Quark-gluon plasma as perfect spin fluid with a color charge in a color field in Riemann–Cartan space

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Abstract

We consider a hydrodynamic approach in which a quantum system of interacting quarks and gluons is simulated classically by representing it as a perfect fluid having intrinsic degrees of freedom. Every particle of such fluid is endowed with a spin and a non-Abelian color charge. The relativistic variational theory of such perfect spin fluid with color charge is constructed with allowance for spin-polarization chromomagnetic effects in an external Yang–Mills field and in a Riemann–Cartan space with curvature and torsion. On the basis of the method of Cartan exterior forms and extended Lie derivatives, a hydrodynamic equation of the Euler type is obtained. This equation is used to obtain a self-consistent set of equations of motion for a classical test particle with a spin and a color charge in a color field combined with a gravitational field characterized by curvature and torsion. The resulting equations generalize the Wong equation, which describes the motion of a particle with an isospin, and the Tamm–Good and Bargmann–Michel–Telegdi equations, which describe the evolution of a charged-particle spin in an electromagnetic field.

1. Introduction

The commonly accepted approach to the description of elementary particles relies on quantum field theory. In this approach, a quantum field is treated as an infinite set of interacting oscillators. Thereby, a complicated field-theoretical construction is approximated by a mechanical medium whose properties can be studied by applying the existing mathematical formalism. One more mechanical model, based on the hydrodynamic description of a medium, underlies quantum field theory, was developed [1]–[5]. This approach displayed a high predictive power both in QED and in elementary-particle theory, including the Landau hydrodynamic model of multi-particle hadron production and the possibility of describing various phases of quark-gluon plasma on the basis of fluid dynamics [6].

Within the such hydrodynamic approach, we consider a version in which a quantum system of interacting quarks and gluons is simulated classically by representing it as a perfect fluid having intrinsic degrees of freedom, the spin and the non-Abelian color charge of particles forming the fluid. The classical nonrelativistic chromohydrodynamic model of a fluid was proposed in [1] without allowing for spin properties. In this study, the relativistic variational theory of a perfect spin fluid with color charge in an external Yang–Mills field is constructed in the Riemann–Cartan space U_4 with curvature and torsion. The approach used here, which is based on the formalism of exterior forms, generalizes that from [2], where some simplifications were made, and that from [7], where a different mathematical formalism was employed. The objective of this study is to deduce the canonical energy-momentum tensor for the model of fluid under study. A mathematically correct derivation of this tensor is desirable because it can be used as a basis for obtaining the classical equations of motion for a color particle that generalize Wong's equations [8] to the case of the color $SU(3)$ group and which take into account the particle spin. Concurrently, the model in which a hydrodynamic description of quark-gluon plasma on the basis of the Landau model is generalized to include explicitly the structure of the QCD vacuum and the color charge of quarks [6] can be substantiated with the aid of this energy-momentum tensor.

2. Model of perfect fluid with spin and non-Abelian color charge

As the dynamic variables describing the fluid, we take the null-form fields ψ and their conjugates $\bar{\psi}$, which are transformed by the representations of the direct product of the Lorentz group and $SU(3)$ color group. An element of the fluid has a vector of four-dimensional velocity $\vec{u} = u^a \vec{e}_a$, whose corresponding three- and one-forms of velocity may be taken, respectively, as $u = \vec{u}\rfloor\eta = u^a \eta_a$ and $*u = u_a\theta^a = g(\vec{u}, \cdots)$, with $u \wedge *u = c^2\eta$, which is the conventional condition for the squared velocity: $g(\vec{u},\vec{u}) = -c^2$. Here θ^a are the basis one-forms, η is the volume four-form. $\rfloor$ denotes the inner product, and $*$ denotes the Hodge dualization. We also introduce the three- and two-form fields $\eta_b = \vec{e}_b \rfloor \eta = *\theta_b$ and $\eta_{ab} = \vec{e}_b \rfloor \eta_a = *(\theta_a \wedge \theta_b)$, respectively. The basis $\vec{e}_a$ is supposed to be nonholonomic orthogonal with $g(\vec{e}_a, \vec{e}_b) = g_{ab} = \mathrm{diag}(1,1,1,-1)$.

The internal-energy density ε of the fluid depends on the extensive (additive) thermodynamic parameters – the number of particles per unit volume (concentration) n and the entropy per particle s, which are calculated in the intrinsic frame of reference – and on the quantities describing the internal degrees of freedom of the fluid particle, namely, the spin tensor $S_{ab} = \bar{\psi} M_{ab} \psi$ and the color charge $J_m = \bar{\psi} I_m \psi$, where M_{ab} and I_m are the generators of the corresponding representations of the Lorentz and $SU(3)$ color groups, respectively. The quantities characterizing the fluid (including ψ and $\bar{\psi}$) are considered to be averaged over a certain elementary three-volume that contains what is referred to as a fluid element.

Furthermore, the fluid element in itself represents a statistical subsystem with a sufficiently large number of particles to consider it as a quasi-closed system whose properties coincide with the statistical properties of the fluid in a macroscopic state. For the fluid element, the first law of thermodynamics has the following form:

$$d\varepsilon(n,s,S_{ab},J_m) = \frac{\varepsilon + p}{n} dn + nTds + \frac{1}{2} n\omega^{ab} dS_{ab} + n\omega^m dJ_m \ , \tag{2.1}$$

where p is the hydrodynamic pressure in the fluid. In (2.1), the coefficient multiplying dn has the meaning of the chemical potential, and ω^{ab} and ω^m describe, respectively, the possible spin and color-charge exchanges between fluid elements.

We suppose that the laws of conservation of the number of particles and entropy [which can be expressed by the equalities $d(nu) = 0$ and $d(nsu) = 0$, where d is the operator of exterior differentiation] are valid. The first of these equations ensures the continuity of flow lines of the fluid particles. The second equation expresses the constancy of entropy along the flow lines of the fluid and thus constitutes the second law of thermodynamics for adiabatic flow of the fluid.

The fact that the spin tensor meets the Frenkel condition $S_{ab}u^b = 0$ expresses the fundamental physical property of this tensor, its spacelike character. In terms of the exterior forms, this condition can by written by $(\vec{e}_a \rfloor \mathcal{S}) \wedge u = 0$, where the spin two-form $\mathcal{S} = (1/2) S_{ab} \theta^a \wedge \theta^b$ has been introduced.

The four-form of Lagrangian density, whose integral determines the action of the theory, is chosen by us in the form

$$\begin{aligned} &\mathcal{L}_{fluid} = L_{fluid}\, \eta = -\varepsilon(n,s,\psi,\bar{\psi})\eta + n\bar{\psi} D\psi \wedge u - \chi n J_m \mathcal{F}^m \wedge *\mathcal{S} + \\ &+ n\lambda_1(u \wedge *u - c^2\eta) + nu \wedge d\lambda_2 + n\lambda_3 u \wedge ds + n\zeta^a (\vec{e}_a \rfloor \mathcal{S}) \wedge u \ . \end{aligned} \tag{2.2}$$

In (2.2), n, s, ψ, $\bar{\psi}$, and u are regarded as independent variables describing the perfect fluid dynamics. In doing so, one should take into account that, in view of (2.1), the dependence of the internal energy on ψ and $\bar{\psi}$ is realized only through its dependence on the spin tensor and color charge: $\varepsilon = \varepsilon(n,s,\bar{\psi} M_{ab}\psi, \bar{\psi} I_m \psi)$. The constraints imposed on the independent variables are taken into account by using the undefined Lagrange multipliers λ_1, λ_2, λ_3 and ζ^a. The second term in (2.2) is a kinetic term in which D designates the operation of exterior covariant differentiation with respect to both gauge groups [the Lorentz and $SU(3)$ color groups]. The term with coupling constant χ in (2.2), in which $\mathcal{F}^m = (1/2) F^m{}_{ab} \theta^a \wedge \theta^b$ designates the two-form of strength of the non-Abelian gauge color field, describes the possible spin polarization chromomagnetic effects.

On variation of (2.2) over the undefined Lagrange multipliers, we obtain the corresponding constraints, and on variation over the dynamic variables describing the perfect fluid, we obtain the variational equations of motion of the perfect fluid with non-Abelian color charge. Using these equations, together with the values of the undefined Lagrange multipliers determined by means of these equations, it is possible to show that the four-form of Lagrangian density (2.2) is proportional to the hydrodynamic pressure in the fluid $\mathcal{L}_{fluid} = p\eta$. This indicates that the variational theory of perfect spin fluid with internal color charge transforms into the variational theory of a conventional perfect fluid as a result of correct passage to the limit.

We use the equations of motion and the identity $DM_{ab} = 0$ to deduce the law that governs changes of the spin tensor of the fluid particle:

$$u \wedge DS_{ab} + \frac{2}{c^2} S_{[a}{}^{c} u_{b]} \dot{u}_c \eta = -2 S_{[a}{}^{f} \Pi^c_{b]} \Pi^d_f (\chi F^m{}_{cd} J_m + \omega_{cd}) \eta \, . \tag{2.3}$$

Here, the projection tensor is designated by $\Pi^c_b = \delta^c_b + c^{-2} u^c u_b$ and the Newtonian dot signifies the differentiation that is defined for an arbitrary object Φ by the equality $\dot{\Phi}^a{}_b = *(u \wedge D\Phi^a{}_b)$. With the zero right-hand side, the law governing changes of the spin tensor (2.3) takes the form of the corresponding Weyssenhoff–Raabe law for the perfect spin fluid.

To derive the equations for the non-Abelian gauge color field, we should add the four-form of the color-field Lagrangian density to the four-form of Lagrangian density (2.2):

$$\mathcal{L}_{matter} = \mathcal{L}_{fluid} + \mathcal{L}_{field} \, , \quad \mathcal{L}_{field} = -\frac{\alpha}{2} \mathcal{F}^m \wedge *\mathcal{F}_m \, . \tag{2.4}$$

Here, α is the coupling constant. Lifting and lowering the indices like m at the gauge field, which are transformed by an adjoint representation of the $SU(3)$ color group, are effected with the metric tensor $g_{mn} = -(1/2) c_m{}^p{}_q c_n{}^q{}_p$, where $c_m{}^p{}_q$ are the structure constants of the $SU(3)$ group.

The equations for the field are obtained by variation of (2.4) over the one-form A^m of potential of the gauge color field:

$$D(\alpha *\mathcal{F}_m + \chi n J_m *\mathcal{S}) = n J_m u \, . \tag{2.5}$$

This equation must be supplemented with the Bianchi identity for a non-Abelian gauge field $D\mathcal{F}^m = 0$.

Using the equation of motion for the fluid and identity $DI_m = 0$, we derive the law that governs changes of the non-Abelian color charge:

$$u \wedge DJ_m = -c_m{}^p{}_n J_p (\chi \mathcal{F}^n \wedge *\mathcal{S} + \omega^n \eta) \, . \tag{2.6}$$

The derived classical equations of motion for the non-Abelian color field, which take into account spin and color degrees of freedom, form a self-consistent set. Equation (2.6) shows that, in the case of a non-Abelian field, the conservation of color charge along a fluid flow line may be violated because of the spin-chromomagnetic interaction. It is notable that, in the case of an Abelian field, the conservation of electric charge along a fluid flow line always takes place, since in that case $c_m{}^p{}_n = 0$.

According to the general principles of the field theory for the Riemann–Cartan space geometric properties of space-time are defined by the canonical energy-momentum tensor and the spin moment tensor of the matter. In the exterior form formalism, the role of the latter tensor is played by the three-form of spin moment $\Delta^a{}_b$, which is defined as a variational derivative of (2.4) with respect to the connection one-form $\Gamma^b{}_a$ of a Riemann–Cartan space U_4,

$$\Delta^a_b = -\frac{\delta \mathcal{L}_{matter}}{\delta \Gamma^b{}_a} = \frac{1}{2} n S^a{}_b u \tag{2.7}$$

The role of the canonical energy-momentum tensor is played by the energy-momentum three-form Σ_a, which is defined as a variational derivative of the Lagrangian density four-form (2.4) with respect to the basis one-form θ^a. On calculating Σ_a for the Lagrangian density (2.4) with the use of the equations of motion of the fluid and the expressions for the undefined Lagrangian multipliers, we arrive at

$$\Sigma_a = -\frac{\delta \mathcal{L}_{matter}}{\delta \theta^a} = \Sigma^{fluid}_a + \Sigma^{field}_a \, , \tag{2.8}$$

$$\Sigma^{fluid}_a = p\eta_a + n\left(\pi_a + \frac{p}{nc^2} u_a\right) u + \chi n (\vec{e}_a \rfloor \mathcal{F}^m J_m) \wedge *\mathcal{S} \, , \tag{2.9}$$

$$\begin{aligned} \Sigma^{field}_a &= \alpha \left(F^m{}_{ac} F_m{}^{bc} - \frac{1}{4} \delta^b_a F^m{}_{cd} F_m{}^{cd} \right) \eta_b = \\ &= \frac{\alpha}{2} \left((\vec{e}_a \rfloor \mathcal{F}^m) \wedge *\mathcal{F}_m - \mathcal{F}^m \wedge (\vec{e}_a \rfloor *\mathcal{F}_m) \right) \, . \end{aligned} \tag{2.10}$$

In these expressions, the dynamic momentum per particle in a fluid element is introduced:

$$\pi_a \eta = -\frac{1}{nc^2} *u \wedge \Sigma^{fluid}_a \, , \quad \pi_a = \frac{1}{c^2} \varepsilon^* u_a - \frac{1}{c^2} S_a{}^c \left(\dot{u}_c + u^b (\chi J_m F^m{}_{bc} + \omega_{bc}) \right) \, , \tag{2.11}$$

Here ε^* denotes the effective energy per fluid particle, $\varepsilon^*\eta = \varepsilon_o\eta + \chi J_m\mathcal{F}^m \wedge *\mathcal{S}$, where $\varepsilon_o = \varepsilon/n$. This expression generalizes the corresponding result obtained for the case of an electromagnetic field [9] to the non-Abelian case. We see that the role of the fluid pressure p is twofold: first, it prevents the compression of the fluid [the first term in (2.9)], and, second, it effectively increases the mass of the fluid element [the second term in (2.9)].

3. Equation of motion of a particle with spin and color charge

It is well known in the theory of electromagnetic fields that the equations of motion of charged particles follow from the energy-momentum conservation law of the system together with the field equations, and, in turn, the energy-momentum conservation law is a consequence of the invariant properties of the system. In the field theory in U_4 space, a similar statement is valid, but the laws of conservation have a more complicated form [10]:

$$D\Sigma_a = (\vec{e}_a \rfloor \mathcal{T}^b) \wedge \Sigma_b - (\vec{e}_a \rfloor \mathcal{R}^c{}_b) \wedge \Delta^b{}_c\ , \qquad D\Delta_{ab} = -\theta_{[a} \wedge \Sigma_{b]}\ , \tag{3.1}$$

Here, $\mathcal{T}^b$ is the torsion two-form and $\mathcal{R}^c{}_b$ is the curvature two-form of U_4 space. The first of these equations is a consequence of invariance of the system's Lagrangian under spacetime diffeomorphisms, and the second equation is a consequence of local Lorentz invariance and is a generalization of the law of conservation of the spin momentum. Both of these equations become identities if and only if the equations of motion of the system in U_4 space hold. Thus, these equations are in fact just another representation of the laws of motion.

We use the first of equations (3.1) to derive the equation of motion of the perfect spin fluid with color charge in the form generalizing the Euler hydrodynamic equation. Here we use the Lie derivative $\pounds_{\vec{v}}$ generalized to arbitrary local gauge groups [including the $SU(3)$ color group]; that is,

$$D \circ i_{\vec{v}} = \pounds_{\vec{v}} - i_{\vec{v}} \circ D\ , \tag{3.2}$$

where $i_{\vec{v}}$ is the operator of contraction with a vector $\vec{v}$. The calculations must rely on the continuity equation $d(nu) = 0$; on the relation

$$D(\chi n J_m *\mathcal{S}) = nJ_m u - D(\alpha *\mathcal{F}_m)\ , \tag{3.3}$$

which follows from the equation for the color gauge field (2.5); and on the equations

$$D(\vec{e}_a \rfloor \eta) = D\eta_a = (\vec{e}_a \rfloor \mathcal{T}^b) \wedge \eta_b\ . \tag{3.4}$$

which are valid in U_4 space.

In the calculations, there arises the differential operator

$$\pounds_{\vec{e}_a}\mathcal{F}^m - (\vec{e}_a \rfloor \mathcal{T}^b) \wedge (\vec{e}_b \rfloor \mathcal{F}^m) = (\vec{e}_a \rfloor \nabla)\mathcal{F}^m\ , \tag{3.5}$$

which represents the total covariant derivative of tensor-valued forms and which takes into account all the types of indices [spinorial and tensorial indices associated with the representations of the Lorentz and $SU(3)$ groups, as well as indices associated with the space of p-forms]. It can be shown that the operator in (3.5) satisfies the condition

$$*(\vec{e}_a \rfloor \nabla)\mathcal{F}^m = (\vec{e}_a \rfloor \nabla) *\mathcal{F}^m\ . \tag{3.6}$$

Substituting (2.7)–(2.11) into (3.1) and using equations (3.2)–(3.6), we arrive at the equation of motion for the fluid in the form

$$\begin{aligned} u \wedge D\left(\pi_a + \frac{p}{nc^2}u_a\right) &= \frac{1}{n}\eta(\vec{e}_a \rfloor \nabla)p - (\vec{e}_a \rfloor \mathcal{T}^b) \wedge \left(\pi_b + \frac{p}{nc^2}u_b\right)u - \\ &- \frac{1}{2}(\vec{e}_a \rfloor \mathcal{R}^{bc}) \wedge S_{bc}u - (\vec{e}_a \rfloor \mathcal{F}^m) \wedge J_m u + \chi(\vec{e}_a \rfloor \nabla)\mathcal{F}^m \wedge J_m *\mathcal{S}\ , \end{aligned} \tag{3.7}$$

which represents a generalization of the well-known hydrodynamic Euler equation to the case of a perfect spin fluid with a color charge.

By going over to the limit of zero pressure in equation (3.7), we find that, in U_4 space, the equation of motion of a particle with a spin and a color charge in an external non-Abelian color gauge field has the form

$$u \wedge D\pi_a = -(\vec{e}_a \rfloor \mathcal{F}^m) \wedge J_m u + \chi(\vec{e}_a \rfloor \nabla)\mathcal{F}^m \wedge J_m *\mathcal{S} - \\ -\frac{1}{2}(\vec{e}_a \rfloor \mathcal{R}^{bc}) \wedge S_{bc} u - (\vec{e}_a \rfloor \mathcal{T}^b) \wedge \pi_b \ . \tag{3.8}$$

The first term on the right-hand side of this equation is a generalization of the Lorentz force to the case of a non-Abelian gauge field. The second term is a chromomagnetic analog of the Stern–Gerlach force acting on a magnetic moment in an electromagnetic field (this force is generated by the additional potential energy of a magnetic moment in a magnetic field [11]). The third term on the right-hand side of equation (3.8) represents the Mathisson force arising from the interaction of the particle spin with the curvature of space, while the fourth term is the so-called translational force, which is due to the interaction of the particle dynamical momentum with the torsion of space. The emergence of this force is peculiar to U_4 space.

Equation (3.8) must be supplemented with the law that governs spin evolution,

$$u \wedge DS_{ab} = 2\pi_{[a}u_{b]}\eta - 2S_{[a}^{\ \ c}(\chi\eta_{b]c} \wedge \mathcal{F}^m J_m + \omega_{b]c}\eta) \ , \tag{3.9}$$

which follows from equations (2.3), (2.11), and with the law that governs the evolution of the particle color charge (2.6).

Equations (3.8), (3.9), (2.6) describe the motion of a particle with a spin and a color charge in the presence of spin-chromomagnetic interaction. They generalize the well-known Wong equations [8] to the case of the $SU(3)$ color group and take into account the particle spin, which may be responsible for the possible interaction between the spin and the chromomagnetic component of the color field and for the additional effect of the gravitational field associated with the geometry of Riemann–Cartan space on the motion of the particle being considered.

4. Evolution of the particle spin in a color field in a Riemann–Cartan space

The particle-spin vector (Tamm–Pauli–Lyubanski vector) is defined as

$$\sigma^a = \frac{1}{2c^2}\eta^{abcd}S_{bc}u_d \ , \qquad \sigma = \sqrt{\sigma^a\sigma_a} \tag{4.1}$$

where η^{abcd} are the components of the Levi-Civita antisymmetric tensor (4-form of volume η). In Minkowski spacetime, the evolution of this vector in a slowly varying external electromagnetic field is governed by the Bargmann–Michel–Telegdi equation [12]. However, this equation takes no account of the effect of the spin on the trajectory of the particle. A more precise equation that describes the motion of the particle spin in a non-uniform electromagnetic field and which takes into account the effect of the spin on the motion of the particle was derived by Good who extended the Tamm equation [13], [14], [11].

By using equations (3.8) and (3.9), which describe the motion of a particle with a spin and a color charge, we will now extend the Tamm–Good and Bargmann–Michel–Telegdi equations to the case of the motion of such a particle in an external color (generally nonuniform) field in U_4 Riemann–Cartan space [7]. To this end, we recast the dynamical momentum of the particle [it is given by (2.11)] into the form

$$\pi_a = m^* u_a - \lambda_a \ , \qquad m^* = \frac{\varepsilon^*}{c^2} \ , \tag{4.2}$$

$$\lambda_a = \frac{1}{c^2}S_a^{\ c}w_c \ , \qquad w_c = \dot{u}_c + u^b(\chi J_m F^m{}_{bc} + \omega_{bc}) \ . \tag{4.3}$$

Using definitions (4.1) and (4.3) and considering that the spin tensor, which is the inverse of that in (4.1), is given by

$$S_{ab} = \frac{1}{c}\eta_{abcd}\sigma^c u^d \ , \tag{4.4}$$

we can easily prove that the vectors u^a, σ^a, and λ^a are mutually orthogonal.

Differentiating the spin vector (4.1), we arrive at

$$u \wedge D\sigma^a = \frac{1}{m^* c^2} u^a \sigma^b u \wedge D(\pi_b - \lambda_b) + \sigma^b \Pi^{ac}(\chi \eta_{bc} \wedge \mathcal{F}^m J_m + \omega_{bc}\eta) , \tag{4.5}$$

where $\Pi^{ac} = g^{ac} + (1/2c^2)u^a u^c$ is the projection tensor. To derive this relation, we used equation (3.9), which governs the evolution of the spin tensor; the Frenkel condition $S_{ab}u^b = 0$; expression (4.4) for the spin tensor in terms of the spin vector; the prescription $\eta_{abcd}\eta^{ijkd} = -6\delta^i_{[a}\delta^j_b\delta^k_{c]}$ for contracting indices of the Levi–Civita tensor; and the result obtained by differentiating expression (4.2) for the dynamical momentum,

$$m^* u \wedge Du_a = u \wedge D(\pi_a + \lambda_a) + u_a u \wedge Dm^* . \tag{4.6}$$

Differentiating expression (4.3) for λ_a and taking into account equation (3.9) and the orthogonality conditions $\sigma^a u_a = 0$, $\sigma^a \pi_a = 0$ and $\sigma^a S_{ab} = 0$, we obtain

$$\sigma^a u \wedge D\lambda_a = \lambda^a \sigma^b (\chi \eta_{ab} \wedge \mathcal{F}^m J_m + \omega_{ab}\eta) . \tag{4.7}$$

Substituting this equation and the equation (3.8) of the motion of a particle into (4.5) and taking into account equation (4.2), we arrive at

$$\begin{aligned} &u \wedge D\sigma^a = -\sigma^b(\chi \eta^a{}_b \wedge \mathcal{F}^m J_m + \omega^a{}_b \eta) - \\ &-\frac{u^a \sigma^b}{m^* c^2}[(\vec{e}_b \rfloor \mathcal{F}^m) \wedge J_m u - \pi^c(\chi \eta_{bc} \wedge \mathcal{F}^m J_m + \omega_{bc}\eta) - \\ &-\chi(\vec{e}_b \rfloor \nabla)\mathcal{F}^m \wedge J_m *\mathcal{S} + \frac{1}{2}(\vec{e}_b \rfloor \mathcal{R}^{cd}) \wedge S_{cd} u + (\vec{e}_b \rfloor \mathcal{T}^c) \wedge \pi_c u] . \end{aligned} \tag{4.8}$$

Equation (4.8) represents the sought extension of the Tamm–Good and Bargmann–Michel–Telegdi equations to the case of a particle having a spin and a color charge and moving in an external nonuniform non-Abelian color field in a Riemann–Cartan space.

5. Conclusion

On the basis of the method of Cartan exterior forms and the variational approach, a relativistic variational theory of the perfect spin fluid with color charge in an external non-Abelian Yang–Mills field was derived in a Riemann–Cartan space U_4 with curvature and torsion, which takes into account the spin polarization chromomagnetic effects. We have given a consistent derivation of the classical energy-momentum tensor for a perfect fluid having intrinsic spin and color degrees of freedom and interacting with a non-Abelian color gauge field.

Then on the basis of the generalized Lie derivatives, a hydrodynamic equation of the Euler type that describes the motion of an element of a perfect spin fluid with a color charge in a non-Abelian color gauge field associated with the $SU(3)$ group and a gravitational field described by the geometry of a Riemann–Cartan space with curvature and torsion has been derived from the law of energy-momentum quasiconservation. The "fluid + field" system being considered is closed because the gravitational field is generated by the energy-momentum tensor of the fluid and because the color field is induced by a current of the non-Abelian color charge of the fluid.

The constructed theory may form the basis of the model that hydrodynamically describes the quark-gluon plasma and differs from the Landau model in that it takes into account in an explicit form the structure of quantum-chromodynamics vacuum and color charge of quarks [6]. Moreover, on the basis of the theory of perfect dilation-spin fluid developed in [15], [16] this model can be generalized to the case of a fluid possessing the dilation charge, apart from the spin and the color charge, and thus provide the foundation for studies of the non-perturbed gluon condensate up to the phase transition from hadron matter to the quark-gluon plasma.

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