Loop quantum gravity effects on the high energy cosmic ray spectrum

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Abstract

Recent observations on ultra high energy cosmic rays (those cosmic rays with energies greater than $\sim 4 \times 10^{18}$ eV) suggest an abundant flux of incoming particles with energies above 1×10^{20} eV. These observations violate the Greisen-Zatsepin-Kuzmin cutoff. To explain this anomaly we argue that quantum-gravitational effects may be playing a decisive role in the propagation of ultra high energy cosmic rays. We consider the loop quantum gravity approach and provide useful techniques to establish and analyze constraints on the loop quantum gravity parameters arising from observational data. In particular, we study the effects on the predicted spectrum for ultra high energy cosmic rays and conclude that is possible to reconcile observations.

1 Introduction

In the present discussion we are concerned with the observation of ultra high energy cosmic rays (UHECR), i.e. those cosmic rays with energies greater than $\sim 4 \times 10^{18}$ eV. Although not completely clear, it has been suggested that these high energy particles are possibly heavy nuclei [1] (we will assume here that they are protons) and, by virtue of the isotropic distribution with which they arrive to us, that they originate in extragalactic sources.

The propagation of UHECR in intergalactic space is subject to interaction with the cosmic microwave background radiation (CMBR). Its presence produces friction on UHECR making them release energy in the form of secondary particles. This affects their possibility to reach great distances. A first estimation of the characteristic distance that UHECR can reach before losing most of their energy was simultaneously made in 1966 by K. Greisen [2] and G. T. Zatsepin & V. A. Kuzmin [3]. They showed that the observation of cosmic rays with energies greater than 4×10^{19} eV should be greatly suppressed. This energy is usually referred as to the GZK cutoff energy. A few years later, F.W. Stecker [4] computed the mean life time for protons as a function of their energy, giving a more accurate perspective of the energy dependence of the cutoff and showing that cosmic rays with energies above 1×10^{20} eV should not travel more than ~ 100 Mpc. More detailed approaches to the GZK-cutoff feature have been made since these first estimations. For example V. Berezinsky & S.I. Grigorieva [5], V. Berezinsky *et al.* [6] and S.T. Scully & F.W. Stecker [7] have made progress in the theoretical study of the spectrum $J(E)$ (i.e. the flux of arriving particles as a function of the observed energy E) that UHECR should present. As a result, the GZK cutoff exists in the form of a suppression in the predicted flux of cosmic rays with energies above $\sim 8 \times 10^{19}$ eV.

Currently there are two main different sets of data for the observed flux $J(E)$ in its most energetic sector ($E > 4 \times 10^{18}$ eV). On one hand we have the observations from the High Resolution Fly's Eye (HiRes) collaboration group [8], which seem to be consistent with the predicted theoretical spectrum and, therefore, with the presence of the GZK cutoff. Meanwhile, on the other hand, we have the observations from the Akeno Giant Air Shower Array (AGASA) collaboration group [9], which reveal an abundant flux of incoming cosmic rays with energies above 1×10^{20} eV. The appearance of these high energy events is opposed to the predicted GZK cutoff, and represents a great challenge that has motivated a vast amount of new ideas and mechanisms to explain the phenomenon [10]. If the AGASA observations are correct then, since there are no known active objects in our neighborhood (let us say within a radius $R \simeq 100$

Mpc) able to act as sources of such energetic particles and since their arrival is mostly isotropic (without any privileged local source), we are forced to conclude that these cosmic rays come from distances larger than 100 Mpc. This is commonly referred as the Greisen-Zatsepin-Kuzmin (GZK) anomaly.

One of the interesting notions emerging from the possible existence of the GZK anomaly is that, since ultra high energy cosmic rays involve the highest energy events registered up to now, then a possible framework to understand and explain this phenomena could be of a quantum-gravitational nature [11, 12, 13]. This possibility is indeed very exciting if we consider the present lack of empirical support for the different approaches to the problem of gravity quantization. In the context of the UHECR phenomena, all these different approaches motivated by different quantum gravity formulations, have usually converged on a common path to solve and explain the GZK anomaly: the introduction of effective models for the description of high energy particle propagation. These effective models, pictures of the yet unknown full quantum gravity theory, offer the possibility to modify conventional physics through new terms in the equations of motion (now effective equations of motion), leading to the eventual breakup of fundamental symmetries such as Lorentz invariance (expected to be preserved at the fundamental level). These Lorentz symmetry breaking mechanisms are usually referred as Lorentz invariance violations (LIV's) if the break introduce a privileged reference frame, or Lorentz invariance deformations (LID's), if such a reference frame is absent [14]. Its appearance on theoretical and phenomenological grounds (such as high energy astrophysical phenomena) has been widely studied, and offers a large and rich amount of new signatures that deserve attention [15].

To deepen the above ideas, we have adopted the loop quantum gravity (LQG) theory [16]. It is possible to study LQG through effective theories that take into consideration matter-gravity couplings. In this line, in the works of J. Alfaro *et al.* [17], the effects of the loop structure of space at the Planck level are treated semiclassically through a coarse-grained approximation. An interesting feature of these methods is the explicit appearance of the Plank scale l_p and the appearance of a new length scale $\mathcal{L} \gg l_p$ (called the "weave" scale), such that for distances $d \ll \mathcal{L}$ the quantum loop structure of space is manifest, while for distances $d \geq \mathcal{L}$ the continuous flat geometry is regained. The presence of these two scales in the effective theories has the consequence of introducing LIV's to the dispersion relations $E = E(p)$ for particles with energy E and momentum p. It can be shown that these LIV's can significantly modify the kinematical conditions for a reaction to take place. For instance, as shown in detail in [18], a consequence for the UHECR phenomenology is that the kinematical conditions for a reaction between a primary cosmic ray and a CMBR photon can be modified, leading to new effects and predictions such as an abundant flux of cosmic rays well beyond the GZK cutoff energy (explaining in this way the AGASA observations).

In this work we provide some techniques to establish and analyze new constraints on the LQG parameters (or any LIV parameters coming from other theories). Also, we attempt to predict a modified UHECR spectrum consistent with the AGASA observations. The results shown here are presented in detail in a previous work by J. Alfaro and G. Palma [13]. This paper is organized as follows: In section 2 we give a brief derivation of the conventional spectrum and analyze it with AGASA observations. In section 3 we present a short outline of loop quantum gravity and its effective description of fermionic and bosonic fields. In section 4 we analyze the effects of LQG corrections on the threshold conditions for the main reactions involved in the UHECR phenomena to take place. In section 5 we show how the modified kinematics can be relevant to the theoretical spectrum $J(E)$ of cosmic rays (we will present the obtained modified spectrum). In section 6 we give some final remarks.

2 Ultra High Energy Cosmic Rays

Here we review the main steps in the derivation of the UHECR spectrum. This presentation will be useful and relevant to understand how LQG corrections affect the predicted flux of cosmic rays. The following material is mainly contained in the works of F.W. Stecker [4], Berezinsky *et al.* [6] and S.T. Scully & F.W. Stecker [7].

2.1 General Description

Two simple and common assumptions used in the development of the cosmic ray spectrum are: 1) sources are uniformly distributed in the Universe, and 2) the generation flux $F(E_g)$ of emitted cosmic rays from the sources is well described by a power law behavior of the form $F(E_g) \propto E_g^{-\gamma_g}$, where E_g is the energy of the emitted particle and γ_g is the generation index.

One of the main quantities for the computation of the UHECR spectrum is the energy loss $-E^{-1}dE/dt$. This quantity describes the rate at which a cosmic ray loses energy, and takes into consideration two chief contributions: the energy loss due to the redshift attenuation and the energy loss due to collisions with the CMBR photons. This last contribution depends, at the same time, on the cross sections σ and the inelasticities K of the interactions produced during the propagation of protons in the extragalactic medium, as well as on the CMBR spectrum. The most important reactions taking place in the description of proton's propagation are the pair creation and the photo-pion production:

$$p + \gamma \to p + e^- + e^+, \qquad \text{and} \qquad \mathrm{p} + \gamma \to \mathrm{p} + \pi. \tag{1}$$

The photo-pion production happens through several channels (for example the baryonic Δ and N, and mesonic ρ and ω resonance channels, just to mention some of them) and is the main responsible in the appearance of the GZK cutoff.

2.2 Some Kinematics

To study the interaction between protons and the CMBR, it is useful to distinguish between three reference systems; the laboratory system $\mathcal{K}$ (which we identify with the Friedman Robertson Walker (FRW) co-moving reference system), the center of mass (c.m.) system $\mathcal{K}^*$, and the system where the proton is at rest $\mathcal{K}'$. In terms of these systems, the photon energy will be expressed as ω in $\mathcal{K}$ and as ϵ in $\mathcal{K}'$. The relation between both quantities is simply $\epsilon = \gamma\omega(1 - \beta\cos\theta)$, where $\gamma = E/m_p$ is the Lorentz factor relating $\mathcal{K}$ and $\mathcal{K}'$, E and m_p are the energy and mass of the incident proton, $\beta = \sqrt{1-\gamma^{-2}}$, and θ is the angle between the momenta of the photon and the proton measured in the laboratory system $\mathcal{K}$. To determine the total energy $E^*_{\rm tot} = E^* + \epsilon^*$ in the c.m. system, it is enough to use the invariant energy squared $s \equiv E^2_{\rm tot} - p^2_{\rm tot}$ (where $E_{\rm tot} = E + \omega$ and $p_{\rm tot}$ are the total energy and momentum in the laboratory system). In this way, we have $E^{*\,2}_{\rm tot} = s = m_p^2 + 2m_p\epsilon$. As a consequence, the Lorentz factor γ_c which relates the $\mathcal{K}$ reference system with the $\mathcal{K}^*$ system, is

$$\gamma_c = \frac{E+\omega}{\sqrt{s}} \simeq \frac{E}{(m_p^2 + 2m_p\epsilon)^{1/2}}. \tag{2}$$

Let us consider the relevant case in which the reaction between the proton and the CMBR photon is of the type

$$p + \gamma \to a + b, \tag{3}$$

where a and b are two final particles of the collision. The final energies of these particles are easily determined by the conservation of energy-momentum. In the $\mathcal{K}^*$ system these are

$$E^*_{a,\,b} = \frac{1}{2\sqrt{s}}(s + m^2_{a,\,b} - m^2_{b,\,a}). \tag{4}$$

Transforming this quantity to the laboratory system, and averaging with respect to the angle between the directions of the final momenta, it is possible to find that the final average energy of a (or b) in the laboratory system is

$$\langle E_{a,\,b}\rangle = \frac{E}{2}\left(1 + \frac{m^2_{a,\,b} - m^2_{b,\,a}}{s}\right). \tag{5}$$

The inelasticity K of the reaction is defined as the average fractional difference $K = \Delta E/E$, where $\Delta E = E - E_{\rm f}$ is the difference between the initial energy E and final energy $E_{\rm f}$ of the proton (in a single collision with the CMBR photons). For the particular case of the emission of an arbitrary particle a (that is to say $p + \gamma \to p + a$), expression (5) allows us to write

$$K_a(s) = \frac{1}{2}\left(1 + \frac{m_a^2 - m_p^2}{s}\right), \tag{6}$$

where K_a is the inelasticity of the described process. This is one of the main quantities involved in the study of the UHECR spectrum.

2.3 Mean Life $\tau(E)$

To derive the UHECR spectrum it is imperative to know the mean life $\tau(E)$ of the cosmic ray (or proton) with energy E propagating in space, due to the attenuation of its energy by the interactions with the CMBR photons. The mean life $\tau(E)$ is defined through the relation $\tau(E)^{-1} = \left(-E^{-1}dE/dt\right)_{\rm col}$, where the label "col" refers to the fact that the energy loss is due to the collisions with the CMBR photons. It is possible to show that the mean life $\tau(E)$ can be written in the form

$$\tau(E)^{-1} = -\frac{kT}{2\pi^2\gamma^2}\int_{\epsilon_{\rm th}}^{\infty} d\epsilon\, \sigma(\epsilon)K(\epsilon)\epsilon \ln[1 - e^{-\epsilon/2\gamma kT}], \tag{7}$$

where $K(\epsilon) = \Delta E/E$ is the inelasticity (with ΔE the difference between the initial and final energies of the proton before and after each collision), $\sigma(\epsilon)$ is the scattering cross section with the CMBR photons, KT is the temperature of the CMBR bath, γ the lorentz factor, and $\epsilon_{\rm th}$ is the threshold photon energy for reactions to take place.

2.4 Energy Loss and Spectrum

The energy loss suffered by a very energetic proton during its journey, from a distant source to our detectors, is not only produced by the collisions that it has with CMBR at a particular epoch. There is also a decrease in its energy due to the redshift attenuation produced by the expansion of the Universe. At the same time, such expansion will affect the collision rate through the attenuation of the photon gas density, which can be understood as a cooling of the CMBR through the relation $T = (1 + z)T_0$, where z is the redshift and T_0 is the temperature of the background at the present time. To calculate the spectrum we need to consider the rate of energy loss during any epoch z of the Universe.

For the present discussion, we consider a flat Friedman Robertson Walker (FRW) universe dominated by matter. The above assumptions give rise to the following relation between the time coordinate t and the redshift z: $dt = -dz/H_0(1 + z)^{5/2}$, where H_0 is the Hubble constant at present time. Since the momentum of a free particle in a FRW space behaves as $p \propto (1 + z)$, we will have, with the additional consideration $p \gg m$ (where m is the particle mass), that the energy loss due to redshift is

$$\left(-\frac{1}{E}\frac{dE}{dt}\right)_{\rm cr} = H_0(1 + z)^{3/2}. \tag{8}$$

On the other hand, the energy loss due to collisions with the CMBR evolves as the background temperature changes (recall $T = (1 + z)T_0$). This evolution can be parameterized through z and is given by

$$\left(-\frac{1}{E}\frac{dE}{dt}\right)_{\rm col} = (1 + z)^3\tau([1 + z]E)^{-1}. \tag{9}$$

Thus, the total energy loss can be expressed considering both contributions (using z instead of t):

$$\frac{1}{E}\frac{dE}{dz} = (1 + z)^{-1} + H_0^{-1}(1 + z)^{1/2}\tau([1 + z]E)^{-1}. \tag{10}$$

Equation (10) can be integrated numerically to provide the energy $E_g(E, z)$ of a proton generated by the source in a z epoch and that will be detected with a energy E here on Earth. Let us express this solution formally by: $E_g(E, z) = \lambda(E, z)E$.

It is also possible to manipulate equation (10) to obtain an expression for the dilatation of the energy interval dE_g/dE. To accomplish this it is necessary to integrate (10) with respect to z and then differentiate it with respect to E to obtain an integral equation for dE_g/dE. The solution of such an equation is found to be

$$\frac{dE_g(z_g)}{dE} = (1 + z_g)\exp\left[\int_0^{z_g}\frac{dz}{H_0}(1 + z)^{1/2}\frac{db(E')}{dE'}\right]. \tag{11}$$

where $E' = (1 + z)\lambda(E, z)E$.

The total flux $dj(E)$ of emitted particles from a volume element $dV = R^3(z)r^2 dr d\Omega$, in the epoch z and coordinate r, measured from Earth at present time with energy E is

$$dj(E)dE = \frac{F(E_0, z)dE_0 n(z)dV}{(1+z)4\pi R_0^2 r^2}, \tag{12}$$

where $j(E)$ is the particle flux per energy, $F(E_0, z)dE_0$ the emitted particle flux within the range $(E_0, E_0 + dE_0)$, and $n(z)$ the density of sources in z. As previously mentioned, it is convenient to study the emission flux with a power law spectrum of the type $F(E) \propto E^{-\gamma_g}$. It can be shown that with such assumption, the relation between the emission flux and the total luminosity L_p of the source is $F(E) = (\gamma_g - 2)L_p E^{-\gamma_g}$. To describe the evolution of the sources we shall also use a power law behavior. This will be done through the relation

$$L_p(z)n(z) = (1+z)^{(3+m)}\mathcal{L}_0, \tag{13}$$

where $\mathcal{L}_0 = L_p(0)n(0)$. In this way, $m = 0$ corresponds to the case in which sources do not evolve. If we consider that $R_0 = (1+z)R(z)$ and $R(z)dr = dt$ for flat spaces (and $v \simeq 1$ for very energetic particles) and integrating (12) from $z = 0$ to some $z = z_{max}$ for which sources are not relevant for the phenomena, it is possible to obtain

$$j(E) = (\gamma_g - 2)\frac{1}{4\pi}\frac{\mathcal{L}_0}{H_0}E^{-\gamma_g}\int_0^{z_{\max}} dz_g(1+z_g)^{m-5/2}\lambda^{-\gamma_g}(E, z_g)\frac{dE_g(z_g)}{dE}. \tag{14}$$

The above expression constitutes the spectrum of UHECR. The volumetric luminosity $\mathcal{L}_0$ and the γ_g and m indexes are free parameters that must be fixed observationally.

2.5 Ultra High Energy Cosmic Rays Spectrum

To accomplish the computation of the theoretical spectrum we need information about the dynamical processes taking place in the propagation of protons along the CMBR. As we already emphasized, the most important reactions taking place in the description of a proton's propagation are, the pair creation $p + \gamma \to p + e^- + e^+$, and the photo-pion production $p + \gamma \to p + \pi$. This last reaction is mediated by several channels. The main channels are $N + \pi$, $\Delta + \pi$, R, $N + \rho(770)$, $N + \omega(782)$. The total cross sections and inelasticities of these processes are well known and can be used in (7) to compute the main time life of protons as a function of their energy. Then, with the help of expressions (11) and (14), we can finally find the predicted spectrum for UHECR. Fig.1 shows the obtained spectrum $J(E)$ of UHECR and the AGASA observed data. Again, we have selected for the theoretical spectrum $J(E)$ the idealized case $E_{\max} = \infty$. To reconcile the data of the low energy region ($E < 4 \times 10^{19}$ eV), where the pair creation dominates the energy loss, it is necessary to have a generation index $\gamma_g = 2.7$ (with the additional supposition that sources do not evolve) and a volumetric luminosity $\mathcal{L}_0 = 4.7 \times 10^{51}$ergs/Mpc^{3}yr. It can be seen that for events with energies $E > 4 \times 10^{19}$ eV, where the energy loss is dominated by the photo-pion production, the predicted spectrum does not fit the data well. To have a statistical sense of the discrepancy between observation and theory, we can calculate the Poisson probability P of an excess in the five highest energy bins. This is $P = 1.1 \times 10^{-8}$. Another statistical measure is provided by the Poisson χ^2 [19]. Computing this quantity for the eight highest energy bins, we obtain $\chi^2 = 29$. These quantities show how far the AGASA measurements are from the theoretical prediction given by the curve of Fig.1. Other more sophisticated models have also been analyzed in detail [6], nevertheless, it has turned out that the conventional standard model of physics does not have the capacity to reproduce the observations from the AGASA collaboration group in a satisfactory way.

3 Loop Quantum Gravity

Loop quantum gravity is a canonical approach to the problem of gravity quantization. It is based on the construction of a spin network basis, labelled by graphs embedded in a three dimensional insertion Σ in space-time. A consequence of this approach is that the quantum structure of space-time will be of a polymer-like nature, highly manifested in phenomena involving the Planck scale l_p.

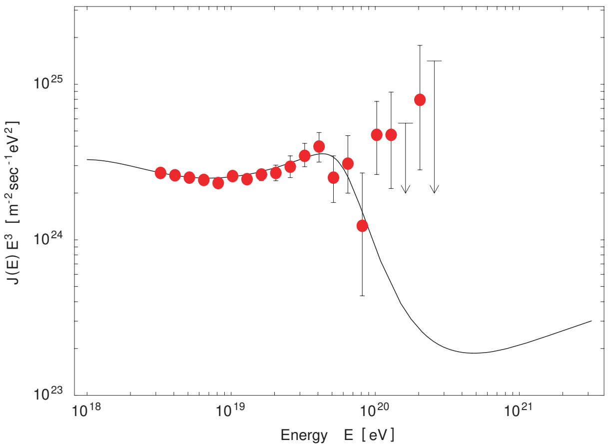

Figure 1: UHECR spectrum and AGASA observations. The figure shows the UHECR spectrum $J(E)$ multiplied by E^3, for uniform distributed sources, without evolution, and with a maximum generation energy $E_{\max} = \infty$. Also shown are the AGASA observed events. The best fit for the low energy sector ($E < 4 \times 10^{19}$ eV) corresponds to $\gamma_g = 2.7$.

The former brief description of loop quantum gravity allows us to realize how complicated a full treatment of a physical phenomena could be when the quantum nature of gravity is considered, even for a flat geometry. It is possible, however, to introduce a loop state which approximates a flat 3-metric on Σ at length scales greater than the length scale $\mathcal{L} \gg l_p$. For pure gravity, this state is referred to as the weave state $|W\rangle$, and the length scale $\mathcal{L}$ as the weave scale. A flat weave $|W\rangle$ will be characterized by $\mathcal{L}$ in such a way that for distances $d \ll \mathcal{L}$ the quantum loop structure of space is manifest, while for distances $d \geq \mathcal{L}$ the continuous flat geometry is regained. With this approach, for instance, the metric operator $\hat{q}_{ab}$ satisfies

$$\langle W|\hat{q}_{ab}|W\rangle = \delta_{ab} + \mathcal{O}(l_p/\mathcal{L}). \tag{15}$$

A generalization of the former idea, to include matter fields, is also possible. In this case, the loop state represents a matter field ψ coupled to gravity. Such a state is denoted by $|W, \psi\rangle$ and, again, is simply referred to as the weave. As before, it will be characterized by the weave scale $\mathcal{L}$ and the hamiltonian operators $\hat{H}_\psi$ are expected to fulfill a relation analog to (15), that is, we shall be able to define an effective hamiltonian H_ψ such that

$$H_\psi = \langle W, \psi|\hat{H}_\psi|W, \psi\rangle. \tag{16}$$

An approach to this task has been performed by J. Alfaro *et al.* [17] for 1/2-spin fermions and the electromagnetic field. In this approach the effects of the loop structure of space at the Planck level are treated semiclassically through a coarse-grained approximation [21]. This method leads to the natural appearance of LIV's in the equations of motion derived from the effective hamiltonian. The key feature here is that the effective hamiltonian is constructed from expectation values of dynamical quantities from both the matter fields and the gravitational field. In this way, when a flat weave is considered, the expectation values of the gravitational part will appear in the equations of motion for the matter fields in the form of coefficients with dependence in both scales, $\mathcal{L}$ and l_p. When a flat geometry is considered, the expectation values can be interpreted as vacuum expectation values for the considered matter fields.

A significant discussion is whether the Lorentz symmetry is present in the full LQG theory or not [22]. For the present work, we shall assume that Lorentz symmetry is indeed present in the full LQG theory. This assumption, jointly with the consideration that the new corrective coefficients are vacuum expectation values, leads us to consider that the Lorentz symmetry is spontaneously broken in the effective theory level.

Specially important in the development of the effective equations of motion for matter fields is that they are only valid in a homogenous and isotropic system. From the point of view of a spontaneous symmetry breakup such a system is unique and, therefore, a privileged reference frame). It is possible then to put the equations of motion (and therefore the dispersion relations) in a covariant form through the introduction of a four-velocity vector explicitly denoting the existence of a preferred system. From the cosmological point of view, such a privileged system does exist, and corresponds to the CMBR co-moving reference system. For that reason, we shall assume that the preferred system denoted by the presence of LIV's is the same CMBR co-moving reference frame, and will use it as the laboratory system.

In what follows we briefly summarize the obtained dispersion relations for both, fermions and bosons.

3.1 Modified Dispersion Relations

The dispersion relation for fermions is found to be [17]:

$$E_{\pm}^2 = p^2 + 2\alpha p^2 + \eta p^4 \pm 2\lambda p + m^2, \tag{17}$$

where the $\pm$ signs correspond to the helicity state of the described particle (note that these signs are produced by parity violation terms in the equations of motion). Additionally, we have defined the parameters α, η and λ in such a way that they depend on the scales $\mathcal{L}$ and l_p in the following way:

$$\alpha = \kappa_\alpha (l_p/\mathcal{L})^2, \qquad \eta = \kappa_\eta\, l_p^2, \qquad \text{and} \qquad \lambda = \kappa_\lambda\, \mathrm{l_p}/2\mathcal{L}^2, \tag{18}$$

where κ_α, κ_η and κ_λ are adimensional parameters of order 1.

In the case of bosonic particles the dispersion relation consists of

$$E^2 = p^2 + 2\alpha p^2 + \eta p^4 + m^2, \tag{19}$$

where again we have defined parameters α and η as:

$$\alpha = \kappa_\alpha (l_p/\mathcal{L})^2, \qquad \text{and} \qquad \eta = \kappa_\eta\, \mathrm{l_p^2} \tag{20}$$

where κ_α, κ_η and κ_λ are adimensional parameters of order 1.

As shown in detail in [13], only the α correction will be relevant in the computation of the spectrum. Therefore, in the rest of this work we shall only consider the following dispersion relation for both, fermions and bosons:

$$E^2 = p^2 + 2\alpha p^2 + m^2. \tag{21}$$

To conclude, let us mention that the dispersion relation (21) will be used for the physical description of electrons, protons, neutrons, Δ and N baryonic resonances, as well as for mesons π, ρ and ω.

4 Threshold Conditions

A useful discussion around the effects that LIV's can have on the propagation of UHECR can be raised through the study of the threshold conditions for the reactions to take place [18]. To simplify our subsequent discussions, let us use the following notation for the modified dispersion relations

$$E^2 = p^2 + f(p) + m^2, \tag{22}$$

where $f(p)$ is the deformation function of the momentum p.

A decay reaction is kinematically allowed when, for a given value of the total momentum $\vec{p}_0 = \sum_{\text{initial}} \vec{p} = \sum_{\text{final}} \vec{p}$, one can find a total energy value E_0 such that $E_0 \geq E_{\min}$. Here $E_{\min}$ is the minimum value attainable by the total energy of the decaying products for a given total momentum $\vec{p}_0$. To find $E_{\min}$, it is enough to take the individual decay product momenta to be collinear with respect to the total momentum $\vec{p}_0$ and with the same direction. To see this, we can variate E_0 with the appropriate restrictions

$$E_0 = \sum_i E_i(p_i) + \xi_j (p_0^j - \sum_i p_i^j), \tag{23}$$

where ξ_j are Lagrange multipliers, the i index specifies the ith particle and the j index the jth vectorial component of the different quantities. Doing the variation, we obtain

$$\frac{\partial E_i}{\partial p_i^j} \equiv v_i^j = \xi_j. \tag{24}$$

That is to say, the velocities of all the final produced particles must be equal to ξ. Since the dispersion relations that we are treating are monotonously increasing in the range of momenta $p > \lambda$, the momenta can be taken as being collinear and with the same direction of the initial quantity $\vec{p}_0$.

In this work, we will focus on those cases in which two particles (say a and b) collide to subsequently decay in the aforementioned final states. For the present discussion, particles a and b have momenta $\vec{p}_a$ and $\vec{p}_b$ respectively, and the total momentum of the system is $\vec{p}_0$. It is easy to see from the dispersion relations that we are considering, that the total energy of the system will depend only on $p_a = |\vec{p}_a|$ and $p_b = |\vec{p}_b|$. Therefore, to obtain the threshold condition for the mentioned kind of process, we must find the maximum possible total energy $E_{\max}$ of the initial configuration, given the knowledge of p_a and p_b. To accomplish this, let us fix $\vec{p}_a$ and variate the incoming direction of $\vec{p}_b = \hat{n} p_b$ in

$$E_0 \quad = \quad E_a(\vec{p}_0 - p_b\hat{n}) + E_b(p_b) + \chi(\hat{n}^2 - 1). \tag{25}$$

Varying (25) with respect to $\hat{n}$ (χ is a Lagrange multiplier), we find

$$\hat{n}^i = v_a^i p_b / 2\chi. \tag{26}$$

In this way we obtain two extremal situations $\chi = \pm v_a p_b/2$, or simply $\hat{n}^i = \pm v_a^i / v_a$. A simple inspection shows that for the dispersion relations that we are considering, the maximum energy is given by $\hat{n}^i = -v_a^i/v_a$, or in other words, when a frontal collision takes place.

Summarizing the threshold condition for a two particle (a and b) collision and subsequent decay, can be expressed through the following requirements:

$$E_a + E_b \geq \sum_{\text{final}} E_f, \tag{27}$$

with all final particles having the same velocity ($v_i = v_j$ for any to final particles i and j), and

$$p_a - p_b = \sum_{\text{final}} p_f, \tag{28}$$

where the sign of the momenta $\sum_{\text{final}} p_f$ is given by the direction of the highest momentum magnitude of the initial particles.

Our interest in the next subsections is the study of the reactions involved in high energy cosmic ray phenomena through the threshold conditions. To accomplish this goal through simple expressions that are easy-to-manipulate, we shall further use, for the equal velocities condition, the simplification

$$E_b m_a = E_a m_b, \tag{29}$$

valid for the study of parameters coming from the region $f(p) \ll m^2$. This simplification will allow the achievement of bounds over the order of magnitude of the different parameters involved in the modified dispersion relations, which are precisely our main concern.

In the following subsections we study the kinematical effects of LIV's through the threshold conditions for the reactions involved in the propagation of UHECR. Since, in this phenomena, photons are present in the form of low energy particles (the soft photons of the CMBR), the LQG corrections in the electromagnetic sector of the theory can be ignored. LQG corrections to the electromagnetic sector, however, have already been studied for other high energy reactions such as the Mkn 501 γ-rays [12]. Finally, let us recall that in the complete treatment of threshold conditions, it is possible to learn that only the α coefficients will be significant [13].

4.1 Photo-Pion Production $\gamma + p \rightarrow p + \pi$

Let us begin with the photo-pion production $\gamma + p \rightarrow p + \pi$. Considering the corrections provided in the dispersion relation (21) for fermions and bosons, we note that, for the photo-pion production to proceed, the following condition must be satisfied

$$2\,\delta\alpha\, E_\pi^2 + 4E_\pi\omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{30}$$

where E_π is the produced pion energy and $\delta\alpha = \alpha_p - \alpha_\pi$.

4.2 Resonant Production $\gamma + p \rightarrow \Delta$

The main channel involved in the photo-pion production is the resonant production of the $\Delta(1232)$. It can be shown that the threshold condition for the resonant $\Delta(1232)$ decay reaction to occur, is

$$2\,\delta\alpha\, E^2 + 4\omega E \geq m_\Delta^2 - m_p^2\,, \tag{31}$$

where E is the incident proton energy and $\delta\alpha = \alpha_p - \alpha_\Delta$.

4.3 Pair Creation $\gamma + p \rightarrow p + e^+ + e^-$

Pair creation, $\gamma + p \rightarrow p + e^+ + e^-$, is greatly abundant in the sector previous to the GZK limit. When the dispersion relations for fermions are considered for both protons and electrons, it is possible to find

$$\delta\alpha\frac{m_e}{m_p}E^2 + E\omega \geq m_e(m_p + m_e), \tag{32}$$

where E is the incident proton energy and $\delta\alpha = \alpha_p - \alpha_e$.

4.4 Bounds

In order to study the threshold conditions (30), (31) and (32), in the context of the GZK anomaly, we must establish some criteria. Firstly, as we have seen in section 2, the conventionally obtained theoretical spectrum provides a very good description of the phenomena up to an energy $\sim 4 \times 10^{19}$ eV. The main reaction taking place in this well described region is the pair creation $\gamma + p \rightarrow p + e^+ + e^-$ and, therefore, no modifications are present for this reaction up to $\sim 4 \times 10^{19}$ eV. As a consequence, and since threshold conditions offer a measure of how modified kinematics is, we will require that the threshold condition (32) for pair creation not be substantially altered by the new corrective terms.

Secondly, we have the GZK anomaly itself, which we are committed to explain. Since for energies greater than $\sim 8 \times 10^{19}$ eV the conventional theoretical spectrum does not fit the experimental data well, we shall require that LQG corrections be able to offer a violation of the GZK-cutoff. The dominant reaction in the violated $E > 8 \times 10^{19}$ region is the photo-pion production and, therefore, we further require that the new corrective terms present in the kinematical calculations be able to shift the threshold significantly to preclude the reaction.

We begin our analysis with the threshold condition for pair production. In this case we have:

$$\delta\alpha\,\frac{m_e}{m_p}E^2 + E\omega \geq m_e(m_p + m_e), \tag{33}$$

with $\delta\alpha = \alpha_p - \alpha_e$. As is clear from the above condition, the minimum soft-photon energy $\omega_{\min}$ for the pair production to occur, is

$$\omega_{\min} = \frac{m_e}{E}(m_p + m_e) - \delta\alpha\,\frac{m_e}{m_p}E. \tag{34}$$

It follows therefore that the condition for a significant increase or decrease in the threshold energy for pair production becomes $|\delta\alpha| \geq m_p(m_p + m_e)/E^2$. In this way, if we do not want kinematics to be modified up to a reference energy $E_{\mathrm{ref}} = 3 \times 10^{19}$, we must impose the following constraint

$$|\alpha_p - \alpha_e| < \frac{(m_p + m_e)m_p}{E_{\mathrm{ref}}^2} = 9.8 \times 10^{-22}. \tag{35}$$

Similar treatments can be found for the analysis of other astrophysical signals like the Mkn 501 γ-rays [23], when the absence of anomalies is considered.

Let us now consider the threshold condition for the photo-pion production. Taking only the α correction, we have

$$2\,\delta\alpha\, E_\pi^2 + 4E_\pi \omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{36}$$

It is possible to find that for the above condition to be violated for all energies E_π of the emerging pion, and therefore no reaction to take place, the following inequality must hold

$$\alpha_\pi - \alpha_p > \frac{2\omega^2(m_p + m_\pi)}{m_\pi^2(2m_p + m_\pi)} = 3.3 \times 10^{-24}\,[\omega/\omega_0]^2\,. \tag{37}$$

where $\omega_0 = KT = 2.35 \times 10^{-4}$ eV is the thermal CMBR energy. If we repeat these steps for the $\Delta(1232)$ resonant decay, we obtain the following condition

$$\alpha_\Delta - \alpha_p > \frac{2\omega^2}{m_\Delta^2 - m_p^2} = 1.7 \times 10^{-25}\,[\omega/\omega_0]^2\,. \tag{38}$$

To estimate a range for the weave scale $\mathcal{L}$, let us use as a reference energy $\omega_{\rm ref} = \omega_{\rm min}$, where $\omega_{\rm min}$ is the minimum energy for the reaction to take place, in inequality (36), when the condition for a significant increase in the threshold condition is taken into account (for a primordial proton reference energy $E_{\rm ref} = 2 \times 10^{20}$, this is $\omega_{\rm min} \sim 2.9 \times \omega_0$), and join the results deduced from the mentioned requirements. Assuming that the κ_α parameters are of order 1, as well as the difference between them for different particles, we can estimate —for the weave scale $\mathcal{L}$— the preferred range

$$2.6 \times 10^{-18}\ \text{eV}^{-1} \leq \mathcal{L} \leq 1.6 \times 10^{-17}\ \text{eV}^{-1}, \tag{39}$$

where the lefthand and righthand sides come from bounds (35) and (37) respectively (since the $\Delta(1232)$ is just one channel of the photo-pion production, we shall not consider it to set any bound).

If no GZK anomaly is confirmed in future experimental observations, then we should state a stronger bound for the difference $\alpha_\pi - \alpha_p$. Using the same assumptions to set the restriction (35) when the primordial proton reference energy is $E_{\rm ref} = 2 \times 10^{20}$ eV, it is possible to find

$$|\alpha_\pi - \alpha_p| < 2.3 \times 10^{-23}. \tag{40}$$

In terms of the length scale $\mathcal{L}$, this last bound may be read as

$$\mathcal{L} \geq 1.7 \times 10^{-17}\ \text{eV}^{-1}, \tag{41}$$

which is a stronger bound over $\mathcal{L}$ than (35), offered by pair creation.

5 Modified Spectrum

In this section we show the way in which the LQG correction α affects the prediction for the theoretical cosmic ray spectrum. Our approach will be centered on the supposition that the LQG corrections to the main quantities for the calculation —such as cross sections and inelasticities of processes— are, in a first instance, kinematical corrections, and that the Lorentz symmetry is spontaneously broken. These assumptions will allow us to introduce the adequate corrections when a modified dispersion relation is known.

5.1 Kinematics

Having a spontaneous Lorentz symmetry breaking we can use the still valid Lorentz transformations to express physical quantities observed in one reference system, in another one. This is possible since, under a spontaneous symmetry breaking, the group representations of the broken group preserves its transformation properties. In particular, it will be possible to relate the observed 4-momenta in different reference systems through the usual rule

$$p'_\mu = \Lambda_\mu{}^\nu p_\nu, \tag{42}$$

where $p_\mu = (-E, \vec{p})$ is an arbitrary 4-momentum expressed in a given reference system $\mathcal{K}$, p'_μ is the same vector expressed in another given system $\mathcal{K}'$, and $\Lambda_\mu{}^\nu$ is the usual Lorentz transformation connecting both systems. Such a transformation will keep invariant the scalar product

$$p^\mu p_\mu = -E^2 + p^2, \tag{43}$$

as well as any other product. Let us illustrate, for transformation (42), the situation in which $\mathcal{K}'$ is a reference system with the same orientation of $\mathcal{K}$ and which represents an observer with velocity $\vec{\beta}$ with respect to $\mathcal{K}$. In this case $\Lambda_\mu{}^\nu$ shall correspond to a boost in the $\hat{\beta} = \vec{\beta}/|\beta|$ direction, and expression (42) will be reduced to

$$E' = \gamma(E - \vec{\beta} \cdot \vec{p}), \qquad \text{and} \qquad \vec{\mathrm{p}}\,' = \gamma(\vec{\mathrm{p}} - \vec{\beta}\mathrm{E}), \tag{44}$$

where $\gamma = (1 - \beta^2)^{-1/2}$. A particular case of this transformation will be that in which $\vec{\beta}$ has the same direction as $\vec{p}$, and $\mathcal{K}'$ corresponds to the c.m. reference system, that is to say, the system in which $\vec{p}\,' = 0$. In such a case we will have $\vec{\beta} = \vec{p}/E$ and $\gamma = E/(E^2 - p^2)^{-1/2}$, jointly with the relation

$$E' = E/\gamma = (E^2 - p^2)^{1/2}. \tag{45}$$

In other words, the c.m. energy of a particle with energy E and momentum $\vec{p}$ in $\mathcal{K}$ will correspond to the invariant $(E^2 - p^2)^{1/2}$. Furthermore, such energy is the minimum measurable energy by an arbitrary observer; this can be confirmed by solving equation $\partial E'/\partial\beta = 0$ from the relation (44) and by verifying that the solution is $\beta = p/E$. This allows us to interpret $E' = (E^2 - p^2)^{1/2}$ as the rest energy of the given particle. To simplify the notation and the ensuing discussions, let us introduce the variable $s = (E')^2$, where E' is given by (45).

So far in our analysis, relativistic kinematics has not been modified. Nevertheless, a difference with the conventional kinematical frame is that in the present theory the product (43) will not be independent of the particle's energy; conversely, we will have the general expression

$$p^\mu p_\mu = -f_a(E, \vec{p}) - m_a^2, \tag{46}$$

where $f_a(E, \vec{p})$ is a function of the energy and the momentum, that represents the LIV provided by the LQG effective theories. Let us note that expression (46) is just the modified dispersion relation

$$E^2 = p^2 + f_a(E, \vec{p}) + m_a^2. \tag{47}$$

To be consistent $f_a(E, \vec{p})$ must be invariant under Lorentz transformations and, therefore, can be written as a scalar function of the energy and the momentum.

We have already made mention of the fact that LIV's inevitably introduce the appearance of a privileged system; in the present discussion we will choose as such a system the isotropic system (which by assumption is the co-moving CMBR system), and will express $f_a(E, \vec{p})$ in terms of E and $\vec{p}$ measured in that system. As may be expected in this situation, f_a will be a function only of the energy E and the momentum norm $p = |\vec{p}|$, since no trace of a vectorial field could be allowed when isotropy is imposed. For example, in the particular case of the dispersion relation for a fermion, the function $f_a(E, \vec{p})$ depends uniquely on the momentum, and can be written as $f_a(p) = 2\alpha_a p^2$. For simplicity, we shall continue using $f_a(p)$ instead of $f_a(E, \vec{p})$.

Through the recently introduced notation and the use of expression (45), the c.m. energy of an a particle with mass m_a and deformation $f_a(p)$ will be

$$s_a^{1/2} = \sqrt{f_a(p) + m_a^2}. \tag{48}$$

Of course, the validity of this interpretation will be subordinate to those cases in which $s_a = f_a(p) + m_a^2 > 0$, or, equivalent, to those states with a time-like 4-momentum. In the converse, particles with energies and corrections such that $s_a = f_a(p) + m_a^2 \leq 0$ will be described by light-like physical states if the equality holds, or space-like physical states if the inequality holds.

A new effect provided by LIV's is that, if a reference system where $p = 0$ exists, then in that system the particle will not be generally at rest. To understand this it is sufficient to verify that in general the velocity follows $v = \partial E/\partial p \neq p/E$, and therefore does not vanish at $p = 0$. Returning to the equations in (44), we can see that when $\beta = v = \partial E/\partial p$, the following result is produced

$$\frac{\partial E'}{\partial p'} = \gamma_v \left(\frac{\partial E}{\partial p} - v \right) \frac{\partial p}{\partial p'} = 0, \tag{49}$$

where $\gamma_v = (1 - v^2)^{-1/2}$. Result (49) shows that the velocity of the system where the particle is at rest is effectively v. There emerges, then, an important distinction between the phase velocity $\beta = p/E$ of the c.m. system of a particle, and the group velocity $v = \partial E/\partial p$ of the same particle.

The above results, for a single particle, can be easily generalized to a system of many particles. For instance, the total 4-momentum of a system of many particles $p^\mu_{\rm tot} = \sum_i p^\mu_i$ will transform through the rule (42), and the scalar product $(p^\mu p_\mu)_{\rm tot}$ will be an invariant under Lorentz transformations (as well as any other product). As in the case of individual particles, we define $\sqrt{s}$ as the total rest energy measured in the system of c.m. That is to say: $s = E^2_{\rm tot} - p^2_{\rm tot}$. In the case in which we have a system composed of a proton with energy E and momentum p, and a photon (from the CMBR) with energy ω and momentum k (all these quantities are measured in the laboratory isotropic system $\mathcal{K}$), the s quantity shall acquire the form

$$\begin{aligned} s &= (E + \omega)^2 - (\vec{p} + \vec{k})^2 \\ &= (E' + \omega')^2 - (\vec{p}\,' + \vec{k})^2, \end{aligned} \tag{50}$$

where the E' and p' quantities are measured in an arbitrary reference system. In particular we are interested in the system where the proton momentum $\vec{p}\,'$ is null; that is to say, the system in which $E' = \sqrt{s_p}$. If ϵ is the photon energy in such a system, then

$$\begin{aligned} s &= (\sqrt{s_p} + \epsilon)^2 - \epsilon^2 \\ &= 2\sqrt{s_p}\,\epsilon + s_p\,, \end{aligned} \tag{51}$$

where we have used the dispersion relation $\omega = k$ (or $\epsilon = k\,'$) for the CMBR photons. These results will allow us to express the main kinematical quantities in term of ϵ and $s_p = f_p(p) + m_p^2$. For example, the Lorentz factor that connects the $\mathcal{K}$ system with the $\mathcal{K}'$ system where $p\,' = 0$, will be

$$\gamma = E/\sqrt{s_p}\,. \tag{52}$$

Meanwhile, the Lorentz factor connecting $\mathcal{K}$ with the c.m. system (that in which $\vec{p}\,' + \vec{k} = 0$), will be

$$\gamma_c = \frac{E + \omega}{s_p + 2\sqrt{s_p}\,\epsilon} \simeq \frac{E}{s_p + 2\sqrt{s_p}\,\epsilon}. \tag{53}$$

As a last comment, let us note that to the first order in the expansion of the dispersion relations in terms of the scales $\mathcal{L}$ and l_p, when we consider high energy processes such that $p^2 \gg f(p) + m^2$ we can freely interchange the momentum p by the energy E in the deviation function $f(p)$. That is to say, we may consider as a valid relation the following expression

$$E^2 = p^2 + f(E) + m^2, \tag{54}$$

where we have made the replacement $f(p) \to f(E)$. This procedure will greatly simplify the next discussions.

5.2 Modified Inelasticity: $p + \gamma \to p + x$

Following the same methods of section 2, let us obtain a modified inelasticity K for a process of the type $p + \gamma \to p + x$, where x is an emitted particle that, in the present physical problem in which we are interested, can be a π, ρ or ω meson. We note that the dispersion relation for the emerging proton (after a collision with a photon) can be written in the form:

$$E_p^2 - p_p^2 = f_p(E_p) + m_p^2, \tag{55}$$

where E_p is the final proton energy. Since the left side of (55) is invariant under Lorentz transformations, we can write

$$(E_p^*)^2 - (p_p^*)^2 = f_p(E_p) + m_p^2, \tag{56}$$

where the * denotes the quantities measured in the c.m. system. On the other hand, in such a system, the following conservation relations of energy and momentum are satisfied:

$$E_p^* + E_x^* = \sqrt{s} \quad \text{and} \quad (\mathrm{p}_\mathrm{p}^*)^2 = (\mathrm{p}_\mathrm{x}^*)^2. \tag{57}$$

Substituting both quantities in relation (55), we obtain

$$2\sqrt{s}E_p^* = s + s_p(E_p) - s_x(E_x). \tag{58}$$

In the same way, we also have the energy conservation relation in the laboratory system: $E_p + E_x = E_{\mathrm{tot}}$. Using the definition for the inelasticity $K_x = \Delta E/E$ for a process, where $\Delta E = E_i - E_f \simeq E_{\mathrm{tot}} - E_f$, it is possible to rewrite the energy conservation relation in the laboratory system in terms of K_x through expressions $E_x = K_x E$, and $E_p = (1 - K_x)E$, where E is the initial energy of the initial proton. Having done this, equation (58) now acquires the form:

$$2\sqrt{s}E_p^* = s + s_p[(1 - K_x)E] - s_x[K_x E]. \tag{59}$$

To simplify the development of the inelasticity, let us write the former relation as $E_p^* = F(E, K_x)$, where $F = F(E, K_x)$ is defined through

$$F = \frac{1}{2\sqrt{s}}(s + s_p[(1 - K_x)E] - s_x[K_x E]). \tag{60}$$

On the other side, the Lorentz transformation rules give us the relation between the proton energies in the laboratory system and the c.m. system respectively. This relation is $E_p/\gamma_c = E_p^* + \beta_c\sqrt{E_p^{*2} - s_p(E_p)}\cos\theta$. Joining this expression with (60), it is possible to find the general equation for K_x:

$$(1 - K_x)\sqrt{s} = \Big(F(E, K_x) + \sqrt{F^2(E, K_x) - s_p[(1 - K_x)E]}\cos\theta\Big). \tag{61}$$

It should be noted, however, that the solution for K_x from (61) will depend in the θ angle. For this reason, once this last equation is resolved, it is convenient to define the total inelasticity K as the average of K_x with respect to the θ angle. That is to say

$$K = \frac{1}{\pi}\int_0^\pi K_x \, d\theta. \tag{62}$$

It is relevant to mention that now, as opposed to result (6), the inelasticity K will be a function of both, the energy E of the initial proton and the energy ϵ of the CMBR photon.

5.3 The $m_a^2 \to s_a = m_a^2 + f_a(E)$ Prescription

Let us recall our interpretation relative to the fact that $s_a^{1/2} = (f_a(E_a) + m_a^2)^{1/2}$ can be understood as the rest energy of a particle a, as a function of the energy E_a that it has in the laboratory system $\mathcal{K}$. As we have already emphasized, such an interpretation will be valid for particles with time like 4-momenta.

In the reactions given between high energy protons and the photons of the CMBR, the whole scenario consists of the collision between two particles, p and γ, with the subsequent production of a certain number of final particles. Let us suppose that a is one of these particles in the final state. The knowledge of the inelasticity K for the reaction will allow us to estimate the average energy $\langle E_a \rangle$ with which such a particle emerges (since K provides the average fraction of energy with which such a particle is produced). That is to say, on average, the rest energy of the final particle a will be $s_a^{1/2} = [f_a(\langle E_a \rangle) + m_a^2]^{1/2}$. Moreover, the knowledge of the inelasticity K will allow us to express s_a as a function of the energy E of the initial proton:

$$s_a = s_a(E). \tag{63}$$

Following our previous interpretation, we can view the recently described process as a reaction between a proton with mass $s_p^{1/2}$, which loses energy emitting particles a with mass $s_a^{1/2}$ calculated in the previous form. This idealized reasoning gives us a clear prescription to kinematically modify those dynamical quantities with which we must work and where energy conservation is involved. This prescription is:

$$m_a^2 \to s_a(E) = f_a(E) + m_a^2, \tag{64}$$

where we have expressed correction f_a as a function of the initial energy of the incident proton.

Prescription (64) establishes the notion of an effective mass which is dependent on the initial energetic content of a reaction. As a consequence, given the explicit knowledge of the dependence that a cross section has on the masses and energies of the involved states, to obtain the modified version, it will be appropriate to use the discussed prescription.

5.4 Redshift

Another important problem related to the introduction of LIV's in the dispersion relations is whether the redshift relation for the propagation of particles in a FRW universe is modified. This could be of great relevance because of the large distances involved in cosmic ray propagation and, therefore, the possible cumulative effects. In the case of a FRW universe with scale factor R, it is possible to find that the propagation of particles with momentum $p^2 = g^{ij} p_i p_j$ (with g_{ij} the spatial part of the FRW metric) obeys the following equation

$$\frac{d}{d\tau}(pR) = 0, \tag{65}$$

where τ is a proper parameter describing the path followed by the particle. This result is regardless of any particular dispersion relation.

5.5 Spectrum and Results

Introducing the above modifications to the different quantities involved in the propagation of protons (like the cross section σ and inelasticity K), we are able to find a modified version for the UHECR energy loss due to collisions. Since the only relevant correction for the GZK anomaly is α, we focused our analysis on the particular case $f(p) = 2\alpha p^2$. To simplify our model we restricted our treatment to the case $\alpha > 0$ (consistent with the effective mass interpretation) and used only $\alpha_m \neq 0$, where α_m is assumed to have the same value for mesons π, ρ and ω.

Fig.2 shows the modified energy loss $\tau(E)$ for UHECR obtained for different values of α_m. It can be seen therefore how the corrections can affect the main life time of protons propagating through the CMBR, allowing a strong improvement in the distances that protons can reach before loosing their characteristic energy (for energies greater than 1×10^{20} eV). The effects that the LQG corrections have on the propagation of UHECR are manifest through a decay of the energy loss in the range $E \sim 1 \times 10^{20}$ eV. To understand this, recall relation (36) for the threshold condition of photo-pion production:

$$2\,\delta\alpha\, E_\pi^2 + 4 E_\pi \omega \geq \frac{m_\pi^2 (2m_p + m_\pi)}{m_p + m_\pi}. \tag{66}$$

As we saw in section 4, the condition for a significant increase or decrease in the energy threshold can be calculated as $|\delta\alpha| \geq (2m_p + m_\pi)(m_p + m_\pi)/2E^2$. Therefore, for a given value of $\delta\alpha > 0$, the energy at which the LIV effects start to take place is

$$E^2 = \frac{1}{2\delta\alpha}(2m_p + m_\pi)(m_p + m_\pi). \tag{67}$$

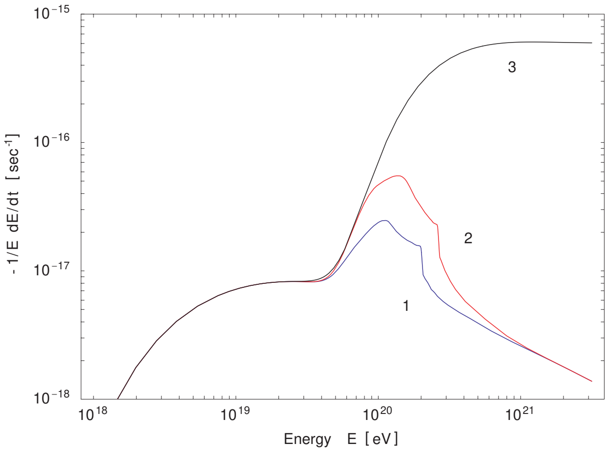

Figure 2: Modified energy loss for UHECR due to collisions. The figure shows the case $\alpha_m \neq 0$, for three different values of the weave scale $\mathcal{L}$. Curve 1: $\alpha_m = 9 \times 10^{-23}$ ($\mathcal{L} \simeq 8.6 \times 10^{-18}$ eV^{-1}); curve 2: $\alpha_m = 5 \times 10^{-23}$ ($\mathcal{L} \simeq 1.2 \times 10^{-17}$ eV^{-1}); curve 3: $\alpha_m = 0$ (without modifications).

In the case $\alpha_m = 9 \times 10^{-23}$ (curve 1 of Fig.2), this energy is $E = 1.1 \times 10^{20}$ eV, while in the case $\alpha_m = 5 \times 10^{-23}$ (curve 2) this corresponds to $E = 1.5 \times 10^{20}$ eV. Beyond these energy scales, at about $E \sim 2 \times 10^{20}$ eV, a sharp decay is observed in the behavior of the curve. This is due to the fact that the modified inelasticity K will strongly constraint the energy-momentum phase space accessible to the final states depending on the initial energy E that the primary proton carries (recall that now K is a function of the energy E of the incident proton and the energy ϵ of the CMBR photon).

Also, we can find the modified version of the UHECR spectrum for $\alpha_m \neq 0$. Fig.3 shows the AGASA observations and the predicted UHECR spectrum. The Poisson probabilities of an excess in the five highest energy bins for the three curves are $P_1 = 3.6 \times 10^{-4}$, $P_2 = 2.6 \times 10^{-4}$ and $P_3 = 2.3 \times 10^{-4}$. The Poisson χ^2 for the eight highest energy bins are ${\chi_1}^2 = 10$, ${\chi_2}^2 = 10.9$ and ${\chi_3}^2 = 11.2$ respectively. The possibility of reconciling the data with finite maximum generation energies is significant given that conventional models require infinite maximum generation energies $E_{\max}$ for the best fit. For the lower part of the spectrum (under $E = 4 \times 10^{19}$ eV), the parameters under consideration leave the spectrum completely unaffected. This is due to the fact that in such a region the dominant reaction is the pair production, which has not being modified to obtain the spectrum. A more accurate study on this issue would require the computation of a modified inelasticity for the pair creation. Meanwhile, we must content ourself with the semiqualitative criteria given in section 4 to rule out the parameters.

6 Conclusions

We have seen how the kinematical analysis of the different reaction taking place in the propagation of ultra high energy protons can set strong bounds on the parameters to the theory. In comparison with our previous work, we have eliminated some previously open possibilities by the particular study of the pair creation $p+\gamma \to p+e^+ +e^-$, in the energy region where this reaction dominates the proton's interactions with the CMBR. In this way, the only possibility still open is the correction α. If this is the case, a favored region for the scale length $\mathcal{L}$ estimated through the threshold analysis would be

$$2.6 \times 10^{-18} \ \text{eV}^{-1} \leq \mathcal{L} \leq 1.6 \times 10^{-17} \ \text{eV}^{-1}.$$

Similarly, the kinematical corrections can be studied in more detail when their effects are considered in the theoretical spectrum. In this regard, we have seen how to develop a modified version of the inelasticity for the photo-pion production, and its implications in the mean life time of a high energy proton as well as on the spectrum. To accomplish this last task we have only assumed a spontaneous Lorentz symmetry break up in the effective equations of motion, allowing the use of Lorentz transformations on the dispersion

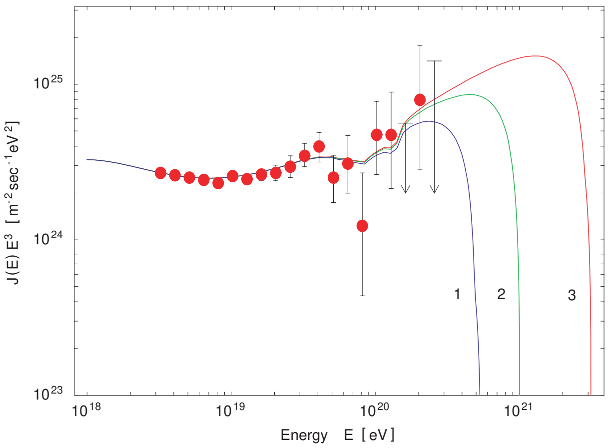

Figure 3: Modified UHECR spectrum and AGASA observations. The figure shows the modified spectrum $J(E)$ multiplied by E^3, for uniform distributed sources and without evolution, for the case $\alpha_m = 1.5 \times 10^{-22}$ ($\mathcal{L} \simeq 6.7 \times 10^{-18}$ eV^{-1}). Three different maximum generation energies $E_{\max}$ are shown. These are, curve 1: 5×10^{20} eV; curve 2: 1×10^{21} eV; and curve 3: 3×10^{21} eV.

relations. Therefore, result (61) can be used in a more general context than the special case offered by the LQG framework.

Future experimental developments like the Auger array, the Extreme Universe Space Observatory (EUSO) and Orbiting Wide-Angle Light Collectors (OWL) satellite detectors, will increase the precision and phenomenological description of UHECR. On the more theoretical side, progress in the direction of a full effective theory, with a systematic method to compute any correction with a known value for each coefficient, is one of the next steps in the "loop" quantization programme [25]. Therefore, it is important to trace a phenomenological understanding of the possible effects that could arise as well as the constraints on LQG, in the high and low energy regimens (for other phenomenological studies of LQG effects, see for example [26]).

Recently[27] the effective theory approach have been under criticism.

In [28], the type of terms we used to explain AGASA data are computed within the Standard model, depending on one arbitrary parameter. The criticism of [27] does not apply to this formalism.

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