Domain wall generation by gravity and fermions

A. A. Andrianov[♭,♯], V. A. Andrianov[♭], P. Giacconi[♯] and R. Soldati[♯]
[♭] *V.A.Fock Department of Theoretical Physics, SPbGU, Sankt-Petersburg, Russia*
[♯] *Dipartimento di Fisica and INFN, Universitá di Bologna, 40126 Bologna, Italy*

Abstract

We present a non-compact *4 + 1* dimensional model with a local strong four-fermion interaction supplementing it with gravity. In the strong coupling regime it reveals the spontaneous translational symmetry breaking which eventually leads to the formation of thick 3-branes, embedded in the AdS_5 manifold. To describe this phenomenon we construct the appropriate low-energy effective Action and find kink-like vacuum solutions in the quasi-flat Riemannian metric. We establish the relation among the bulk five dimensional gravitational constant, the brane Newton's constants and the curvature of AdS_5 space-time, the compositeness scale of the scalar matter and the symmetry breaking scale.

1 Introduction

The idea about that our *3 + 1* dimensional world might be allocated on a brane in a multi-dimensional space-time has recently attracted much interest [1]-[6], giving new tools to solve the long standing mass and scale hierarchy problems in particle theory. New extra dimensional physics could manifest itself in accessible experiments and observations, when the size of extra dimensions is relatively large or even infinite (see review articles [7]–[11]).

The thick (or fat) brane (or domain wall) formation and the trapping of light particles in its layer might be obtained [12]–[17] by a number of particular background scalar and/or gravitational fields living in the multi-dimensional bulk, when their vacuum configuration has a non-trivial topology, thereby generating zero-energy states localized on the brane.

Respectively, the mechanism of how such background fields might emerge and further induce the spontaneous breaking of translational symmetry is worthy to be elaborated[18, 19].

In this talk we describe a non-compact *4 + 1* dimensional fermion model [18] with a local strong four-fermion interaction providing the spontaneous breaking of translational symmetry. This model involves two four-component fermion bi-spinor field $\psi_j(X)$ defined on a five dimensional Minkowski space-time and coupled to a scalar field $\Phi(X)$. The extra-dimension coordinate is assumed to be space-like,

$$X^A = (x^\mu, z)\ , \quad x^\mu = (x^0, x^1, x^2, x^3)\ , \quad (\eta^{AA}) = (+,-,-,-,-)$$

and the subspace of coordinates x^μ eventually corresponds to the four dimensional Minkowski space. The extra-dimension size is supposed to be infinite (or large enough).

The fermion wave function is then described by the Dirac equation

$$[\,i\gamma^A\partial_A \mp \Phi(X)\,]\psi_j(X) = 0\ , \quad \gamma^A = (\gamma^\mu, i\gamma_5)\ , \quad \{\gamma^A, \gamma^B\} = 2\eta^{AB}\ , \tag{1}$$

γ^μ, γ_5 being a standard set of four dimensional Dirac matrices in the chiral (or Weyl) representation.

The trapping of light fermions on a four dimensional hyper-plane – the domain wall $\equiv$ 3-brane – localized in the fifth dimension at $z = z_0$ can be promoted by a certain *topological*, z-dependent configuration of the v. e. v. of scalar field $\langle\Phi(X)\rangle_0 = \varphi(z)$ – for instance $\varphi(z) = M\tanh(Mz)$ – due to the appearance of zero-modes with a certain chiralities in the spectrum of the four dimensional Dirac operator [1, 7].

As we aim to build up a light Dirac fermion, we need two different chiralities for the same shape of scalar background. This is why the minimal set of fermions over the five dimensional space-time has to include [16, 18] two proto-fermions $\psi_1(X), \psi_2(X)$. In order to generate left- and right-handed parts of a four dimensional Dirac bi-spinor as zero modes, those fermions have to couple to the scalar field $\Phi(X)$ with opposite charges.

In addition to the trapping scalar field, a further one is required to supply light domain wall fermions with a mass. Its coupling must mix left and right chiralities as the mass term breaks the chiral invariance. Thus we introduce two types of four-fermion self-interactions to reveal two composite scalar fields with a proper coupling to fermions. These two scalar fields acquire mass spectra similar to fermions with light counterparts located on the domain wall. The dynamical scheme of creation of domain wall particles turns out to be quite economical and few predictions on masses and decay constants of fermion and boson particles have been derived [18]. However the allocation of matter on the domain wall certainly leads to strong gravitational effects.

2 Model for brane generation by fermions

For technical convenience we define the classical Lagrange density of our model in the five-dimensional Euclidean space,

$$\mathcal{L}^{(5)}(\overline{\Psi},\Psi) = \overline{\Psi}\; i\,\partial\!\!\!/\Psi + \frac{g_1}{4N\Lambda^3}\left(\overline{\Psi}\tau_3\Psi\right)^2 + \frac{g_2}{4N\Lambda^3}\left(\overline{\Psi}\tau_1\Psi\right)^2, \tag{2}$$

where $\Psi(X)$ is an eight-component five-dimensional fermion field and $\partial\!\!\!/ \equiv \gamma^A\partial_A$ with five Dirac matrices $\widehat{\gamma}^A \equiv \gamma^A \otimes \mathbf{1}_2$ borrowed from the $3+1$ dimensional space. In turn, $\tau_a \equiv \mathbf{1}_4 \otimes \sigma_a,\ a = 1,2,3$ are generalized Pauli matrices acting on the bi-spinor components $\psi_i(X)$ of $\Psi(X)$. It contains two dimensional coupling constants expressed in units of the compositeness scale Λ. The latter one plays the role of a cut-off for virtual fermion energies and momenta.

We supplement it with a (partially induced) gravitational field $g_{AB}(X)$ and bosonize in terms of auxiliary scalar fields $\Phi(X)$ and $H(X)$,

$$S(\Phi,H,\overline{\Psi}_l,\Psi_l,g) = \int_{\mathcal{M}_5} d^5X\sqrt{g}\;\left[\mathcal{L}^{(5)}_{\mathrm{fermion}} + \mathcal{L}^{(5)}_{\mathrm{boson}}\right]\;;\quad g\equiv \det(g_{AB})\;, \tag{3}$$

where the fermion part is defined in the Euclidean five-dimensional space as

$$\mathcal{L}^{(5)}_{\mathrm{fermion}} = i\overline{\Psi}_l\left[\,\widehat{\gamma}_k e^A_k\left(\partial_A + \omega_A\right) + \tau_3\Phi + \tau_1 H\,\right]\Psi_l \equiv i\overline{\Psi}_l\left(\nabla\!\!\!\!/ + \tau_3\Phi + \tau_1 H\right)\Psi_l\;, \tag{4}$$

in terms of the pentad–fields $e^i_A(X)$ and spin connection ω_A (see [20, 21, 19] for definitions and technical details). As well the bosonic Euclidean Lagrange density

$$\mathcal{L}^{(5)}_{\mathrm{boson}} = N\Lambda^3\left(\frac{\Phi^2}{g_1} + \frac{H^2}{g_2}\right) - \frac{\Lambda}{\mathcal{G}}\left(\varepsilon\frac{R}{2} - \lambda_0\right)\;. \tag{5}$$

where the conventional notations are used for the scalar curvature R, the bare gravitational, $\mathcal{G}$ and cosmological, λ_0 constants. $\varepsilon = \pm 1, 0$ is introduced to specify different options – gravity, antigravity or induced gravity, for the bare gravitational interaction.

3 Low-energy effective action

All interactions lead coherently, first, to the discrete symmetry breaking and, further on, to the breaking of translational invariance.Namely, for sufficiently strong couplings, this system undergoes the phase transition to the state in which the condensation of fermion-antifermion pairs does spontaneously break – partially or completely – the so-called τ-symmetry: $\Psi \longrightarrow \tau_1\Psi;\ \Phi \longrightarrow -\Phi$; and $\Psi \longrightarrow \tau_3\Psi;\ H \longrightarrow -H$. This phenomenon can be established from the low-energy effective action with kinetic terms for composite scalar fields generated by high-energy fermions (in the one-loop approximation and using the invariant separation of high-and low-energy scales [22]),

$$\begin{aligned}
\mathcal{L}^{(5)}_{\mathrm{low}} &= i\overline{\Psi}_l(X)\left[\nabla\!\!\!\!/ + \tau_3\Phi(X) + \tau_1 H(X)\right]\Psi_l(X) \\
&+ \frac{N\Lambda}{4\pi^3}\Bigg(\partial_A\Phi(X)\partial^A\Phi(X) + \partial_A H(X)\partial^A H(X) - 2\Delta_1\Phi^2(X) - 2\Delta_2 H^2(X) \\
&+ \left[\,\Phi^2(X) + H^2(X)\,\right]\left\{\Phi^2(X) + H^2(X) + \frac{R(X)}{6}\right\}\Bigg) \\
&- \frac{\Lambda}{2\kappa\mathcal{G}}\left\{\,R(X) - 2\lambda\,\right\} \\
&+ \frac{N\Lambda}{2880\pi^3}\left\{5R^2 - 8R_{AB}R^{AB} - 7R_{ABCD}R^{ABCD}\right\}
\end{aligned} \tag{6}$$

where the two mass scales Δ_i characterize the deviations from the critical point

$$\Delta_i(g_i) = \frac{2\Lambda^2}{9g_i}\,(g_i - g_i^{\rm cr})\,; \quad g_1^{\rm cr} = g_2^{\rm cr} = 9\pi^3\ . \tag{7}$$

The *dressed* cosmological constant λ and the gravitational low-energy dimensionless parameter κ do arise as a net effect of the interplay between the classical and fermion induced contributions:

$$\lambda = \kappa\lambda_0 + \frac{N\Lambda^4}{75\pi^3}\,\mathcal{G}\kappa\ , \quad \kappa = \frac{1}{\varepsilon + N\Lambda^2\mathcal{G}/54\pi^3}\,. \tag{8}$$

where κ must be positive for the gravitational interaction to be attractive. One can consider different scenarios: namely, `Fundamental gravity`, when the bare and dressed gravitational couplings are comparable $\varepsilon = 1$, $\kappa \simeq 1$, or `Induced gravity`, when the bare gravitational Action is either absent or irrelevant whilst the fermion induced gravitational Action is dominant $\varepsilon = 0$ and/or $\kappa \ll 1$.

4 Brane generation due to spontaneous breaking of translational symmetry

The search for classical vacuum configurations of gravity and scalar fields is performed by analyzing the low-energy effective Action (6), restricting ourselves to the class of conformal-like metrics (warped geometries) with the flat Minkowski hyperplanes at each point along the fifth coordinate,

$$ds^2 = g_{AB}(X)\,dX^A\,dX^B = \exp\{-2\rho(z)\}\,dx_\mu dx_\mu + dz^2 \tag{9}$$

with the Euclidean signature.

The generalized Einstein equations in the presence of scalar matter can be cast into the form,

$$\rho'' = \frac{\overline{\kappa}}{M^2}\left\{\Phi'^2 + H'^2 + \frac{1}{2}\left(\rho'' - \frac{1}{3}\frac{d^2}{dz^2} - \frac{1}{3}\rho'\frac{d}{dz}\right)\left(\Phi^2 + H^2\right)\right\}\ . \tag{10}$$

as well as, into the following equation,

$$\begin{aligned} 2M^2\lambda_{\rm eff} \quad &= \quad \Phi'^2 + H'^2 + 2\Delta_1\Phi^2 + 2\Delta_2 H^2 - \left(\Phi^2 + H^2\right)^2 \\ &+ \quad \left(2\rho'^2 - \frac{4}{3}\rho'\frac{d}{dz}\right)\left(\Phi^2 + H^2\right) - \frac{4M^2}{\overline{\kappa}}\,\rho'^2 \end{aligned} \tag{11}$$

which actually represents the integral of motion (after Eqs. of motion for scalar fields are taken into account), *i.e.* the rescaled five dimensional cosmological constant plays the role of an integration constant. In these equations we have neglected by terms originated from the last part of the Lagrangian density (6) quadratic in the curvature as they do not contribute into the leading order in the weak gravitational coupling expansion.

The pair of equations for scalar matter fields completes the system (10), (11),

$$\Phi'' = 2\Phi\left(\Phi^2 + H^2\right) - 2\Delta_1\Phi + 4\rho'\,\Phi' + \frac{2}{3}\,\Phi\left(2\rho'' - 5\rho'^2\right)\ , \tag{12}$$

$$H'' = 2H\left(\Phi^2 + H^2\right) - 2\Delta_2 H + 4\rho'\,H' + \frac{2}{3}\,H\left(2\rho'' - 5\rho'^2\right)\ . \tag{13}$$

The joint solution of all these equations is properly found within the weak-gravity approximation, *i.e.* assuming a relatively small five-dimensional gravitational constant. The latter one is conveniently defined after introducing the natural symmetry breaking scale $\Delta_1 = M^2$. The characteristic scale M of the translational symmetry breaking – inverse thickness of a brane – can be conceivably much less than the compositeness scale Λ , keeping consistently the brane formation and particle localization phenomena in the low-energy region.

Respectively the gravitational low–energy dimensionless parameter has been introduced as

$$\overline{\kappa} \;\equiv\; \frac{N\kappa}{6\pi^3}\, M^2\mathcal{G} \;\ll\; 1 \tag{14}$$

as well as the effective cosmological constant is

$$\lambda \;\equiv\; 3\overline{\kappa}\,\lambda_{\mathrm{eff}} \;. \tag{15}$$

As in [18] one can discover, to the leading order in $\bar{\kappa}$, kink–like solutions for Eqs. (12),(13): namely,

$$(J) \quad \Phi_J \equiv \langle\Phi(X)\rangle_0 = M\tanh(Mz)\;, \quad H_J \equiv \langle H(X)\rangle_0 = 0\;; \tag{16}$$

$$(K) \quad \Phi_K \equiv \langle\Phi(X)\rangle_0 = M\tanh(\beta z)\;, \quad H_K \equiv \langle H(X)\rangle_0 = \mu\,\mathrm{sech}(\beta z)\;, \tag{17}$$

where

$$\mu = \sqrt{2\Delta_2 - M^2} \ll M\;, \qquad \beta = \sqrt{M^2 - \mu^2}\;. \tag{18}$$

The solution (K) exists only for $\Delta_2 < M^2 < 2\Delta_2$ and it coincides with the extremum (J) in the limit $\Delta_2 \to M^2/2,\ \mu \to 0,\ \beta \to M$. Both of them give consistently

$$\lambda \simeq \frac{N\mathcal{G}\kappa M^4}{4\pi^3} = \frac{3}{2}\bar{\kappa}M^2\;, \tag{19}$$

to the leading order in $\bar{\kappa}$. In the same approximation the conformal factor is found to be,

$$\begin{aligned} \rho(z) &= \frac{2\overline{\kappa}}{3}\left(1 + \frac{\mu^2}{2M^2}\right)\ln\cosh(\beta z) + \mathcal{O}(\bar{\kappa}^2) \;\overset{|z|\to\infty}{\sim}\; k|z|\;; \\ k &\equiv \frac{2}{3}\,\overline{\kappa}\beta\left(1 + \frac{\mu^2}{2M^2}\right) \;\approx\; \frac{2}{3}\,\overline{\kappa}M\;. \end{aligned} \tag{20}$$

Evidently, this solution approaches the symmetric Anti-de-Sitter (AdS) metric for large z .

5 Spectrum and interaction of particles trapped on the brane

Let us summarize the structure [18] of the spectrum and of the interaction of the light matter states trapped on a brane. The kinetic operators of the two scalars $\Phi(X)$ and $H(X)$ and of the spinor field $\Psi(X)$ do exhibit normalizable zero-modes in the extra dimension, in the vicinity of the vacuum background (16) or (17), at the scaling point $M^2 = \Delta_1 = 2\Delta_2$ or $\mu = 0$. Those zero-modes $\phi_0(z)$, $h_0(z)$ and $\psi_0(z)$, respectively, are localized at the origin of the z-axis, with a localization width $\sim 1/M$ and, at ultra-low energies, the fluctuations of the matter fields can be parametrized as follows: namely,

$$\Phi(X) \simeq \langle\Phi(X)\rangle_0 + \phi(x)\phi_0(z)\;; \quad H(X) \simeq \langle H(X)\rangle_0 + h(x)h_0(z)\;; \quad \Psi(X) \simeq \psi(x)\,\psi_0(z)\;. \tag{21}$$

For these states the ultra-low-energy effective Lagrange density (in the Euclidean space) is generated

$$\begin{aligned} \mathcal{L}^{(4)} \;&=\; i\overline{\psi}(x)\,[\,\partial\!\!\!/ + g_f\,h(x)\,]\,\psi(x) + \frac{1}{2}[\,\partial_\mu\phi(x)\,]^2 + \frac{1}{2}[\,\partial_\mu h(x)\,]^2 \\ &+\; \lambda_1\phi^4(x) + \lambda_2\phi^2(x)h^2(x) + \lambda_3 h^4(x)\;, \end{aligned} \tag{22}$$

with the ultra-low energy effective couplings given by

$$g_f = \frac{\pi}{4}\sqrt{\zeta}\;, \quad \lambda_1 = \frac{18}{35}\,\zeta\;, \quad \lambda_2 = \frac{2}{5}\,\zeta\;, \quad \lambda_3 = \frac{1}{3}\,\zeta\;, \qquad \zeta \equiv \frac{M\pi^3}{\Lambda N}\;. \tag{23}$$

Once gravity is switched on, it can be shown that the zero-modes remain localizable and therefore the AdS vacuum solution does not play any dominant role concerning the determination of the coupling constants in eq. (22).

Let us estimate all the scales and the coupling constants we have previously introduced. This can be achieved by imposing the observed value of the Newton's constant and the Newton's gravitational law within the presently available limits in modern experiments [23]. It is adopted that both the scale of compositeness and the translational symmetry breaking scale must be naturally very high, in between several TeV's and the GUT scale 10^{15} GeV.

One can find the relation between the five dimensional and brane gravity constants using the factorized Riemannian metric

$$ds^2 = \exp\{-2\rho(z)\}\, g_{\mu\nu}(x)dx^\mu dx^\nu + dz^2 \ . \tag{24}$$

For this metric the effective four dimensional Einstein-Hilbert Action, at the leading order, becomes

$$\begin{aligned} S[\,g\,] = \ & - \ \frac{\Lambda}{2\kappa\mathcal{G}} \int d^5X \sqrt{g(X)}\, \{R(X) \ - \ 2\lambda\} \\ \simeq \ & - \ \frac{\Lambda}{2\kappa\mathcal{G}} \int d^4x \sqrt{g(x)}\, R(x) \int_{-\infty}^{+\infty} dz \ \exp\{-2\rho(z)\} \\ & - \ \frac{\Lambda}{\kappa\mathcal{G}} \int d^4x \sqrt{g(x)} \int_{-\infty}^{+\infty} dz \ \exp\{-4\rho(z)\} \left\{6[\,\rho'(z)\,]^2 - \lambda\right\} \\ \equiv \ & - \ \frac{1}{16\pi G_N} \int d^4x \sqrt{g(x)}\, \{R(x) \ - \ 2\Lambda_{\mathrm{grav}}\} \ , \end{aligned} \tag{25}$$

whence we eventually get the Planck mass scale $M_P \sim 1.22 \times 10^{19}$ GeV/c^2 which corresponds to the Newton's gravitational constant

$$M_P^2 = G_N^{-1} \equiv \frac{8\pi\Lambda}{\kappa\mathcal{G}} \int_{-\infty}^{+\infty} dz \ \exp\{-2\rho(z)\}; \quad G_N \ \simeq \ \frac{\pi^2\,\overline{\kappa}^{\,2}}{2N\Lambda M} \ . \tag{26}$$

Remarkably, the full value of the cosmological constant, including the gravitational as well as the matter vacuum energy densities, exactly vanishes to all orders in the perturbative expansion in powers of $\overline{\kappa}$,

$$\begin{aligned} \Lambda_{\mathrm{cosmo}} \ \equiv \ & \frac{2N\Lambda G_N}{\pi^2} \int_{-\infty}^{+\infty} dz \ \exp\{-4\rho(z)\} \Big\{ 2M^2\lambda_{\mathrm{eff}} - (4/\overline{\kappa})\, M^2\rho'^2(z) \\ + \ & \Phi'^2(z) + H'^2(z) - 2\Delta_1\Phi^2(z) - 2\Delta_2 H^2(z) + \left[\,\Phi^2(z) + H^2(z)\,\right]^2 \\ + \ & \frac{2}{3}\left[\,\Phi^2(z) + H^2(z)\,\right]\left[\,2\rho''(z) - 5\rho'^2(z)\,\right] \Big\} = \ 0 \ , \end{aligned} \tag{27}$$

where the vacuum expectation values (12) and (13) of the scalar fields together with the field equation (10) of the conformal factor have been suitably taken into account. It makes consistently endorsed the *ansatz* for the flat Minkowski's hyperplanes.

Let us find the relations among the AdS curvature scale $k \simeq 2\overline{\kappa}\, M/3$, the Planck mass M_P and the spontaneous symmetry breaking scale M . We recall that the characteristic parameter of ultra-low energy dynamics of light fermions and scalar fields is ζ from Eq. (23). Therefore, the relationship among the three scales M_P, k and M reads

$$k^2 M_P^2 = \frac{8\pi}{9\zeta}\, M^4 \ , \quad \bar{\kappa} = \frac{2\pi}{\sqrt{\zeta}}\, (M/M_P). \tag{28}$$

in accordance with Eqs. (14) and (20).

Our first scenario – `fundamental gravity` – is selected to have a principally detectable dynamics of scalar fields and of the Higgs-Yukawa coupling to fermions in the brane world. Let us therefore analyze the regime where $M/\Lambda \simeq \zeta \sim 0.1 \div 0.3$, at least, not much less. In this case, the experimental bound $k > 10\ \mathrm{mm}^{-1} = 2{\cdot}10^{-3}$ eV [23] turns out to be compatible with the lower bound [9, 10] for the localization scale $M > (2 \div 3)$ TeV, which makes somewhat challenging to produce new physics related to the fifth dimension at the next generation of colliders. The corresponding cut-off is $\Lambda > 10 \div 20$ TeV so that, from Eq. (28), perturbation theory is controlled by the very tiny constant $\overline{\kappa} > 10^{-15}$.

The second scenario under consideration is that one of the `induced gravity`. Let us adopt the induced gravity relations,

$$\kappa \simeq 54\pi^3/N\Lambda^2\mathcal{G} \ll 1, \quad \overline{\kappa}^{1/2} \simeq 3M/\Lambda \ll 1.$$

Then

$$kM_P^2 = 4N\Lambda^3/27\pi^3, \quad k^5M_P^4 = 128N^2M^9/27\pi^6.$$

In particular, for a lower experimental bound $M \geq 2$ TeV , $k \geq 10^{-10}$ GeV one finds $\Lambda \geq 10^9$ GeV , $\zeta \sim M/\Lambda \sim 10^{-7}$, $\overline{\kappa} \sim 10^{-12}$. This means that the light particle interaction is highly suppressed and the only particle interaction which is left is the gravitational one. Although the Higgs-like particles may be involved into the gauge boson interaction and be observable by gauge boson mediation, it turns out that branons [24], *i.e.* the quanta of the field ϕ, decouple from any other kind of matter that makes them a perfect candidate for the dark matter/energy, depending on their mass.

As a summary of these studies, we conclude that:
a) on the one hand, fundamental gravity in five dimensions appears to be more challenging for future experiments, because the phenomenologically acceptable large values of AdS_5 curvature $k \sim 10^{-3}$ eV together with the relatively low translational symmetry breaking scales $M \sim 1 \div 3$ TeV turn out to be compatible with a weak, but not vanishing, coupling of branons to Higgs-like scalars and fermions. However, it appears that the lower are the values of the AdS_5 curvature k, the higher is the threshold for new physics M and the weaker is the interaction among spinor and scalar matter, in such a way to move branons to the dark side of the universe.
b) On the other hand, induced gravity leads to decoupling of branons from other matter in a wide range of acceptable scales and coupling constants, thus putting them straightforwardly to the dark matter realm.
c) In any case, the dimensionless parameter which characterizes the strength of the gravitational interaction is very small, of the order $\overline{\kappa} \leq 10^{-8}$. This feature does justify the use of the perturbation theory both in the calculation of vacuum field configurations and gravity background, as well as in the derivation of mass spectrum of localized particles.

This work was supported by Grant INFN/IS-PI13. Two of us (A.A.A. and V.A.A.) were supported by Grants RFBR 04-02-26939 and Grant UR.02.01.299.

References

[1] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B125 (1983) 136, 139

[2] K. Akama, Lect. Notes Phys. 176 (1982) 267 [hep-th/0001113];
M. Visser, Phys. Lett. B159 (1985) 22 [hep-th/9910093] .

[3] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B436 (1998) 257

[5] M. Gogberashvili, Mod. Phys. Lett. A14 (1999) 2025; Int. J. Mod. Phys. D11 (2002) 1639 [hep-ph/0001109].

[6] L. Randall, R. Sundrum, Phys. Rev. Lett.83 (1999) 3370, 4690.

[7] V.A. Rubakov, Phys. Usp. 44 (2001) 871 ; Phys. Usp. 46 (2003) 211 .

[8] I. Antoniadis, in: Beatenberg 2001, *High-energy physics*, p. 301; preprint hep-th/0102202;
S. Forste, Fortsch. Phys. 50 (2002) 221;
E. Kiritsis, Fortsch. Phys. 52 (2004) 200

[9] Yu.A. Kubyshin, preprint hep-ph/0111027,
J. Hewett, M. Spiropulu, Ann. Rev. Nucl. Part. Sci. 52 (2002) 397

[10] F. Feruglio, Eur. Phys. J. C33 (2004) S114

[11] C. Csaki, preprint hep-ph/0404096 .

[12] O. DeWolfe, D. Z. Freedman, S. S. Gubser, A. Karch, Phys. Rev. D62 (2000) 046008

[13] M. Gremm, Phys. Lett. B478 (2000) 434; Phys. Rev. D62 (2000) 044017

[14] C. Csaki, J. Erlich, T. J. Hollowood, Y. Shirman, Nucl. Phys. B581 (2000) 309

[15] G.R. Dvali, G. Gabadadze, M.A. Shifman, Phys. Lett. B497 (2001) 271

[16] S.L. Dubovsky, V.A. Rubakov, P.G. Tinyakov, Phys. Rev. D62 (2000) 105011

[17] M Laine, H.B. Meyer, K. Rummukainen, M. Shaposhnikov, JHEP 0301 (2003) 068; JHEP 0404 (2004) 027

[18] A. A. Andrianov, V. A. Andrianov, P. Giacconi, R. Soldati, JHEP 0307 (2003) 063

[19] A. A. Andrianov, V. A. Andrianov, P. Giacconi, R. Soldati, preprint hep-th/0503115.

[20] G. Cognola, P. Giacconi, Phys. Rev. D39 (1989) 2987

[21] D. Vassilevich, Phys. Rept. 388 (2003) 279

[22] A.A. Andrianov, L. Bonora, Nucl. Phys. B233 (1984) 232, 247

[23] E. G. Adelberger, B. R. Heckel, A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 53 (2003) 77

[24] A. Dobado, A.L. Maroto, Nucl. Phys. B592 (2001) 203;
J. Alcaraz, J.A.R. Cembranos, A. Dobado, A.L. Maroto, Phys. Rev. D67 (2003) 075010