Recursive technique for evaluation of Feynman diagrams

V. V. Andreev
Gomel State University,
Physics Department,
Sovietskaya 104 str., Gomel, Belarus

Abstract

A method is presented in which matrix elements for some processes are calculated recursively. This recursive calculational technique based on the method of basis spinor.

1 Introduction

The possibility of investigation higher and higher energies at present and future colliders, entails the necessity of predicting and calculating with high precision more and more complicated processes. The standard method to obtain a cross section with the fermions in perturbative quantum field theories is to reduce the squared amplitude to a trace from products of γ-matrices. An alternative approach is to calculate the Feynman amplitudes directly. The idea of calculating amplitudes has a long enough history. In 1949 it was suggested in Ref.[1] to calculate a matrix element by means of explicit form of γ-matrices and Dirac spinors.

Various methods of calculating the reaction amplitudes with fermions have been proposed and successfully applied in recent years. The analytical methods of calculating the Feynman amplitudes can be divided in two basic groups. The first group involves the analytical methods that reduce the calculation of S-matrix element to trace calculation. The reduction of a matrix element to a trace calculation from products of γ-matrices underlies a large number of methods (see, Refs.[2, 3] and Refs. [4]-[10] etc.).

The second group involves the analytical methods that practically do not use the operations with traces from products of γ-matrices. The method of the CALCUL group which was used for the calculations of the reactions with massless fermions is the most famous among [11]-[15]. There are generalizations of the CALCUL method for massive Dirac particles both for special choices of the fermion polarization ([13],[16]-[20]) and for an arbitrary fermion polarization [21, 22]. We call the polarization states of fermions in Ref.[13, 16, 20] as Berends-Daverveldt-Kleiss-Stirling or $BDKS$-states. .

Thanks to their easy implementation methods of matrix element calculation have become a basis for modern programs dealing with evaluation cross-sections for various processes.

In the paper we describe an approach to Feynman diagrams is based on the using of an isotropic tetrad in Minkowski space and basis spinors connected with it (see [23, 24]). We propose to use recursion relations as a technique to evaluate the Feynman amplitudes of processes. The advantage of recursive technique is that for calculation of a $n+1$ matrix element of some process can use the calculation of n process. Both for analytic and numerical evaluation this is asset.

2 Isotropic tetrad and basis spinors

Let us introduce the orthonormal four-vector basis in Minkowski space which satisfies the relations:

$$l_0^\mu \cdot l_0^\nu - l_1^\mu \cdot l_1^\nu - l_2^\mu \cdot l_2^\nu - l_3^\mu \cdot l_3^\nu = g^{\mu\nu}, \quad (l_A \cdot l_B) = g_{AB}, \tag{1}$$

where g is the Lorentz metric tensor.

2.1 Isotropic tetrad

With the help of the basis vectors l_A $(A = 0, 1, 2, 3)$ we can define lightlike vectors, which form the isotropic tetrad in Minkowski space (see, [25])

$$b_\rho = \frac{l_0 + \rho l_3}{2}\ ,\ n_\lambda = \frac{\lambda\, l_1 + \mathrm{i} l_2}{2}\ , (\rho, \lambda = \pm 1)\ . \tag{2}$$

¿From Eqs. (1), (2) it follows that

$$(b_\rho \cdot b_{-\lambda}) = \frac{\delta_{\lambda,\rho}}{2}\ , \quad (n_\lambda \cdot n_{-\rho}) = \frac{\delta_{\lambda,\rho}}{2}\ , \quad (b_\rho \cdot n_\lambda) = 0\ , \tag{3}$$

$$g^{\mu\nu} = 2 \sum_{\lambda=-1}^{1} \left[b_\lambda^\mu \cdot b_{-\lambda}^\nu + n_\lambda^\mu \cdot n_{-\lambda}^\nu \right]\ . \tag{4}$$

Also for practical applications it is convenient to introduce the tensor $X^{\mu,\nu}$ which are related to isotropic tetrad vectors (3)

$$X_{A,\,\lambda}^{\mu,\,\nu} = 4\left(b_A^\mu \cdot n_\lambda^\nu - n_\lambda^\mu \cdot b_A^\nu\right)\ , \tag{5}$$

and the contractions with arbitrary four-vectors

$$X_{A,\,\lambda}^{p,\,q} \equiv p_\mu\, q_\nu X_{A,\,\lambda}^{\mu,\,\nu} = 4\left((p \cdot b_A)\,(q \cdot n_\lambda) - (p \cdot n_\lambda)\,(q \cdot b_A)\right)\ . \tag{6}$$

For photons with momentum k^μ and helicity $\lambda = \pm 1$ we use following definition of polarizations in the axial gauge

$$\varepsilon_\lambda\,(k) = \frac{-\sqrt{2}\,\lambda\,[(k \cdot n_{-\lambda})\, b_{-1} - (b_{-1} \cdot k)\ n_{-\lambda}]}{(k \cdot b_{-1})}\ , \tag{7}$$

provided that, the four-vectors k, b_1, b_{-1} are linearly independent.

2.2 Massless basis spinors

By means of the isotropic tetrad vectors (2) we define *massless basis spinors* $u_\lambda\,(b_{-1})$ and $u_\lambda\,(b_1)$

$$\not{b}_{-1} u_\lambda\,(b_{-1}) = 0\ , \quad u_\lambda\,(b_1) \equiv \not{b}_1 u_{-\lambda}\,(b_{-1})\ , \tag{8}$$

$$\omega_\lambda u_\lambda\,(b_{\pm 1}) = u_\lambda\,(b_{\pm 1}) \tag{9}$$

with the matrix $\omega_\lambda = 1/2\,(1 + \lambda\gamma_5)$ and the normalization condition

$$u_\lambda\,(b_{\pm 1})\,\bar{u}_\lambda\,(b_{\pm 1}) = \omega_\lambda\ \not{b}_{\pm 1}. \tag{10}$$

The relative phase between basis spinors with different helicity is given by

$$\not{n}_\lambda u_{-\nu}\,(b_{-1}) = \delta_{\lambda,\nu} u_\lambda\,(b_{-1})\,. \tag{11}$$

The important property of basis spinors (8) is the completeness relation

$$\sum_{\lambda,A=-1}^{1} u_\lambda\,(b_A)\,\bar{u}_{-\lambda}\,(b_{-A}) = I\ , \tag{12}$$

which follows from Eqs.(8), (11). Thus, the arbitrary Dirac spinor can be decomposed in terms of basis spinors $u_\lambda\,(b_A)$.

Using the Eqs.(4),(8) and (11) we can obtain that

$$\gamma^\mu u_\lambda\,(b_A) = 2\ b_A^\mu\ u_{-\lambda}\,(b_{-A}) - 2\ A\ n_{-A\times\lambda}^\mu\ u_{-\lambda}\,(b_A)\ , \tag{13}$$

$$\bar{u}_\lambda\,(b_C)\, u_\rho\,(b_A) = \delta_{\lambda,-\rho}\ \delta_{C,-A}\ , \tag{14}$$

$$\omega_\lambda u_\rho\,(b_A) = \delta_{\lambda,\rho} u_\rho\,(b_A)\,,\ (C, A, \lambda, \rho) = \pm 1\ . \tag{15}$$

and

$$\gamma^\mu \gamma^\nu u_\lambda\,(b_A) = g^{\mu\nu}\ u_\lambda\,(b_A) - A\ X_{A,-A\times\lambda}^{\mu,\nu}\ u_\lambda\,(b_{-A})\ . \tag{16}$$

Eqs.(13)-(15), and also Eq. (16) underlie the method of basis spinors (MBS) [23],[24].

3 Recursion relations for the matrix elements

When evaluating a Feynman amplitude involving the fermions, the amplitude is expressed as sum of terms which have the form

$$\begin{aligned} &\mathcal{M}_{\lambda_p,\lambda_k}(p,s_p,\ k,s_k\ ;Q) = \\ &= \mathcal{M}_{\lambda_p,\lambda_k}([p]\,,[k]\,;Q) = \bar{u}_{\lambda_p}(p,s_p)\,Q\ u_{\lambda_k}(k,s_k)\ , \end{aligned} \tag{17}$$

where λ_p and λ_k are the polarizations of the external particles with four-momentum p, k and arbitrary polarization vectors s_p, s_k. The operator Q is a sum of products of Dirac γ-matrices. The matrix element (17) with Dirac spinors is a scalar function. As such, it should be expressible in terms of scalar functions formed from the spin and momentum four-vectors of the fermions, including p, s_p, k, s_k and of the operator Q.

We will now that in the our approach this matrix element (17) can be represented as linear combinations of the products of the lower-order matrix elements.

3.1 Basic matrix element

Let us consider a special case of a matrix element (17), when $p = b_{-C}$ and $k = b_A$ (basic matrix element), i.e.

$$\mathcal{M}_{-\sigma,\rho}(b_{-C},\ b_A\ ;Q) \equiv \Gamma^{C,A}_{\sigma,\rho}[Q] = \bar{u}_{-\sigma}(b_{-C})\,Q u_\rho(b_A)\ . \tag{18}$$

With the help of the completeness relation (12) we can obtain the recursion formula for $\Gamma^{C,A}_{\sigma,\rho}[Q_1Q_2]$

$$\Gamma^{C,A}_{\sigma,\rho}[Q_1Q_2] = \sum_{D,\lambda=-1}^{1} \Gamma^{C,D}_{\sigma,\lambda}[Q_1]\,\Gamma^{D,A}_{\lambda,\rho}[Q_2]\,. \tag{19}$$

By means of the relations (13) and (14) it is easy to calculate $\Gamma^{C,A}_{\sigma,\rho}$ in terms of the isotropic tetrad vectors. For instance,

$$\Gamma^{C,A}_{\sigma,\rho}[\gamma^\mu] = 2\delta_{\sigma,-\rho}\left(\delta_{-C,A}\,b^\mu_A - A\,\delta_{C,A}\,n^\mu_{-A\times\rho}\right)\ , \tag{20}$$

$$\Gamma^{C,A}_{\sigma,\rho}[\gamma^\mu\,\gamma^\nu] = \delta_{\sigma,\rho}\left(\delta_{C,A}g^{\mu\,\nu} - A\,\delta_{C,-A}X^{\mu,\,\nu}_{A,\,-A\times\rho}\right)\ , \tag{21}$$

where $X^{\mu,\,\nu}_{A,\,-A\times\rho}$ is determined by Eq.(6).

3.2 Decomposition coefficients

The next type of lower-order matrix element (17) is

$$\mathcal{M}_{\rho,\lambda_p}(b_A\ ,[p]\,;I) \equiv \mathcal{M}_{\rho,\lambda_p}(b_A\ ,[p]) = \bar{u}_\rho(b_A)\,u_{\lambda_p}(p,s_p)\ . \tag{22}$$

The matrix element (22) is determined the decomposition coefficients of an arbitrary Dirac spinor $u_{\lambda_p}(p,s_p)$ on basis spinors (8).

The opportunity of calculating the coefficients (22) is founded on that an arbitrary Dirac spinor can be determined through the basis spinor (8) with the help of projection operators $u_{\lambda_p}(p,s_p)\,\bar{u}_{\lambda_p}(p,s_p)$.

Let us consider massive fermions. The Dirac spinors for massive fermion and antifermion with four-momentum p and arbitrary polarization vector s_p can be obtained with the help of the relation

$$w^A_\lambda(p,s_p) = \frac{(\not{p} + Am_p)\,(1+\lambda_p\gamma_5\,\not{s}_p)}{2\sqrt{(b_{-1}\cdot(p+m_ps_p))}}\,u_{-A\times\lambda_p}(b_{-1})\ . \tag{23}$$

The notation $w^A_{\lambda_p}(p,s_p)$ stands for either $u_{\lambda_p}(p,s_p)$ (Dirac spinor of fermion; $A=+1$) or $\upsilon_{\lambda_p}(p,s_p)$ (Dirac spinor of antifermion; $A=-1$). The bispinors $u_\lambda(p,s_p)$ and $\upsilon_\lambda(p,s_p)$ satisfy Dirac equation and spin conditions for massive fermions and antifermions.

The Dirac spinor $u_\lambda(p)$ of massless fermion with momentum p ($p^2 = 0,\ (p \cdot b_{-1}) \neq 0$) and helicity λ is defined by (see, for example, [13])

$$u_\lambda(p) = \frac{\not p}{\sqrt{2\,(p \cdot b_{-1})}} u_{-\lambda}(b_{-1}) \,. \tag{24}$$

Using the Eqs.(13)-(15) and (16) the matrix element (22) is reduced to an algebraic expression in terms of scalar products of isotropic tetrad vectors and physical vectors or in terms of components of four-vectors.

Let us consider massless fermions. Using Eqs.(13)-(14) we obtain, that

$$\mathcal{M}_{\rho,\lambda}(b_A, p) = \delta_{\lambda,-\rho}\sqrt{2}\left[\delta_{A,-1}\sqrt{(p \cdot b_{-1})} - \delta_{A,1}\frac{(p \cdot n_{-\lambda})}{\sqrt{(p \cdot b_{-1})}}\right]. \tag{25}$$

Let us consider massive Dirac massive particles with arbitrary polarization vector s_p. After evaluations we obtain, that the decomposition coefficients for a massive fermion with momentum p, an arbitrary polarization vector s_p and mass m_p can be written as scalar products of tetrad and physical vectors

$$\mathcal{M}_{\rho,\lambda_p}(b_A, p, s_p) = \frac{1}{\sqrt{2\,(b_{-1} \cdot \xi_1^p)}}\left[2\delta_{\lambda,-\rho}\left\{\delta_{A,-1}(b_{-1} \cdot \xi_1^p) + \delta_{A,1}\left(n_{-\lambda_p} \cdot \xi_1^p\right)\right\}\right.$$
$$\left. + \delta_{\lambda,\rho}\left\{\delta_{A,1}\frac{m_p}{2} + \delta_{A,-1}\frac{X_{-1,-\lambda_p}^{\xi_{-1}^p,\xi_1^p}}{m_p}\right\}\right] , \tag{26}$$

where the scalar function $X^{p,q}$ is determined by Eq.(6).

3.3 Recursion relation

With the help of completeness relation (12) the amplitude (17) with $Q = Q_2 Q_1$ is expressed as combinations of the lower-order matrix element

$$\mathcal{M}_{\lambda_p,\lambda_k}([p], [k]; Q_2 Q_1) = \sum_{\sigma,A=-1}^{1} \mathcal{M}_{\lambda_p,\sigma}([p], b_A; Q_2)\, \mathcal{M}_{-\sigma,\lambda_k}(b_{-A}, [k]; Q_1) \,. \tag{27}$$

This insertion allows us to “cut” fermion chain (17) into pieces of fermion chains with basis spinors $u_\lambda(b_A)$. Hence the our formalism enables to calculate the blocks of the Feynman diagrams and then to use them in the calculation as scalar functions. All possible Feynman amplitudes can be built up from a set of “building” blocks.

Let us consider the matrix element (17) with an operator

$$Z^{(n)} = Q_n Q_{n-1} \cdots Q_1 Q_0 \tag{28}$$

with $Q_0 = I$. In the Eq.(28) all operators Q_j have an identical mathematical expressions. Using Eq.(27) we find that

$$\mathcal{M}_{\lambda_p,\lambda_k}\left([p], [k]; Z^{(n)}\right) \equiv \mathcal{M}^{(n)}_{\lambda_p,\lambda_k}([p], [k]) =$$
$$= \sum_{\sigma,A=-1}^{1} \mathcal{M}_{\lambda_p,\sigma}([p], b_A)\, \mathcal{M}^{(n)}_{-\sigma,\lambda_k}(b_{-A}, [k]) \,, \tag{29}$$

where matrix element $\mathcal{M}^{(n)}_{-\sigma,\lambda_k}(b_{-A}, [k])$ can be calculated with the help of recursion relation

$$\mathcal{M}^{(n)}_{-\sigma,\lambda_k}(b_{-A}, [k]) = \sum_{\rho,C=-1}^{1} \Gamma^{A,C}_{\sigma,\rho}[Q_n]\, \mathcal{M}^{(n-1)}_{-\rho,\lambda_k}(b_{-C}, [k]) \,. \tag{30}$$

Once scalar functions $\mathcal{M}_{\rho,\lambda_k}(b_A, [k])$ and $\Gamma^{A,C}_{\sigma,\rho}[Q_j]$ are known (see Eqs.(25), (26) and Eqs.(20)-(21)), it is possible to evaluate the higher order of $\mathcal{M}^{(n)}_{-\sigma,\lambda_k}(b_{-A}, [k])$ with the help of recursion relation (30).

4 Examples

Consider the "toy" example

$$\mathcal{M}^{(n)}_{\lambda_p,\lambda_k}(p,s_p,k,s_k) = \bar{u}_{\lambda_p}(p,s_p)\ \not{q}_n\ \not{q}_{n-1}\dots\ \not{q}_1 u_{\lambda_k}(k,s_k)\,, \tag{31}$$

where q_j are some arbitrary four-vectors.

Therefore, we have that

$$Z^{(n)} = \not{q}_n\ \not{q}_{n-1}\dots\ \not{q}_1\ . \tag{32}$$

With the help of Eq.(29) we can obtain the recursion formulas for calculation matrix element (31):

$$\mathcal{M}^{(j)}_{\lambda_p,\lambda_k}([p],[k]) = 2\sum_{\rho,C=-1}^{1}\mathcal{M}_{\lambda_p,\rho}([p],b_C)\Big[(q_j\cdot b_{-C})\,\mathcal{M}^{(j-1)}_{\rho,\lambda_k}(b_C,[k]) -$$
$$-C\ (q_j\cdot n_{C\times\rho})\,\mathcal{M}^{(j-1)}_{\rho,\lambda_k}(b_{-C},[k])\Big]\ , \tag{33}$$

and

$$\mathcal{M}^{(j)}_{\lambda_p,\lambda_k}([p],[k]) = \sum_{\rho,C=-1}^{1}\mathcal{M}_{\lambda_p,\rho}([p],b_C)\Big[(q_j\cdot q_{j-1})\,\mathcal{M}^{(j-2)}_{-\rho,\lambda_k}(b_{-C},[k]) +$$
$$+C\ X^{q_j,\,q_{j-1}}_{-C,C\times\rho}\mathcal{M}^{(j-2)}_{-\rho,\lambda_k}(b_C,[k])\Big]\ , \tag{34}$$

4.1 Photon emission

Let us consider the matrix element where photon emission occurs from the incoming electron

$$\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q\ Z_k) = \bar{u}_{\lambda_2}(p_2,s_{p_2})\,Q\ Z_k\ u_{\lambda_1}(p_1,s_{p_1})\ \text{, where} \tag{35}$$

$$Z_k = e\,\frac{(\not{p}_1 - \not{k} + m_e)\ \not{\varepsilon}_\sigma(k)}{(p_1-k)^2 - m_e^2}\ . \tag{36}$$

Using the algebra of γ-matrix and Dirac equation the operator Z_k rewritten as

$$Z_k = -e\left\{\frac{(p_1\ \varepsilon_\sigma(k))}{(p_1\ k)} - \frac{\not{k}\ \not{\varepsilon}_\sigma(k)}{2\,(p_1\ k)}\right\}\ . \tag{37}$$

Now we get

$$\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q\ Z_k) = -e\frac{(p_1\ \varepsilon_\sigma(k))}{(p_1\ k)}\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q) +$$
$$+\frac{e}{2\,(p_1\ k)}\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q\ \ \not{k}\ \not{\varepsilon}_\sigma(k))\ . \tag{38}$$

For the matrix element (35) with the fermions and photon of arbitrary polarization we have the exact formula in terms of lower-order matrix elements $\mathcal{M}_{\lambda_2,\sigma}([p_2],b_C;Q)$ and $\mathcal{M}_{-\sigma,\lambda_1}(b_C,[p_1])$

$$\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q\ \ \not{k}\ \not{\varepsilon}_\sigma(k)) =$$
$$= -e\frac{(p_1\ \varepsilon_\sigma(k))}{(p_1\ k)}\mathcal{M}_{\lambda_2,\lambda_1}([p_2],[p_1];Q) - \frac{e}{\sqrt{2}\,(p_1\ k)}\times$$
$$\times\sum_{\sigma,C=-1}^{1}\mathcal{M}_{\lambda_2,\sigma}([p_2],b_C;Q)\left\{\mathcal{M}_{-\sigma,\lambda_1}(b_C,[p_1])\,C\ \sigma\,(k\ b_{-C})\,e^{-\mathrm{i}(1-C)\varphi_k}\right\}\ , \tag{39}$$

where $\mathcal{M}_{-\sigma,\lambda_1}(b_C,[p_1])$ is determined by the Eq.(26) for arbitrary fermion polarization or for special cases of fermion polarization.

5 Summary and Acknowledgements

In present paper we have formulated a new effective method to calculate the Feynman amplitudes for various processes with fermions of arbitrary polarizations. In our method it is much easier to keep track of partial results and to set up recursive schemes of evaluation which compute and store for later use subdiagrams of increasing size and complexity.

In our approach of the matrix element calculation: 1. We don't use an explicit form of Dirac spinors and γ–matrices; 2. We don't use the calculation of traces; 3. As well as in the trace methods the matrix element of Feynman amplitudes is reduced to the combination of scalar products of momenta and polarization vectors; 4. Unlike spinor technique in different variants [11]-[14] in this method doesn't use either Chisholm identities, or the presentation of the contraction $\not{p}$ with four vector p and of the polarization vector of bosons through the Dirac spinors; 5. Unlike `Wvd` technique [15],[18] in this method doesn't use special Feynman rules for calculating of the matrix elements.

The recursive algorithms can be easily realized in the various systems of symbolic calculation (Mathematica, Maple, Reduce, Form) and in such packages as `FeynArts`, [26], `FeynCalc` [27], `HIP` [28] and so on.

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References

[1] J.L. Powell, Phys. Rev. **75** (1949) 32
[2] M.V. Galynskii, S.M. Sikach, Phys.Part.Nucl.**29** (1998) 469; Fiz.Elem.Chast.Atom.Yadra **29** (1998) 1133 [arXiv:hep-ph/9910284].
[3] A.L. Bondarev , arXiv:hep-ph/9710398.
[4] E. Bellomo, I1 Nuovo Cimento. Ser.X **21** (1961) 730
[5] A.A. Bogush, F.I. Fedorov, Vesti AN BSSR, ser.fiz.-m.n. **N 2** (1962) 26 (in Russian).
[6] A.A. Bogush, Vesti AN BSSR, ser.fiz.-m.n., **N 2** (1964) 29 (in Russian).
[7] H.W. Fearing, R.R. Sillbar, Phys.Rev. **D6** (1972) 471
[8] F.I. Fedorov, Izvestiya Vyzov. Fizika, **N 2** (1980) 32 (in Russian).
[9] M. Caffo, E. Remiddi, Helvetica Phys. Acta. **55** (1982) 339
[10] R. Vega and J. Wudka, Phys. Rev. **D53** (1996) 5286; *erratum* Phys. Rev. **D56** (1997) 6037
[11] F.A. Berends, P.de Gausmaeceker, R. Gastmans, R. Kleiss, W.Troost and Tai Tsun Wu, Phys.Lett. **B103** (1981) 124
[12] G.R. Farrar and F. Neri, Phys. Lett. **130B** (1983) 109
[13] R. Kleiss, W.J. Stirling, Nucl. Phys. **B262** (1985) 235
[14] Zhan Xu, Da-Hua Zhang, and Lee Chang, Nucl.Phys. **B291** (1987) 392
[15] F.A. Berends and W.T. Giele, Nucl. Phys. **B294** (1987) 700
[16] F.A. Berends, P.H. Daverveldt, and R. Kleiss, Nucl. Phys. **B253** (1985) 441
[17] R. Kleiss, Z.Phys.C. **33** (1987) 433
[18] S. Dittmaier, Phys. Rev. **D59** (1999) 016007
[19] K. Hagiwara, D. Zeppenfeld, Nucl. Phys. **B274** (1986) 1
[20] A. Ballestrero and E. Maina, Phys. Lett. **B350** (1995) 225
[21] T. A.-Gongora, R.G. Stuart, Z.Phys. **C42** (1989) 617
[22] V.V. Andreev, Phys.Rev. **D62** (2000) 014029
[23] V.V. Andreev Yad. Fizika **66** (2003) 410
[24] V.V. Andreev, Nucl.Instr.&Methods in Phys.Research **A502** (2003) 607
[25] V.I. Borodulin, R.N. Rogalyov, and S.R. Slabospitsky, Preprint IHEP 95-90, (1995); [arXiv:hep-ph/9507456].
[26] J. Küblbeck, M. Böhm, and A. Denner, Comp. Phys. Commun. **60**, 165 (1990).
[27] R. Mertig, M.Böhm, and A. Denner, Comp. Phys. Commun. **64** (1991) 345
[28] E.Yehudai, Preprint Fermi-Lab-Pub-92/22-T (1992).