Stringification of chiral dynamics: Wess-Zumino interaction

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Abstract

The QCD hadronic string is supplemented with the reparameterization-invariant boundary interaction to background chiral fields associated with pions in a way compatible with the conformal symmetry. It allows the full reconstruction of the P-even part of the Chiral Lagrangian in a good agreement with the phenomenology of P-even meson interactions. The modification of boundary interaction necessary to induce the parity-odd Chiral Dynamics (WZW action) is outlined.

1 Introduction: pion coupling to QCD string

String description of QCD in the hadronization regime is a long-standing problem with a number of theoretical arguments [1, 2] and phenomenological evidences [3, 4] as well as with the recent lattice simulations [5] in favor to its viability at intermediate energies (hadron masses).

The crucial low-energy phenomenon in QCD which makes influence on Hadron String building for light quarkoniums is Chiral Symmetry Breaking. It determines the QCD vacuum and results in the formation of light (massless in the chiral limit) pseudoscalar mesons. For the string dynamics the background chiral fields $U(x)$ add new couplings [6] involving the string variable $x_\mu(\tau,\sigma)$, on the boundary of the string where flavor is attached. A consistent string propagation in this non-perturbative background has been realized in [6] where it was adjusted to provide the essential property of string theory - conformal invariance.

The hadronic string in the conformal gauge is described by the following conformal field theory action which has four dimensional Euclidean space-time as target space

$$\mathcal{W}_{str} = \frac{1}{4\pi\alpha'}\int d^{2+\epsilon}\sigma \left(\frac{\varphi}{\mu}\right)^{-\epsilon} \partial_i x_\mu \partial_i x_\mu, \tag{1}$$

where for $\epsilon = 0$ one takes

$$x_\mu = x_\mu(\tau,\sigma); \quad -\infty < \tau < \infty, 0 < \sigma < \infty; \quad i = \tau,\sigma \quad \mu = 1, ..., 4.$$

The conformal factor $\varphi(\tau,\sigma)$ is introduced to restore the conformal invariance in $2+\epsilon$ dimensions. The Regge trajectory slope (related to the inverse string tension) is known to be universal $\alpha' \simeq 0.9$ GeV^{-2} [3].

We would like to couple in a chiral invariant manner the matrix in flavor space $U(x)$ containing the meson fields to the string degrees of freedom while preserving general covariance in the two dimensional coordinates and conformal invariance under local scale transformations of the two-dimensional metric tensor. Since the string variable x does not contain any flavor dependence, we introduce two dimensionless Grassmann variables ('quarks') living on the boundary of the string sheet. The boundary quark fields $\psi_L(\tau), \psi_R(\tau)$ transform in the fundamental representation of the light-flavor group $SU(N)$ with $N = 2, 3$. The subscripts L, R are related to the *chiral* spinors. A local hermitian action $S_b = \int d\tau L^{(f)}$ is introduced on the boundary $\sigma = 0$, $-\infty < \tau < \infty$ to describe the interaction with background chiral fields $U(x(\tau)) = \exp(i\pi(x)/f_\pi)$ where $f_\pi \simeq 90 MeV$, the weak pion decay constant, relates the field $\pi(x)$ to a π-meson one.

The boundary Lagrangian is chosen to be reparametrization invariant and in its bare minimal form reads

$$L^{(f)}_{min} = \frac{1}{2} i \left(\bar\psi_L U(1-z)\dot\psi_R - \dot{\bar\psi}_L U(1+z)\psi_R + \bar\psi_R U^+(1+z^*)\dot\psi_L - \dot{\bar\psi}_R U^+(1-z^*)\psi_L\right), \tag{2}$$

where a dot implies a τ derivative. The CP symmetry under transformation,

$$\psi \longrightarrow \gamma_0\psi; \quad \psi_R \longrightarrow \gamma_0\psi_L; \quad \psi_L \longrightarrow \gamma_0\psi_R; \quad U \longrightarrow U^\dagger, \tag{3}$$

requires purely imaginary constants $z = -z^* = \pm i|z|$.

This Lagrangian has been proved [6] to provide the E.o.M. of Chiral dynamics and thereby the Chiral Lagrangian for the parity-even sector if the conformal symmetry of this boundary QFT is reproduced, *i.e.* the renormgroup β functions of new boundary constants vanish. At one-loop it gives the leading E.o.M of the Chiral Dynamics

$$\delta U = \Delta(0)\left[\frac{1}{2}\partial_\mu^2 U - \frac{3+z^2}{4}\partial_\mu U U^\dagger \partial_\mu U\right] = 0, \tag{4}$$

if $z^2 = -1$. The latter condition provides both the integrability of Eq. (4) up to the local (Weinberg) Lagrangian and the unitarity of chiral fields as a constraint.

Meanwhile the one-loop divergences in vertexes with any number of "bosons" − string coordinates $x_\mu(\tau, \sigma = 0)$, require an extension of boundary action to be properly renormalized, The relevant counterterms can be parameterized with three bare constants g_1 , g_2 and g_3,

$$\begin{aligned} \Delta L_{bare} \quad = \quad & \frac{i}{8}(1-z^2)\bar{\psi}_L\Big((g_1 - zg_2)\partial_\nu \dot{U}U^{-1}\partial_\nu U - (g_1 + zg_2)\partial_\nu U U^{-1}\partial_\nu \dot{U} \\ & + 2zg_3\partial_\nu U U^{-1}\dot{U}U^{-1}\partial_\nu U\Big)\psi_R + \text{h.c.} \end{aligned} \tag{5}$$

Renormalization is accomplished by redefining the couplings g_i in the following way

$$g_i = g_{i,r} - \Delta(0). \tag{6}$$

In spite of the fact that the new couplings are dimensional, it turns out [6] that their contribution into the trace of the energy-momentum tensor vanishes once the requirements of unitarity of U are taken into account and therefore conformal invariance is not broken.

At two loops the dim-4 chiral structural constants [7] have been unambiguously calculated in terms of the product of the Regge trajectory slope $\alpha' \simeq 0.9\ \text{GeV}^{-2}$, f_π^2 and certain rational numbers (equivalently they can be characterized by the ratio of f_π^2 to the hadron string tension $T = 1/2\pi\alpha'$),

$$g_{1,r} = -g_{2,r} = -g_{3,r} = 2\alpha'; \tag{7}$$

$$L_1 = \frac{1}{2}L_2 = -\frac{1}{4}L_3 = \frac{f_\pi^2\alpha'}{16} = \frac{f_\pi^2}{32\pi T}. \tag{8}$$

This prediction is basically supported by "abnormal" divergences in two loops with maximal number of vertexes. It fits well the phenomenological values [8].

However the Lagrangian (2) is essentially parity even and thereby does not contain any vertexes which can eventually entail the anomalous P-odd part of the Chiral Dynamics. In our talk we outline the modification of the boundary interaction which might bring the Wess-Zumino-Witten Chiral action and other parity-odd vertexes.

2 Chiral dynamics on the line

To approach the required modification we guess on what might be the form of boundary Lagrangian if one derives it from the essential part of the Chiral Quark Model projecting it on the string boundary. The constituent quark fields control properly the chiral symmetry during the "ein-bein" projection, $Q_L \equiv \xi^\dagger\psi_L, \quad Q_R \equiv \xi\psi_R, \quad \xi^2 \equiv U$. In these variables and in the chiral limit the CQM Lagrangian density and the pertinent E.o.M. read

$$\mathcal{L}_{CQM} = i\bar{Q}\left(\not{\partial} + \not{v} + g_A \not{a}\gamma_5\right)Q; \quad i\left(\not{\partial} + \not{v} + g_A \not{a}\gamma_5\right)Q = 0, \tag{9}$$

where

$$v_\mu \equiv \frac{1}{2}(\xi^\dagger(\partial_\mu\xi) - (\partial_\mu\xi)\xi^\dagger), \qquad a_\mu \equiv -\frac{1}{2}(\xi^\dagger(\partial_\mu\xi) + (\partial_\mu\xi)\xi^\dagger), \tag{10}$$

and $g_A \equiv 1 - \delta g_A$ is an axial coupling constant of quarks to pions. We relegate the effects of constituent quark mass to the gluodynamics encoded in the string interaction. Then one can decouple the left and right components of boundary fields in the process of dim-1 projection.

Let's assume the quark fields to be located on the dim-1 boundary with coordinates $x_\mu \equiv x_\mu(\tau)$. The first step in projection of the E.o.M. (9) can be performed by their multiplication on $\gamma^\mu \dot{x}_\mu$ which leads to the following boundary equations,

$$\{i\left(\partial_\tau + \dot{x}_\mu v^\mu + g_A \gamma_5 \dot{x}_\mu a^\mu\right) + \sigma^{\mu\nu}\dot{x}_\mu\left(\partial_\nu + v_\nu + g_A\gamma_5 a_\nu\right)\} Q = 0;\ \sigma^{\mu\nu} \equiv \frac{1}{2} i[\gamma^\mu \gamma^\nu]. \tag{11}$$

We notice that this projected Dirac-type equation seems to be associated to the boundary action with a Lagrangian of type (2).

Let us restore the current quark basis of fields ψ_L thereby going back to the original chiral fields U,

$$\begin{aligned} &\frac{1}{2}\left\{i\left(\{\partial_\tau, U^\dagger\} + z\dot{U}^\dagger\right) + \sigma^{\mu\nu}\dot{x}_\mu\left(\{\partial_\nu, U^\dagger\} + g_A \partial_\nu U^\dagger\right)\right\}\psi_L = 0; \\ &\frac{1}{2}\left\{i\left(\{\partial_\tau, U\} + z\dot{U}\right) + \sigma^{\mu\nu}\dot{x}_\mu\left(\{\partial_\nu, U\} + g_A \partial_\nu U\right)\right\}\psi_R = 0. \end{aligned} \tag{12}$$

Now the crucial point of the "ein-bein" projection consists of making the quark fields ψ truly one-dimensional. Namely we define their gradient in terms of the tangent vector $\dot{x}_\mu$ and arbitrary matrix functions $f(x_\mu), b(x_\mu)$ of $x_\mu(\tau)$,

$$\begin{aligned} \partial_\nu(f_L\psi_L) + \partial_\nu(b_L)\psi_L &\Rightarrow \frac{\dot{x}_\mu}{\dot{x}_\nu \dot{x}^\nu}\left[\partial_\tau(f_L\psi_L) + \partial_\tau(b_L)\psi_L\right]; \\ \partial_\nu(f_R\psi_R) + \partial_\nu(b_R)\psi_R &\Rightarrow \frac{\dot{x}_\mu}{\dot{x}_\nu \dot{x}^\nu}\left[\partial_\tau(f_R\psi_R) + \partial_\tau(b_R)\psi_R\right]. \end{aligned} \tag{13}$$

Keeping in mind our program we choose the functions $f(x_\mu), b(x_\mu)$ to provide the correct chiral properties, translational and reparameterization invariance (in terms of chiral fields U). As well the operator appeared in projection must be anti-self-adjoint in respect to the dim-4 Dirac scalar product. All these requirements are satisfied by the choice,

$$\{\partial_\nu, U^\dagger\}\psi_L \Rightarrow \frac{\dot{x}_\mu}{\dot{x}_\nu \dot{x}^\nu}\{\partial_\tau, U^\dagger\}\psi_L; \qquad \{\partial_\nu, U\}\psi_R \Rightarrow \frac{\dot{x}_\mu}{\dot{x}_\nu \dot{x}^\nu}\{\partial_\tau, U\}\psi_R. \tag{14}$$

Finally, the projected equations are originated from the boundary Lagrangian,

$$\begin{aligned} L^{(f)} &\equiv \frac{1}{2} i\left\{\bar{\psi}_L\left[\{\partial_\tau, U\} + \widehat{F}^{\mu\nu}\dot{x}_\mu\partial_\nu U\right]\psi_R + \bar{\psi}_R\left[\{\partial_\tau, U^\dagger\} - \widehat{F}^{\mu\nu}_\sharp \dot{x}_\mu \partial_\nu U^\dagger\right]\psi_L\right\}; \\ \widehat{F}^{\mu\nu} &\equiv z g^{\mu\nu} + g_\sigma \sigma^{\mu\nu}; \quad \widehat{F}^{\mu\nu}_\sharp \equiv \gamma_0\left(\widehat{F}^{\mu\nu}\right)^\dagger \gamma_0, \end{aligned} \tag{15}$$

where we have obtained the indications that $z = i g_A$ and $g_\sigma = -i g_A$. Still keeping in mind a certain ambiguity in the projection procedure we intend to consider both constants as arbitrary ones and search for their values from the consistency of the Hadron string with chiral fields on its boundary.

The meaning of purely imaginary z and g_σ is clarified by the CP symmetry (3) of the Lagrangian (15). Indeed it is CP symmetric only if

$$z = -z^*; \quad g_\sigma = -g_\sigma^*. \tag{16}$$

3 Two-dimensional QCD and beyond

The above constructed projection is unambiguously verified in the two-dimensional version of QCD where the bosonization fixes basic coupling constants in the Chiral Lagrangian. As in two dimensions $\gamma_0 = \sigma_1$; $\gamma_1 = -i\sigma_2$, γ_2 (*i.e.* "γ_5") $= \sigma_3$ the Lorentz algebra is generated by $\sigma_{\mu\nu} = i\epsilon_{\mu\nu}\gamma_2$ which must be used in the boundary action (15).

To develop the string perturbation theory we expand the function $U(x)$ in powers of the string coordinate field $x_\mu(\tau) = x_{0\mu} + \tilde{x}_\mu(\tau)$, expand the boundary action in powers of $\tilde{x}_\mu(\tau)$ and look for divergences. At one loop one obtains the following condition to remove the divergences (β-function= 0 to preserve conformal symmetry),

$$-\partial_\mu^2 U + \frac{1}{2}(3 + z^2 - g_A^2)\partial_\mu U U^\dagger \partial^\mu U - i g_A \epsilon_{\mu\nu}\partial^\mu U U^\dagger \partial^\nu U = 0. \tag{17}$$

Unitarity of chiral fields (= local integrability of Eqs. of Motion (17)) constrains the coupling constants to $g_A^2 - z^2 = 1$. The choice in accordance with the QCD bosonization is $z = 0, g_A = 1$. It corresponds to the correct value of the dim-2 anomaly (last term in (17)). Thus in QCD_2 the hadron string induces the WZW action from the vanishing the boundary β function already at one-loop level.

In dim-4 QCD the anomaly and the WZW action have dimension 4 and therefore they are generated by cancellation of two-loop divergences. The antisymmetric tensor $\epsilon_{\mu\nu\rho\lambda}$ in anomalies arises from the well-known algebra of $\sigma_{\mu\nu}$ matrices. At one-loop level the interplay between coupling constants z and g_σ takes place as well with the unitarity condition, $3g_A^2 - z^2 = 1$. But now their values are determined from the consistency (local integrability) of the two-loop equations providing β-function = 0. As well as for the P-even part of the Chiral Lagrangian the P-odd anomaly arises from two-loop contributions with maximal number of vertexes and is described in terms of the product of α', f_π^2 and certain rational numbers. But now the structural constant of anomalous operator must be quantized in units of $1/16\pi^2$ [9]. Therefrom one may arrive to the relation between the Regge trajectory slope α' (or the string tension T) and the pion decay constant f_π, plausibly as follows,

$$f_\pi^2 \simeq \frac{1}{16\pi^2\alpha'} = \frac{T}{8\pi}.$$

Thus the anomaly unambiguously relates the scales of the Goldstone boson physics and of the string dynamics.

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