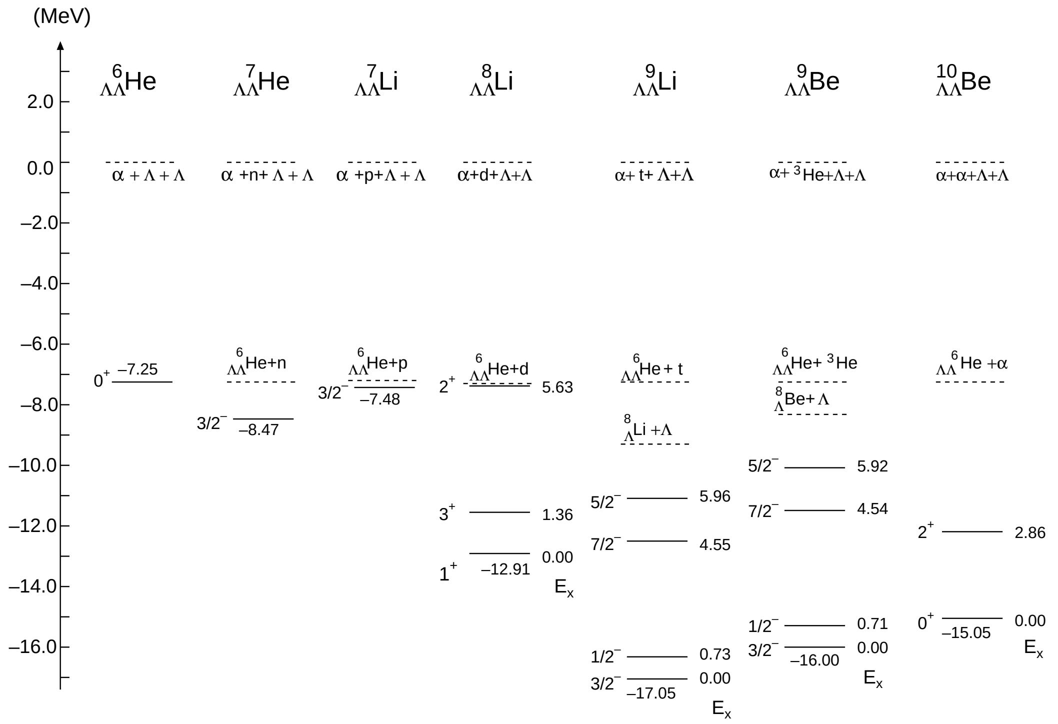

Figure 2: The energy levels of the double-Λ hypernuclei $^{6}_{\Lambda\Lambda}$He, $^{7}_{\Lambda\Lambda}$He, $^{7}_{\Lambda\Lambda}$Li, $^{8}_{\Lambda\Lambda}$Li, $^{9}_{\Lambda\Lambda}$Li, $^{9}_{\Lambda\Lambda}$Be and $^{10}_{\Lambda\Lambda}$Be calculated using the $\alpha + x + \Lambda + \Lambda$ model with $x = 0, n, p, d, t, {}^3$He and α, respectively.

The obtained energy spectra of the double-Λ hypernuclei with $A = 6 - 10$ are summarized in Fig. 2. The bound excited states of double-Λ hypernuclei predicted in the present cluster model are summarized as follows: In $^{7}_{\Lambda\Lambda}$He and $^{7}_{\Lambda\Lambda}$Li, the ground states are both bound but no excited states are predicted. Needless to say, there are no bound excited states in double-Λ hypernuclei with $A \leq 6$ since there is no bound excited state in their core nuclei. The lightest double-Λ hypernucleus that has at least one excited state is $^{8}_{\Lambda\Lambda}$Li. In $^{8}_{\Lambda\Lambda}$Li we predict two $T = 0$ excited states in the bound-state region. One might expect to see a $T = 1, 0^+$ bound excited state in $^{8}_{\Lambda\Lambda}$Li, which would correspond to the $T = 1, 0^+$ state in ^{6}Li at $E_{\rm x} = 3.56$ MeV, but the state is not shown in Fig. 4 because the $T = 1$ state may have five-body structure and is outside the scope of the present cluster model. We predict three bound excited states in $^{9}_{\Lambda\Lambda}$Li ($^{9}_{\Lambda\Lambda}$Be). There is one bound excited state in $^{10}_{\Lambda\Lambda}$Be as mentioned before. It will be challenging to discover these excited states one by one as well as the ground states.

4 Conclusion

It is striking that the calculated $B_{\Lambda\Lambda}$ of the 2^+ excited state in ${}^{10}_{\Lambda\Lambda}$Be, 12.28 MeV, agrees with the experimental value $B^{\rm exp}_{\Lambda\Lambda}$ (${}^{10}_{\Lambda\Lambda}$Be) $= 12.33\pm^{0.35}_{0.21}$ MeV in the *Demachi-Yanagi* event [10]. We therefore interpret this event as the observation of the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be. The agreement suggests that our systematic calculations are predictive for coming events expected to be found in the further analysis of the E373 data, *etc.*

Together with the energy spectrum of each double-Λ hypernucleus, those of the corresponding core nucleus and single-Λ hypernucleus are exhibited side by side in Fig. 1 so one can see clearly that injection of one and two Λ particles leads to stronger binding of the whole system and prediction of more bound states in most systems. In the bound states of any double-Λ hypernucleus, two Λ particles are dominantly coupled to $S = 0$ and hence the spin and parity are the same as those of its nuclear core. Nevertheless the theoretical $B_{\Lambda\Lambda}$ values are of importance to guide the analysis of the emulsion experiments.

In conclusion, it is our intention that these extensive four-body cluster-model calculation should serve to motivate extensive spectroscopic studies of double-Λ hypernuclei.

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