Gauss-Bonnet black holes at new colliders: beyond the dimensionality of space

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Abstract

The Gauss - Bonnet invariant is one of the most promising candidates for a quadratic curvature correction to the Einstein action in expansions of supersymmetric string theory. We study the evaporation of such Schwarzschild - Gauss - Bonnet black holes which could be formed at future colliders if the Planck scale is of order a TeV, as predicted by some modern brane world models. We show that, beyond the dimensionality of space, the corresponding coupling constant could be measured by the LHC. This opens new windows for physics investigation in spite of the possible screening of microphysics due to the event horizon.

1 Introduction

It has recently been pointed out that black holes could be formed at future colliders if the Planck scale is of order a TeV, as is the case in some extra-dimension scenarios [1, 2]. This idea has driven a considerable amount of interest (see *e.g.* [3]). The same phenomenon could also occur due to ultrahigh energy neutrino interactions in the atmosphere [4]. Most works consider that those black holes could be described by the D-dimensional ($D \geq 5$) generalized Schwarzschild or Kerr metrics [5]. The aim of this work is to study the experimental consequences of the existence of the Gauss-Bonnet term (as a step toward quantum gravity) if it is included in the D-dimensional action. This approach should be more general and relies on a real expansion of supersymmetric string theory.

2 Black hole formation at colliders

The "large extra dimensions" scenario [6] is a very exciting way to address geometrically the hierarchy problem (among others), allowing only the gravity to propagate in the bulk. The Gauss law relates the Planck scale of the effective 4D low-energy theory M_{Pl} with the fundamental Planck scale M_D through the volume of the compactified dimensions, V_{D-4}, via: $M_D = \left(M_{Pl}^2/V_{D-4}\right)^{1/(D-2)}$. It is thus possible to set $M_D \sim TeV$ without being in contradiction with any currently available experimental data. This translates into radii values between a fraction of a millimeter and a few Fermi for the compactification radius of the extra dimensions (assumed to be of same size and flat, *i.e.* of toroidal shape). Furthermore, such a small value for the Planck energy can be naturally expected to minimize the difference between the weak and Planck scales, as motivated by the construction of this approach. In such a scenario, at sub-weak energies, the Standard Model (SM) fields must be localized to a 4-dimensional manifold of weak scale "thickness" in the extra dimensions. As shown in [6], as an example based on a dynamical assumption with D=6, it is possible to build such a SM field localization. This is however the non-trivial task of those models.

Another important way for realizing TeV scale gravity arises from properties of warped extra-dimensional geometries used in Randall-Sundrum scenarios [7]. If the warp factor is small in the vicinity of the standard model brane, particle masses can take TeV values, thereby giving rise to a large hierarchy between the TeV and conventional Planck scales [2, 8]. Strong gravitational effects are therefore also expected in high energy scattering processes on the brane.

In those frameworks, black holes could be formed by the Large Hadron Collider (LHC). Two partons with a center-of-mass energy $\sqrt{s}$ moving in opposite directions with an impact parameter less than the horizon radius r_+ should form a black hole of mass $M \approx \sqrt{s}$ with a cross section expected to be of order $\sigma \approx \pi r_+^2$. Thoses values are in fact approximations as suppression effects should be considered [9, 10] and are taken into account in the section 5 of this paper. Although the accurate corss section values are not yet known, a semiclassical analysis of quantum black hole formation is now being constructed and the existence of a closed trapped surface in the collision geometry of relativistic particles in demonstrated. To compute the real probability to form black holes at the LHC, it is necessary to take into account that only a fraction of the total center-of-mass energy is carried out by each parton and to convolve the previous estimate with the parton luminosity [1]. Many clear experimental signatures are expected [2], in particular very high multiplicity events with a large fraction of the beam energy converted into transverse energy with a growing cross section. Depending on the value of the Planck scale, up to approximately a billion black holes could be produced at the LHC.

3 Schwarzschild - Gauss - Bonnet black holes

The classical Einstein theory can be considered as the weak field and low energy limit of a some quantum gravity model which is not yet built. The curvature expansion of string gravity therefore provides an interesting step in the modelling of a quasiclassical approximation of quantum gravity. As pointed out in [11], among higher order curvature corrections to the general relativity action, the quadratic term is especially important as it is the leading one and as it can affect the graviton excitation spectrum near flat space. If, like the string itself, its slope expansion is to be ghost free, the quadratic term *must* be the Gauss - Bonnet combination: $L_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$. Furthermore, this term is naturally generated in heterotic string theories [12] and makes possible the localization of the graviton zero-mode on the brane [13]. It has been successfully used in cosmology, especially to address the cosmological constant problem (see *e.g.* [14] and references therein) and in black hole physics, especially to address the endpoint of the Hawking evaporation problem (see *e.g.* [15] and references therein). We consider here black holes described by such an action:

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left\{R + \lambda(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2) + \ldots\right\},$$

where λ is the Gauss - Bonnet coupling constant. The measurement of this λ term would allow an important step forward in the understanding of the ultimate gravity theory. Following [16], we assume the metric to be of the following form :

$$ds^2 = -e^{2\nu}dt^2 + e^{2\alpha}dr^2 + r^2 h_{ij}dx^i dx^j$$

where ν and α are functions of r only and $h_{ij}dx^i dx^j$ represents the line element of a $(D-2)$-dimensional hypersurface with constant curvature $(D-2)(D-3)$. The substitution of this metric into the action [11] leads to the following solutions :

$$e^{2\nu} = e^{-2\alpha} = 1 + \frac{r^2}{2\lambda(D-3)(D-4)}\times$$
$$\left(1 \pm \sqrt{1 + \frac{32\pi^{\frac{3-D}{2}}G\lambda(D-3)(D-4)M\Gamma(\frac{D-1}{2})}{(D-2)r^{D-1}}}\right).$$

The mass of the black hole can then be expressed [11, 16] in terms of the horizon radius r_+,

$$M = \frac{(D-2)\pi^{\frac{D-1}{2}}r_+^{D-3}}{8\pi G\Gamma\left(\frac{D-1}{2}\right)}\left(1 + \frac{\lambda(D-3)(D-4)}{r_+^2}\right)$$

where Γ stands for the Gamma function. The temperature is obtained by the usual requirement that no conical singularity appears at the horizon in the euclidean sector of the hole solution,

$$T_{BH} = \frac{1}{4\pi}(e^{-2\alpha})' \mid_{r=r_+} = \frac{(D-3)r_+^2 + (D-5)(D-4)(D-3)\lambda}{4\pi r_+\left(r_+^2 + 2\lambda(D-4)(D-3)\right)}.$$

In the case $D = 5$, those black holes have a singular behavior [16] and, depending on the value of λ, can become thermodynamically unstable or form stable relics. For $D > 5$, which is the only relevant hypothesis for this study (as $D = 5$ would alter the solar system dynamics if the Planck scale is expected to lie ~TeV), a quantitatively different evaporation scenario is expected.

4 Flux computation

Using the high-energy limit of multi-dimensional grey-body factors [17], the spectrum per unit of time t and of energy Q can be written, for each degree of freedom, for particles of type i and spin s as:

$$\frac{\mathrm{d}^2 N_i}{\mathrm{d}Q\mathrm{d}t} = \frac{4\pi^2 \left(\frac{D-1}{2}\right)^{\frac{2}{D-3}} \left(\frac{D-1}{D-3}\right) r_+^2 Q^2}{e^{\frac{Q}{T_{BH}}} - (-1)^{2s}}.$$

This is an approximation as modifications might arise when the exact values of the greybody factors are taken into account due to their dependence, in the low energy regime, on both the dimensionality of the spacetime and on the spin of the emitted particle. Fortunately, as demonstrated in the 4-dimensional case [18], the *pseudo-oscillating* behaviour induces compensations that makes the differences probably quantitatively quite small. The mean number of emitted particle can then be written as

$$N_{tot} = \frac{15(D-2)\pi^{\frac{D-9}{2}}\zeta(3)}{\Gamma(\frac{D-1}{2})G} \frac{\frac{3}{4}N_f + N_b}{\frac{7}{8}N_f + N_b} \left[\frac{r_{init+}^{D-2}}{D-2} + 2(D-3)\lambda r_{init+}^{D-4}\right]$$

where N_f and N_b being the total fermionic and bosonic degrees of freedom, r_{init+} is the initial horizon radius of a black hole with mass M_{init} and, interestingly, the ratio of a given species i to the total emission is given by :

$$\frac{N_i}{N_{tot}} = \frac{\alpha_s g_i}{\frac{3}{4}N_f + N_{tot}}$$

where α_s is 1 for bosons and is 3/4 for fermions and g_i is the number of internal degrees of freedom for the considered particles. The mean number of particles emitted by a Schwarzschild - Gauss - Bonnet black hole ranges from 25 to 4.7 depending on the values of λ and D, for $M_D \sim 1$ TeV and $M_{init} \sim 10$ TeV. Those values are decreased to 5 and 1.05 if M_{init} is set at 2 TeV. Figure 1 shows the flux for different values of λ and D.

5 String coupling constant measurement

To investigate the LHC capability to reconstruct the fundamental parameter λ, we have fixed the Planck scale at 1 TeV. Although a small excursion range around this value would not change dramatically our conclusions, it cannot be taken much above, due to the very fast decrease of the number of formed black holes with increasing M_D. Following [1], we consider the number of black holes produced between 1 TeV and 10 TeV with a bin width of 500 GeV (much larger than the energy resolution of the detector), rescaled with the value of r_+ modified by the Gauss - Bonnet term. For each black hole event, the emitted particles are randomly chosen by a Monte-Carlo simulation according to the spectra given in the previous section, weighted by the appropriate number of degrees of freedom. The Hawking radiation takes place predominantly in the S-wave channel [19], so bulk modes can be neglected and the evaporation can be considered as occurring within the brane. As the intrinsic spectrum $\mathrm{d}N_i/\mathrm{dQ}$ is very strongly modified by fragmentation process, only the direct emission of electrons and photons above 100 GeV is considered. We have checked with the Pythia [20] hadronization program that only a small fraction of directly emitted γ-rays and electrons fall within an hadronic jet, making them impossible to distinguish from the background of decay products. Furthermore, the background from standard model Z(ee)+jets and γ+jets remains much lower than the expected signal. The value of the Planck scale is assumed to be known as a clear threshold effect should appear in the data and a negligible uncertainty is expected on this measurement. For each event, the initial mass of the black hole is also assumed to be known as it can be easily determined with the full spectrum of decay products (only 5% of missing energy is expected due to the small number of degrees of freedom of neutrinos and gravitons). The energy resolution of the detector is taken into account and parametrized [21] as $\sigma/E = \sqrt{a^2/E + b^2}$ with $a \approx 10\%\sqrt{\mathrm{GeV}}$ and $b \approx 0.5\%$. Unlike [1], we also take into account the time evolution of the black holes and perform a full fit for each event. Once all the particles have been generated, spectra are reconstructed for all the mass bins and compared with theoretical computations. The values of D and λ compatible with the

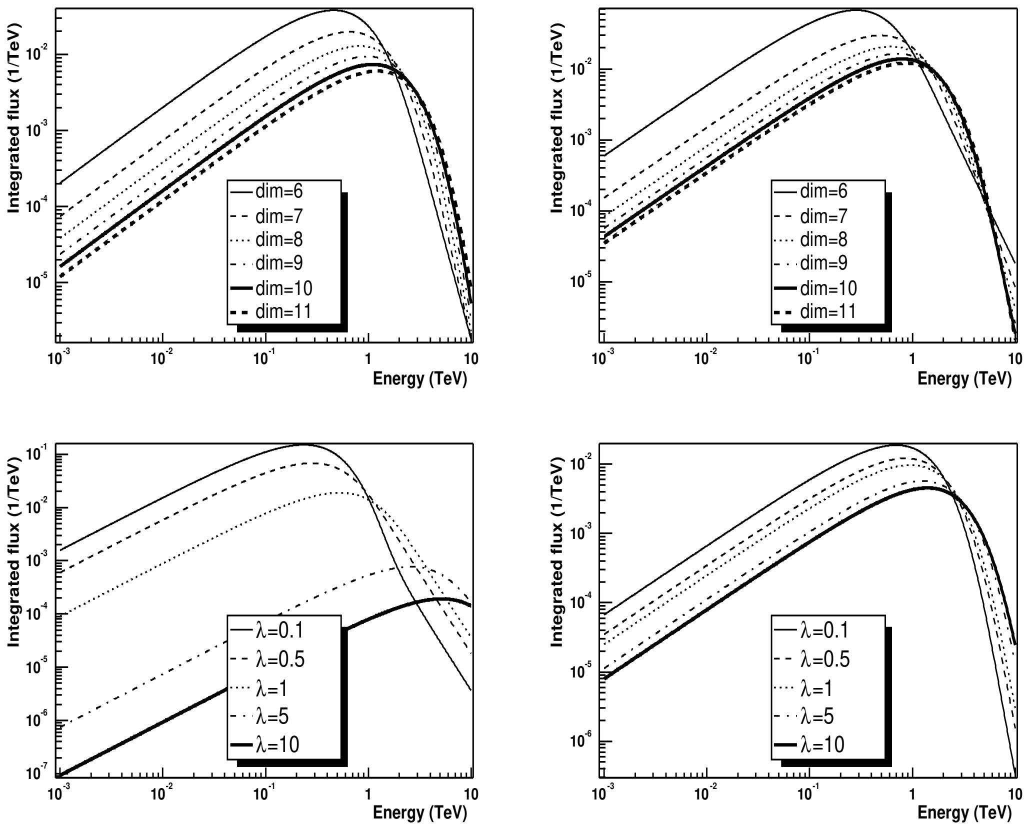

Figure 1: Integrated flux as a function of the total energy of the emitted quanta for an initial black hole mass $M = 10$ TeV. Upper left: $\lambda = 0$, $D = 6, 7, 8, 9, 10, 11$. Upper right : $\lambda = 0, 5$ TeV^{-2}, $D = 6, 7, 8, 9, 10, 11$. Lower left : $D = 6$, $\lambda = 0.1, 0.5, 1, 5, 10$ TeV^{-2}. Lower right : $D = 11$, $\lambda = 0.1, 0.5, 1, 5, 10$ TeV^{-2}.

simulated data are then investigated. Figure 2 shows the $\chi^2/d.o.f.$ for the reconstructed spectra for 2 different couples (λ [TeV^{-2}], D)=(1,10) and (λ [TeV^{-2}], D)=(5,8). The statistical significance of this χ^2 should be taken with care since a real statistical analysis would require a full Monte-Carlo simulation of the detector. Nevertheless, the "input" values can clearly be extracted from the data. Furthermore, it is important to notice that for reasonable values of λ (around the order of the quantum gravity scale, *i.e.* around a TeV^{-2} in our case) it can unambiguously be distinguished between the case *with* and the case *without* a Gauss - Bonnet term.

6 Discussion

In case the Planck scale lies in the TeV range due to extra dimensions, this study shows that, beyond the dimensionality of space, the next generation of colliders should be able to measure the coefficient of a possible Gauss - Bonnet term in the gravitational action. This would allow an important step forward in the construction of a full quantum theory of gravity. It is also interesting to notice that this would be a nice example of the convergence between astrophysics and particle physics in the final understanding of black holes and gravity in the Planckian region.

Then, as studied in [16, 23], a cosmological constant could also be included in the action. On the theoretical side, this would be strongly motivated by the great deal of attention paid to the Anti-de Sitter and, recently, de Sitter / Conformal Field Theory (AdS and dS /CFT) correspondences. On the experimental side, this would open an interesting window as there is no unambiguous relation between the D-dimensional and the 4-dimensional cosmological constants.

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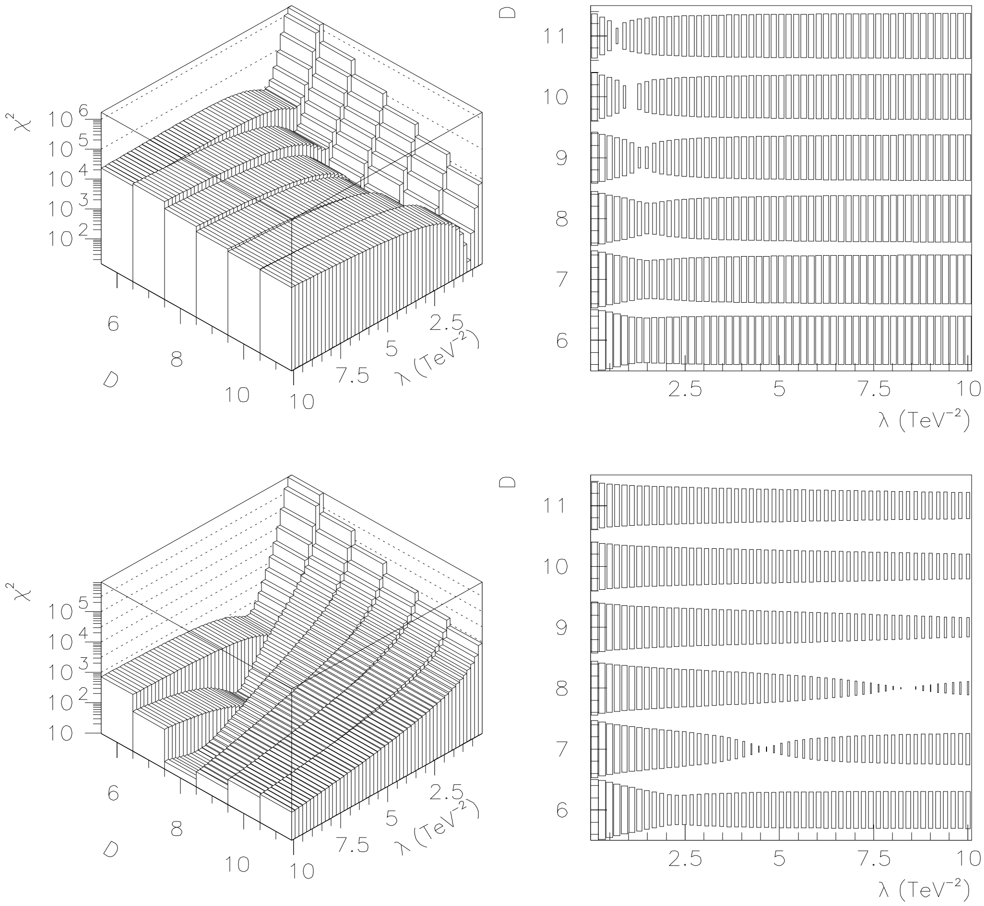

Figure 2: Upper part: values of the $\chi^2/d.o.f.$ for the reconstructed spectra as a function of D and λ for "input" values $\lambda = 1$ TeV^{-2} and $D = 10$; the right side shows rectangles proportional to the logarithm of the $\chi^2/d.o.f.$ Lower part (left and right): values of the $\chi^2/d.o.f.$ for the reconstructed spectra as a function of D and λ for "input" values $\lambda = 5$ TeV^{-2} and $D = 8$; the right side shows rectangles proportional to the logarithm of the $\chi^2/d.o.f.$

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